

$$\chi_{cJ} \rightarrow K^*(892)\bar{K}$$

decays within the
effective theory framework

Nikolay Kivel

PNPI, St. PETERSBURG, Russia

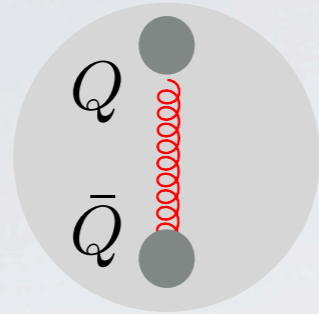
based on

N. Kivel, Eur. Phys. A 54, 2018

Motivation

Why $\chi_{cJ} \rightarrow K^*(892)\bar{K}$ decays ?

$\chi_{cJ}(1P)$



J=0 M=3.42 GeV

J=1 M=3.51 GeV

J=2 M=3.56 GeV

$$X_J(c\bar{c}) \leftrightarrow (n_r + 1)^{(2S+1)} L_J$$

n_r "radial" quantum #

$\vec{J} = \vec{L} + \vec{S}$ ang. mom.

$\vec{S} = \vec{s}_c + \vec{s}_{\bar{c}}$ spin of $c\bar{c}$

$$n_r = 0$$

L=0 $2S+1S_J$ $\eta_c(^1S_0)$ $J/\Psi(^3S_1)$

L=1 $2S+1P_J$ $h_c(^1P_1)$ $\chi_{cJ}(^3P_J)$

Why $\chi_{cJ} \rightarrow K^*(892)\bar{K}$ decays ?



$$J^{PC} = J^{++}$$

$$J=1$$

$$M=3.51 \text{ GeV}$$

final state

$$K(498) \quad J^P = 0^-$$

$$K^*(892) \quad J^P = 1^-$$

amplitude

$$A[\chi_{cJ} \rightarrow K^* K] \sim m_s - m_q$$

is sensitive to
SU(3) breaking

3 amplitudes:

$$\chi_{c1} \rightarrow K K_{\parallel, \perp}^* \quad \chi_{c2} \rightarrow K K_{\perp}^*$$

QCD:

$$\frac{A[\chi_{cJ} \rightarrow K K_{\perp}^*]}{A[\chi_{c1} \rightarrow K K_{\parallel}^*]} \sim \frac{\Lambda}{m_c} \quad m_c \gg \Lambda$$

Why $\chi_{cJ} \rightarrow K^*(892)\bar{K}$ decays ?

BESII, PRD 74, 2006

BESIII, PRD 96, 2017

Branching ratios in units of 10^{-4}

$\chi_{cJ} \rightarrow VP$	$K^*(892)^0 \bar{K}^0 + c.c.$	$K^*(892)^+ \bar{K}^- + c.c.$
χ_{c1}	10 ± 4	15 ± 7
χ_{c2}	1.3 ± 0.28	1.5 ± 0.22

Theoretical description is based on the double expansion with respect to

- small velocity v of heavy quark ($m_Q \rightarrow \infty$): NRQCD & pNRQCD
- small ratio Λ/m_Q of heavy quark : collinear factorisation

There are many observed hadronic decay channels PP, PV, PT, VT
... and many theoretical challenges!

Very large effects beyond the leading order approximation!

Why $\chi_{cJ} \rightarrow K^*(892)\bar{K}$ decays ?

There are many observed hadronic decay channels PP, PV, PT, VT
... and many problems!

Very large effects beyond the leading order approximation!

charm mass is not large enough: $v_c^2 \simeq 0.3$

- large relativistic corrections
- large hadronic corrections

Very special mechanism related with the color-octet component of quarkonia wave function:

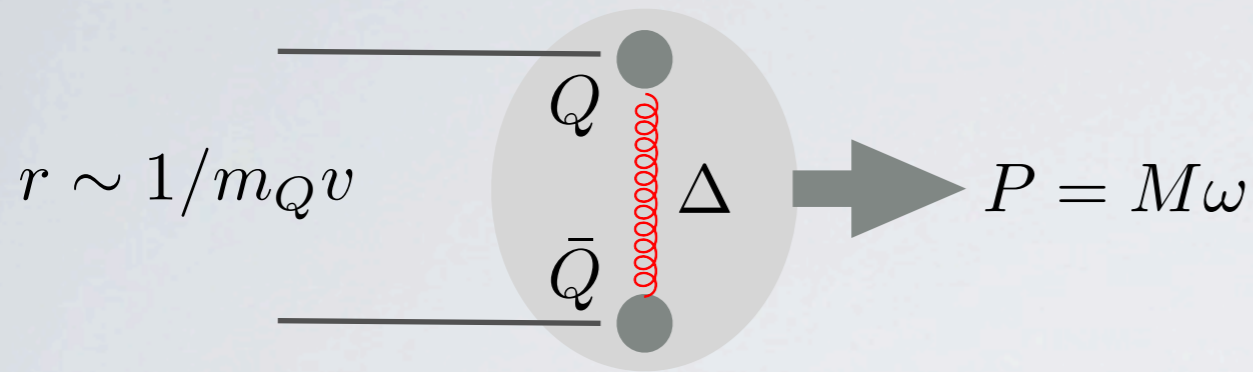
$$\Psi = c_0 |(Q\bar{Q})\rangle + c_8 |(Q\bar{Q})g\rangle + \dots$$

There are many speculations that octet component is especially important for the description of P-wave charmonia

however this mechanism has not been investigated within EFT framework ...

the first consideration for $B \rightarrow \chi_{cJ}K$ Beneke, Vernazza, NP B 2009

Heavy quark-antiquark states: brief introduction



rest frame $\omega = (1, \vec{0})$

relative momentum $\Delta = m_Q v$

$m_Q \gg \Lambda_{QCD}$ $m_Q \rightarrow \infty$ $v \ll c$

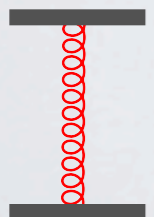
non-relativistic limit

kin. energy of heavy quark $E_{kin} = \vec{p}_Q^2 / 2m_Q \sim m_Q v^2$

virtuality $p_Q^2 - m_Q^2 \sim (m_Q v)^2$

Coulomb limit $m_Q \rightarrow \infty$ $m_Q v^2 \gg \Lambda_{QCD}$

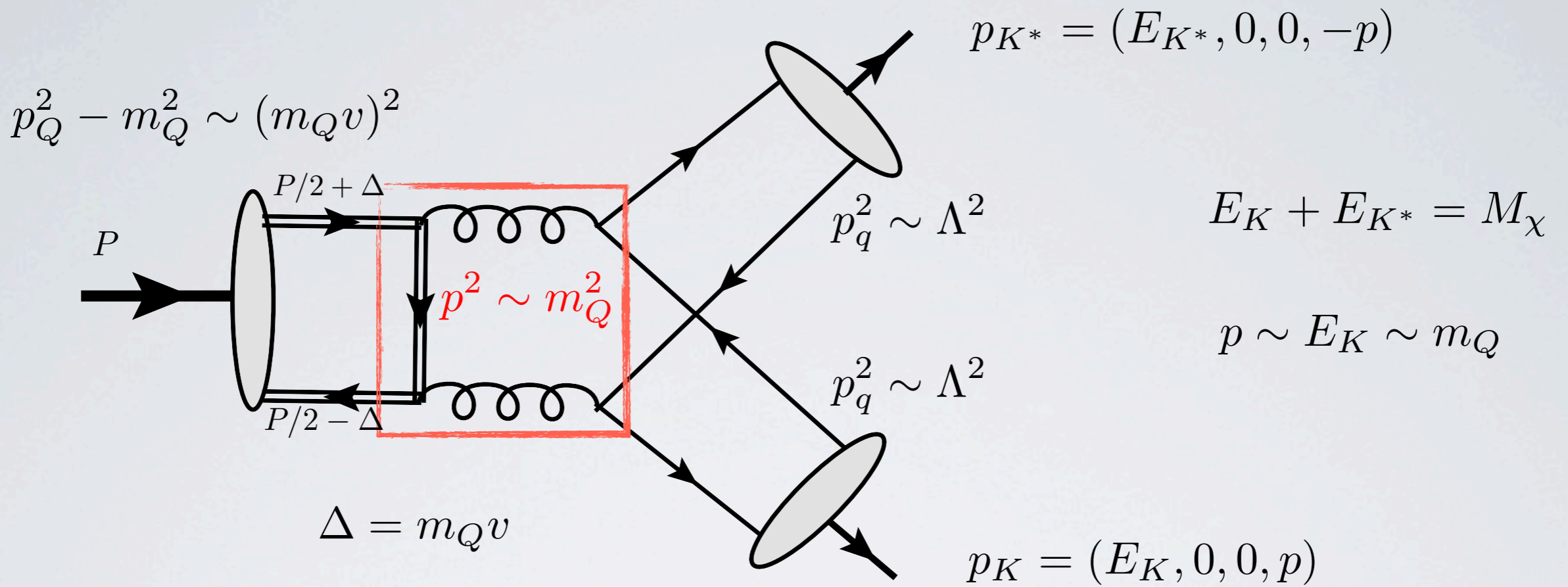
Coulomb binding energies



$$V_S = -\frac{4}{3} \frac{\alpha_s(mv)}{r}$$

$$\alpha_s(mv) \sim v$$

$$E_n = -\frac{4}{9} \frac{1}{n^2} m_Q \alpha_s^2 \sim m_Q v^2$$



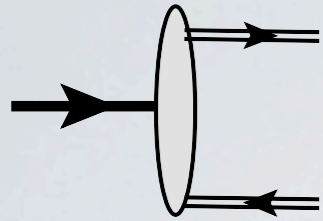
$$\begin{aligned}
 A[\chi_{cJ} \rightarrow K_{\parallel}^* K] &= \int d^3 \Delta \Psi_P^{(J)}(\vec{\Delta}) \mathbf{T}[c(\vec{\Delta}) \bar{c}(\vec{\Delta}) \rightarrow K_{\parallel}^* K] \\
 &\sim \int d^3 \Delta \Psi_P^{(J)}(\vec{\Delta}) \vec{\Delta} \mathbf{T}'[c(0) \bar{c}(0) \rightarrow K_{\parallel}^* K] \\
 &\sim R'_{21}(0) \mathbf{T}'[c(0) \bar{c}(0) \rightarrow K_{\parallel}^* K]
 \end{aligned}$$

$$\Delta \Psi_P^{(J)}(\vec{\Delta}) \sim \sum_m \tilde{R}_{21}(|\vec{\Delta}|) Y_{1m}(\Omega)$$

$$\int d^3 \Delta \Psi_P^{(J)}(\vec{\Delta}) = 0$$

Effective field theory: NRQCD + soft QCD

NRQCD (small v)



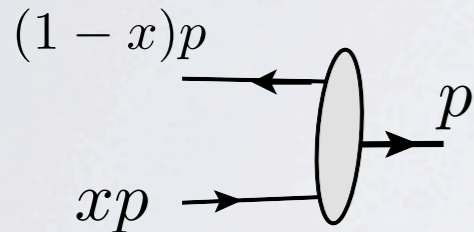
$$\langle 0 | \mathcal{O}(^3P_J) | \chi_{cJ} \rangle \sim \sqrt{M_\chi} R'_{21}(0)$$

Heavy Quark Spin Symmetry:

The matrix elements are not independent (up to v^2) corr's

soft QCD (Λ/m_Q)

DA can be related with the BS wave function



$$\phi_{K,K^*}(x, \mu) = \int_{|k_\perp| < \mu} d^2 k_\perp \Psi_{BS}(x, k_\perp)$$

$$f_K \phi_K(x) = \int d\lambda e^{i\lambda x(pn)} \langle K(p) | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle \quad n^2 = 0 \quad \text{light-like vector}$$

DA describes the momentum-fraction distribution of partons at zero transverse separation in a 2-particle Fock state

Decay amplitude $A[\chi_{c1} \rightarrow K_{\parallel}^* K]$

amplitude $A[\chi_{cJ} \rightarrow K^* K] \sim m_s - m_q$

QCD:

3 amplitudes:

$$\chi_{c1} \rightarrow K K_{\parallel,\perp}^* \quad \chi_{c2} \rightarrow K K_{\perp}^*$$

$$\frac{A[\chi_{cJ} \rightarrow K K_{\perp}^*]}{A[\chi_{c1} \rightarrow K K_{\parallel}^*]} \sim \frac{\Lambda}{m_c}$$

$$A[\chi_{c1} \rightarrow K_{\parallel}^* K] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \frac{f_V^{\parallel} f_P}{m_c^2} \alpha_s^2 \int_0^1 dx \frac{\phi_{K_{\parallel}^*}(x)}{x(1-x)} \int_0^1 dy \frac{\phi_K(y)}{y(1-x)} \frac{y-x}{xy + (1-x)(1-y)}$$

$$x \rightarrow 1-x : \phi(x) \rightarrow \phi(1-x) \Leftrightarrow q \rightarrow \bar{q}$$

$$\pi(q\bar{q}) \quad \phi(x) = \phi(1-x)$$

$$K(q\bar{s}) \quad \phi(x) \neq \phi(1-x) \quad \underline{\phi(x) - \phi(1-x) \sim m_s - m_q}$$

Power counting in EFT:

$$R'_{21}(0) \sim v^4 \quad \frac{f_V^{\parallel} f_P}{m_c^2} \sim \frac{\Lambda^2}{m_c^2} \quad \underline{A[\chi_{c1} \rightarrow K_{\parallel}^* K] \sim v^4 (\Lambda/m_c)^2}$$

Branching ratio $Br[\chi_{c1} \rightarrow K_{\parallel}^* K]$

amplitude $A[\chi_{cJ} \rightarrow K^* K] \sim m_s - m_q$

$$A[\chi_{c1} \rightarrow K_{\parallel}^* K] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \frac{f_V^{\parallel} f_P}{m_c^2} \alpha_s^2 \int_0^1 dx \frac{\phi_{K_{\parallel}^*}(x)}{x(1-x)} \int_0^1 dy \frac{\phi_K(y)}{y(1-x)} \frac{y-x}{xy + (1-x)(1-y)}.$$

Estimates: $Br[\chi_{c1} \rightarrow K_{\parallel}^* K] \simeq (0.2-0.6) \times 10^{-4}$

experiment $Br[\chi_{c1} \rightarrow K_{\parallel}^* K] \simeq (10 \pm 4/15 \pm 7) \times 10^{-4}$

This calculation strongly underestimates the data.

Large effect from $\chi_{c1} \rightarrow K K_{\perp}^*$ notice that

$$Br[\chi_{c2} \rightarrow K_{\perp}^* K] \simeq (1.3 \pm 0.3/1.5 \pm 0.2) \times 10^{-4}$$

QCD: $\frac{A[\chi_{cJ} \rightarrow KK_{\perp}^*]}{A[\chi_{c1} \rightarrow KK_{\parallel}^*]} \sim \frac{\Lambda}{m_c}$

Decay amplitude $A[\chi_{c1} \rightarrow K_{\parallel}^* K]$

$$f_K \phi_K(x) = \int d\lambda e^{i\lambda x(pn)} \langle K(p) | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle$$

$$A[\chi_{cJ} \rightarrow KK_{\perp}^*] \sim \text{Tr}[\Gamma_{K_{\perp}^*} \gamma \Gamma_K \gamma]$$

$$\Gamma_{K_{\perp}^*} \otimes \Gamma_K$$

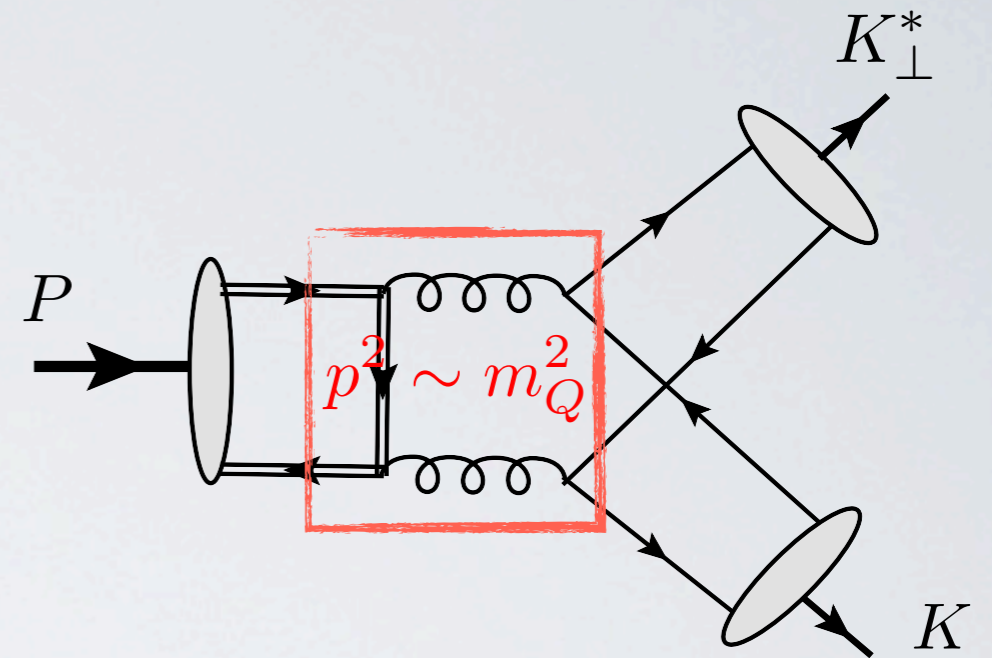
tw2 x tw2 $\sigma^{+\perp} \gamma_5 \otimes \gamma^- \gamma_5$ **vanishes**

tw2 x tw3 $\sigma^{+\perp} \gamma_5 \otimes \sigma^{+-} \gamma_5$ $\left| \langle K(p) | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle \sim f_K m_K^2 / (m_s + m_q) \right.$

tw2 x tw3 $\sigma^{+\perp} \gamma_5 \otimes \gamma_5$

chiral enhanced corrections

tw3 x tw2 $\gamma^{\perp} (\gamma_5) \otimes \gamma^- \gamma_5$ $f_K m_K$



Decay amplitude $A[\chi_{cJ} \rightarrow K_{\perp}^* K]$

$$A[\chi_{cJ} \rightarrow K K_{\perp}^*] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \frac{f_V^{\perp} f_P \mu_P}{m_c^3} \alpha_s^2 \int_0^1 dx \frac{\phi_{K_{\perp}^*}(x)}{x\bar{x}} \int_0^1 dy F(x, y)$$

$$R'_{21}(0) \sim v^4 \quad \frac{f_V^{\perp} f_P \mu_P}{m_c^3} \sim \frac{\Lambda^3}{m_c^3}$$

$$A[\chi_{cJ} \rightarrow K_{\perp}^* K] \sim v^4 (\Lambda/m_c)^3$$

$$\mu_P = m_K^2 / (m_s + m_q)$$

endpoint region $x \rightarrow 1, y \rightarrow 0$

$$\mu_P \int_{1-\delta}^1 dx \frac{\phi_{K_{\perp}^*}(x)}{x\bar{x}} \int_0^{\delta} dy F(x, y) \sim (m_s - m_q) \phi'_{K_{\perp}^*}(1) \int_{1-\delta}^1 dx \int_0^{\delta} dy \frac{1 + \ln y}{[y + (1-x)]^2}$$

$$\sim (m_s - m_q) \phi'_{K_{\perp}^*}(1) \int_0^{\delta} dy \frac{\ln y}{y}$$

same for region $x \rightarrow 0, y \rightarrow 1$

! $\delta \rightarrow 1$

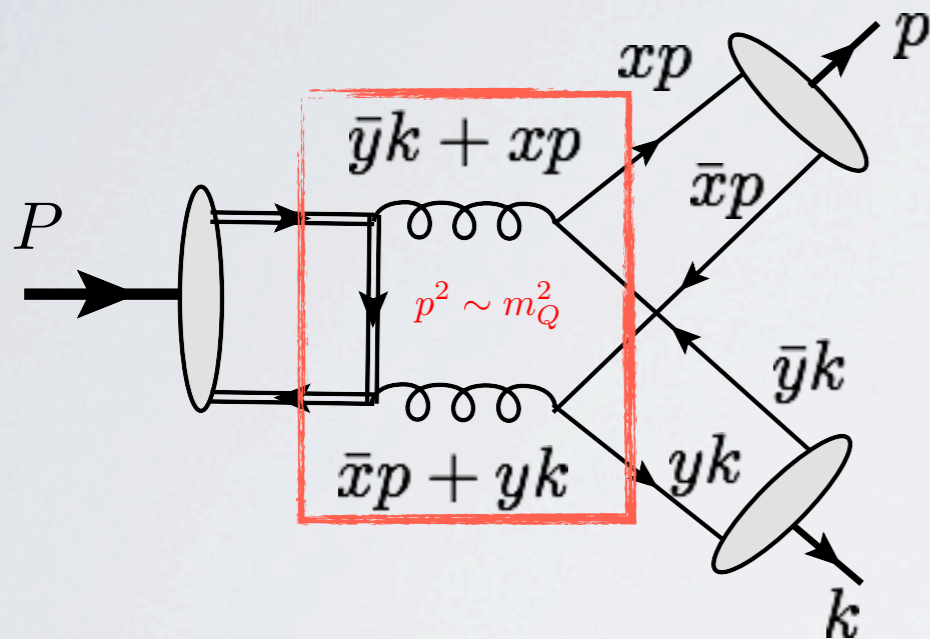
the integral is singular \Rightarrow the factorisation scheme is not well defined!

Endpoint contribution

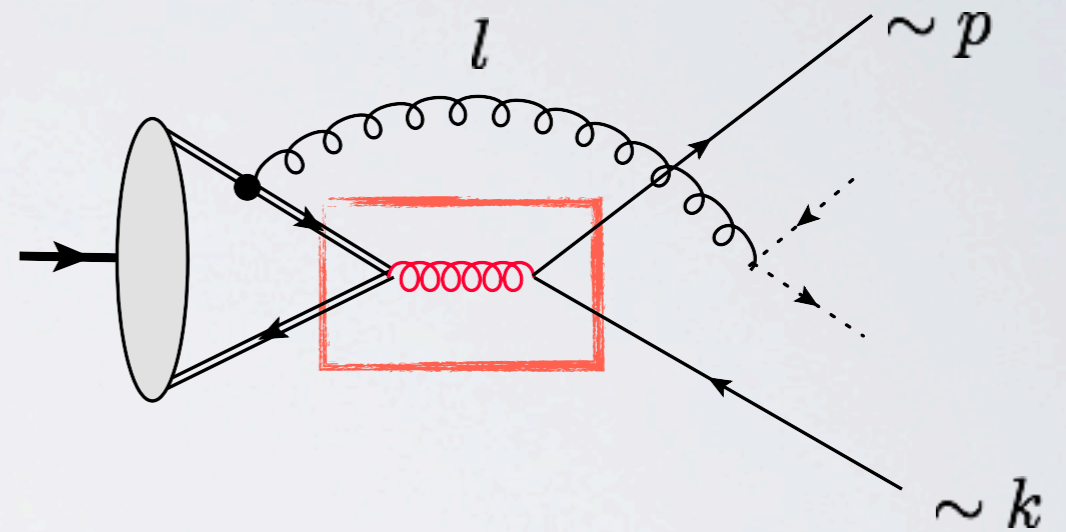
collinear region

$$0 \ll x \ll 1, 0 \ll y \ll 1$$

$$\bar{x} \equiv 1 - x$$



endpoint region $x \rightarrow 1, y \rightarrow 0$
 $x \rightarrow 0, y \rightarrow 1$



long distance exchange between 2 sectors:
 NRQCD & soft QCD

\mathbf{V} and Λ are well separated scales

NRQCD $l \sim m_Q v^2$

soft QCD $l \sim \Lambda$

$$v^2 \sim \Lambda/m_Q$$

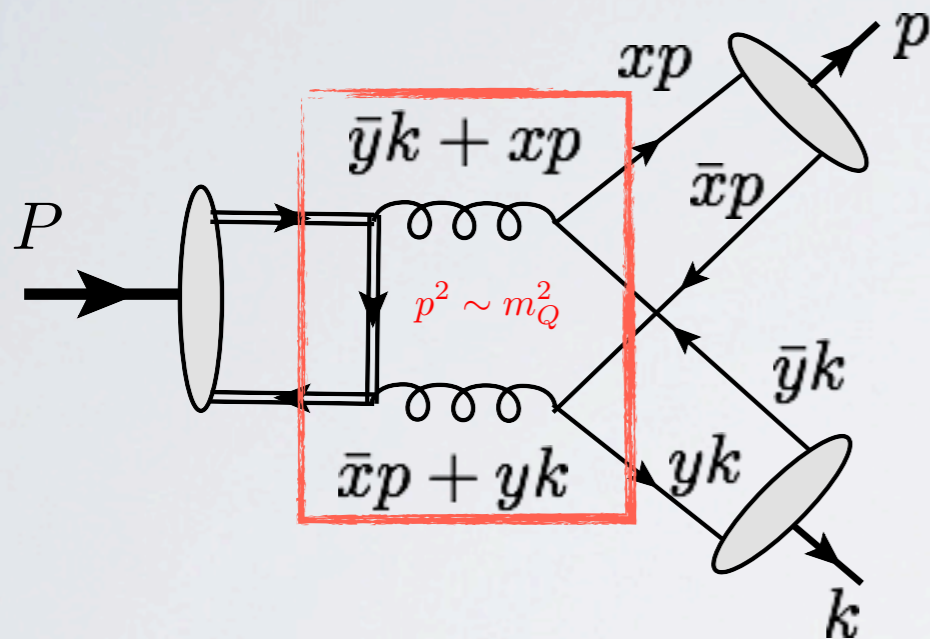
$$v_c^2 \simeq 0.3$$

Endpoint contribution

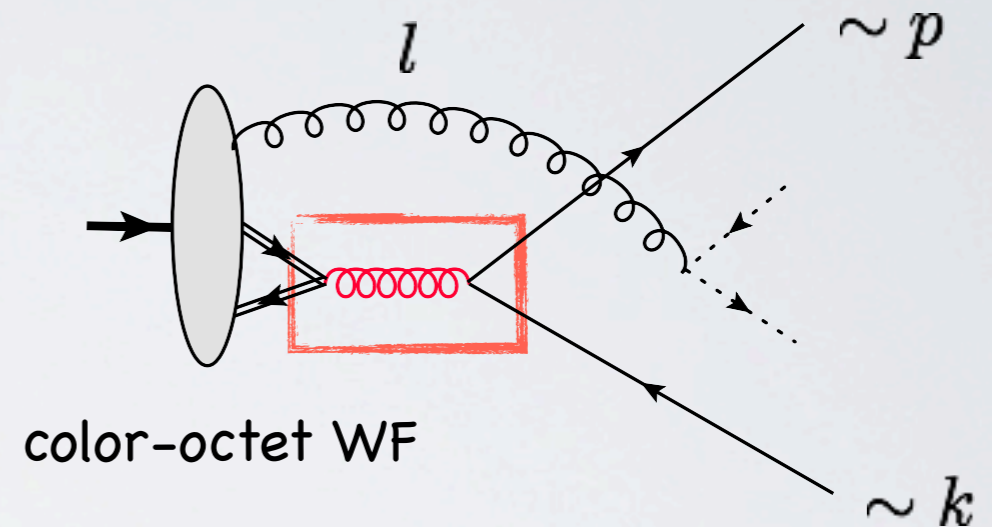
collinear region

$$0 \ll x \ll 1, 0 \ll y \ll 1$$

$$\bar{x} \equiv 1 - x$$



endpoint region $x \rightarrow 1, y \rightarrow 0$
 $x \rightarrow 0, y \rightarrow 1$



long distance exchange between 2 sectors:
 NRQCD & soft QCD

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NRQCD $l \sim m_Q v^2$

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$$v^2 \sim \Lambda/m_Q$$

$$v_c^2 \simeq 0.3$$

Decay amplitude $A[\chi_{cJ} \rightarrow K_{\perp}^* K]$

$$A[\chi_{cJ} \rightarrow K_{\perp}^* K] = \langle 0 | \mathcal{O}(^3P_J) | \chi_{cJ} \rangle \phi_{K^*}^{\perp} * \alpha_s^2 T_h * \phi_K \\ + \langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{cJ} \rangle$$

The soft-overlap contribution can be sufficiently large:

it is less suppressed by α_s

many indications from quarkonium phenomenology

QCD sum rules estimates ...

The IR-singularities in the “factorisable” contribution must be absorbed into renormalization of the “nonfactorisable” term.

Heavy Quark Spin Symmetry: does it work for the soft-overlap matrix elements?

Soft-overlap matrix element in the Coulomb limit

Coulomb limit $m_Q \rightarrow \infty$ $m_Q v^2 \gg \Lambda_{QCD}$

Coulomb binding energies

$$E_n = -\frac{4}{9} \frac{1}{n^2} m_Q \alpha_s^2 \sim m_Q v^2$$

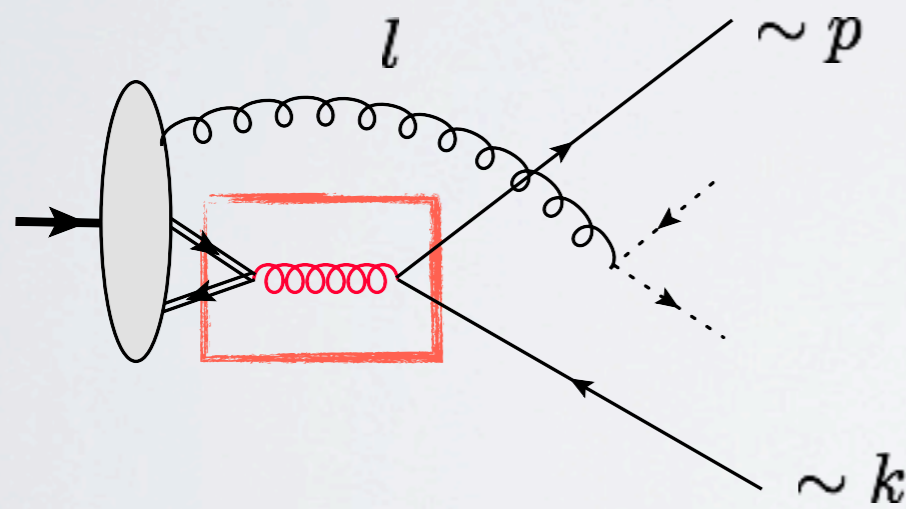


$$V_0 = -\frac{4}{3} \frac{\alpha_s(mv)}{r}$$

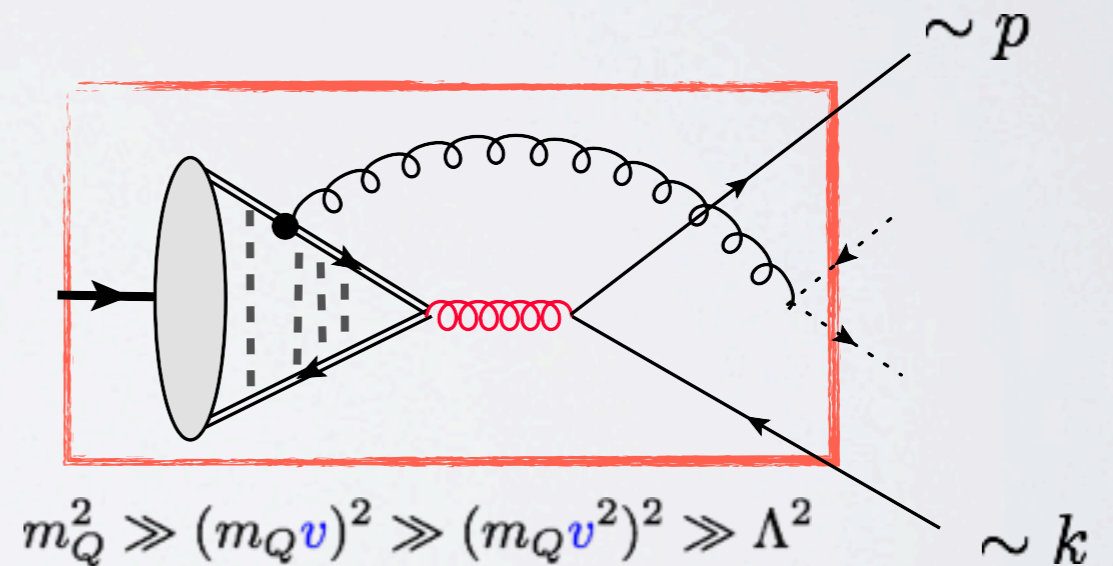
$$V_8 = \frac{1}{6} \frac{\alpha_s(mv)}{r}$$

$$\alpha_s(mv) \sim v$$

$$v^2 \sim \Lambda/m_Q$$



Coulomb: $v^2 \gg \Lambda/m_Q$



$$m_Q^2 \gg (m_Q v)^2 \gg (m_Q v^2)^2 \gg \Lambda^2$$

$$\langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{cJ} \rangle$$

“nonfactorisable”

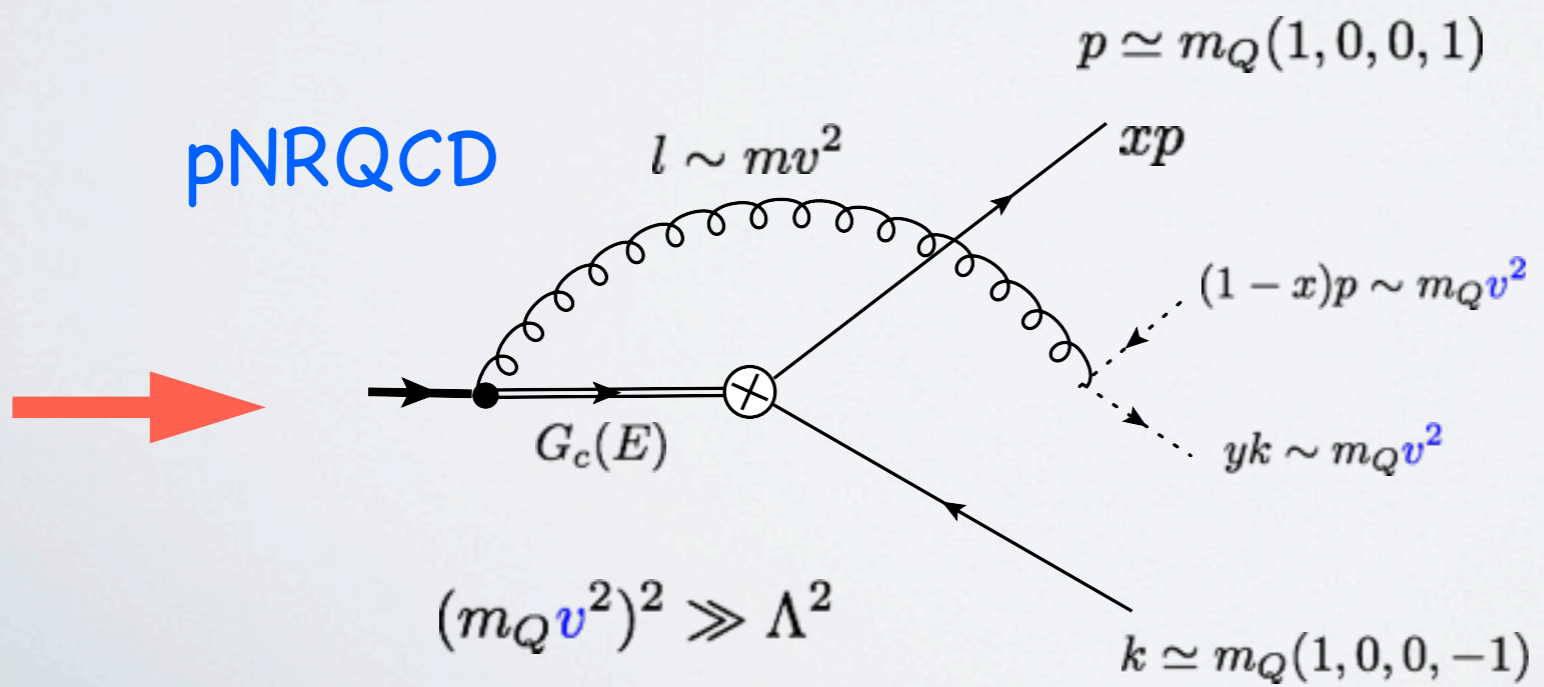
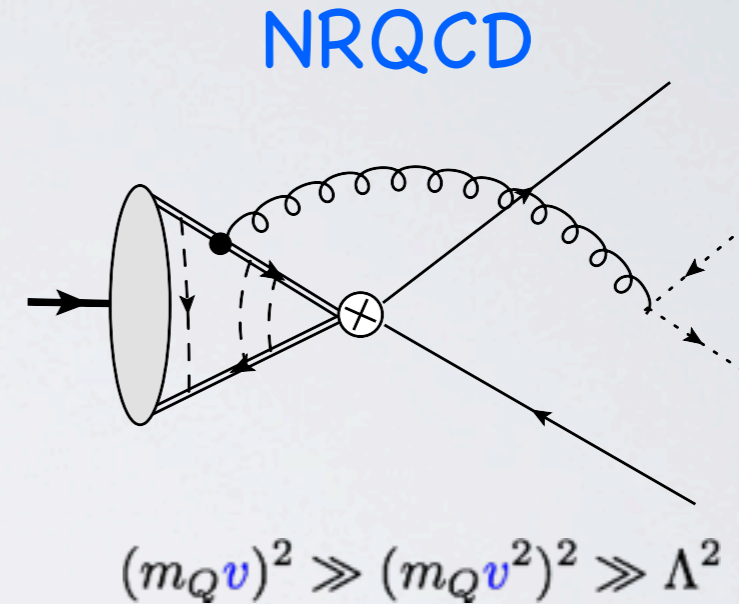
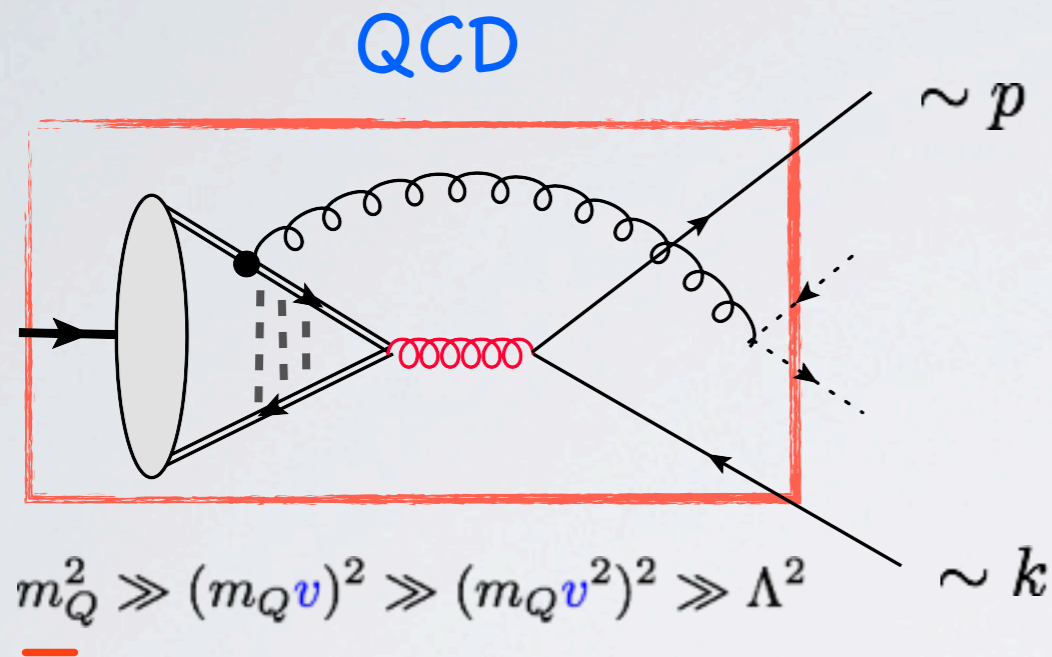
$$\langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{cJ} \rangle$$

“factorisable”

Soft-overlap matrix element in the Coulomb limit

Coulomb: $v^2 \gg \Lambda/m_Q$

$$\langle K_{\perp}^* K | \alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8 | \chi_{cJ} \rangle \sim v^4 (\Lambda/m_Q)^3$$



endpoint region

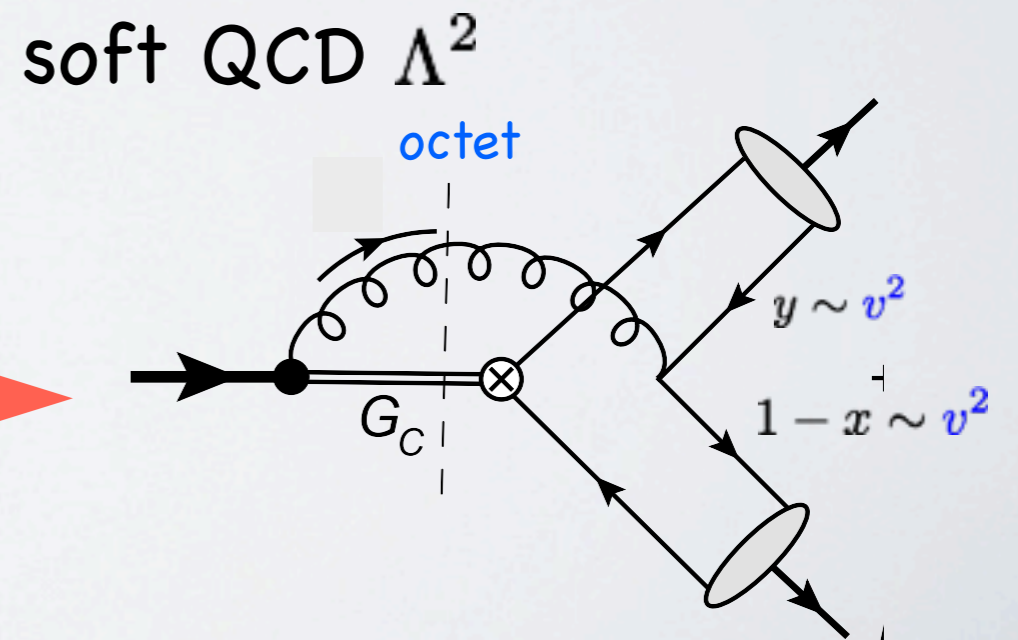
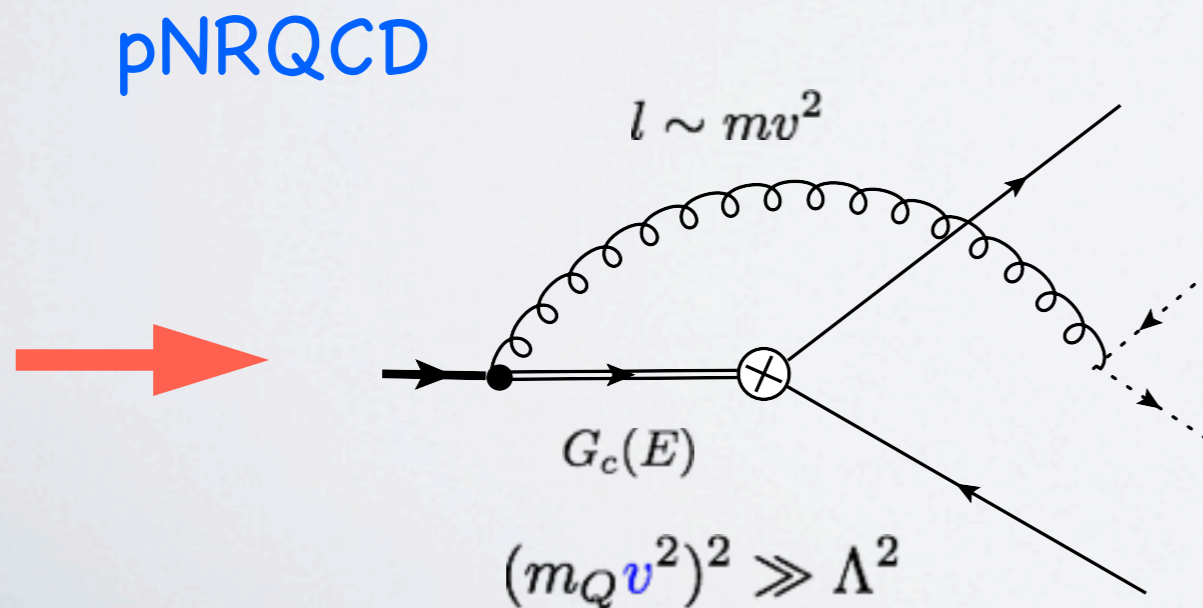
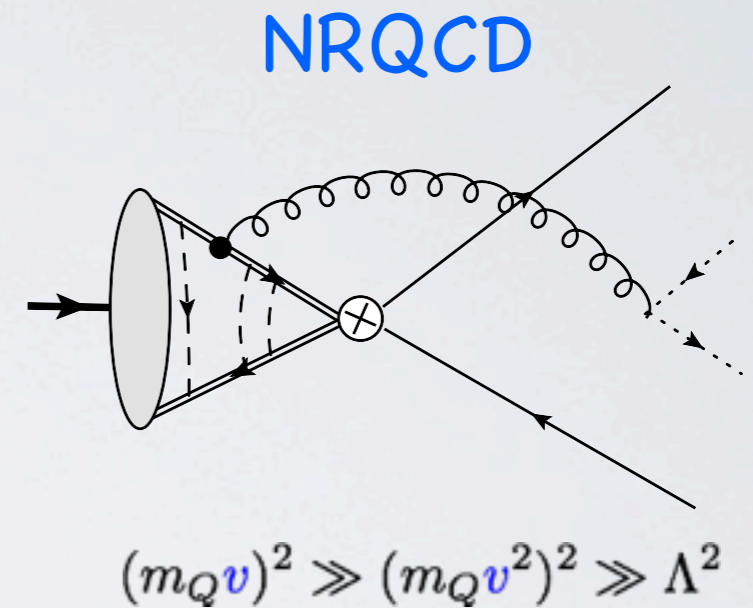
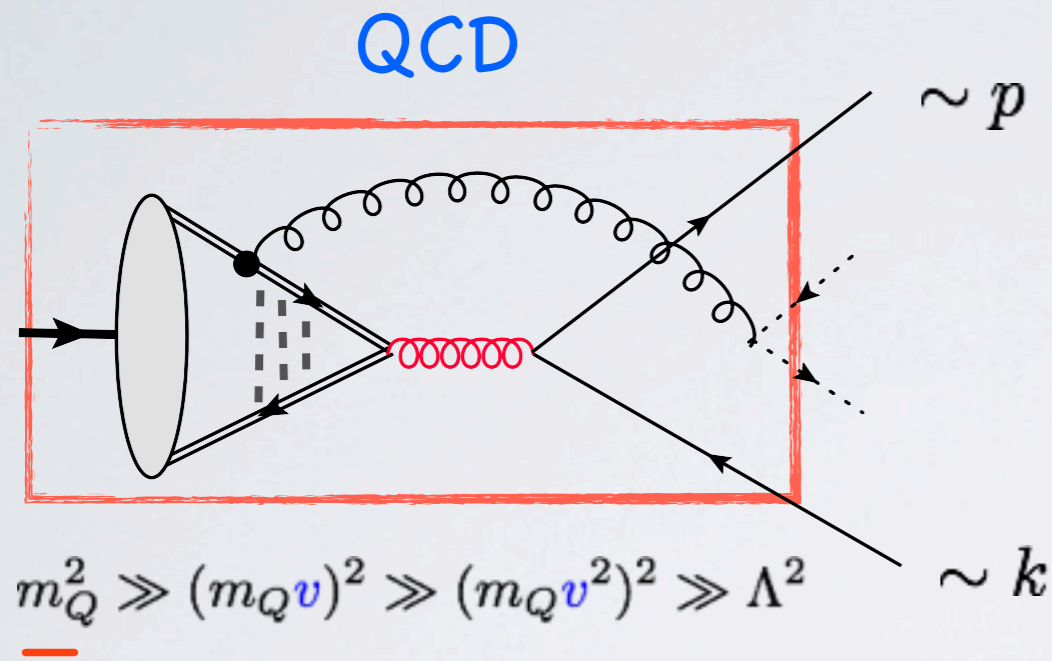
$$y \sim v^2$$

$$1-x \sim v^2$$

soft QCD can be described by DAs

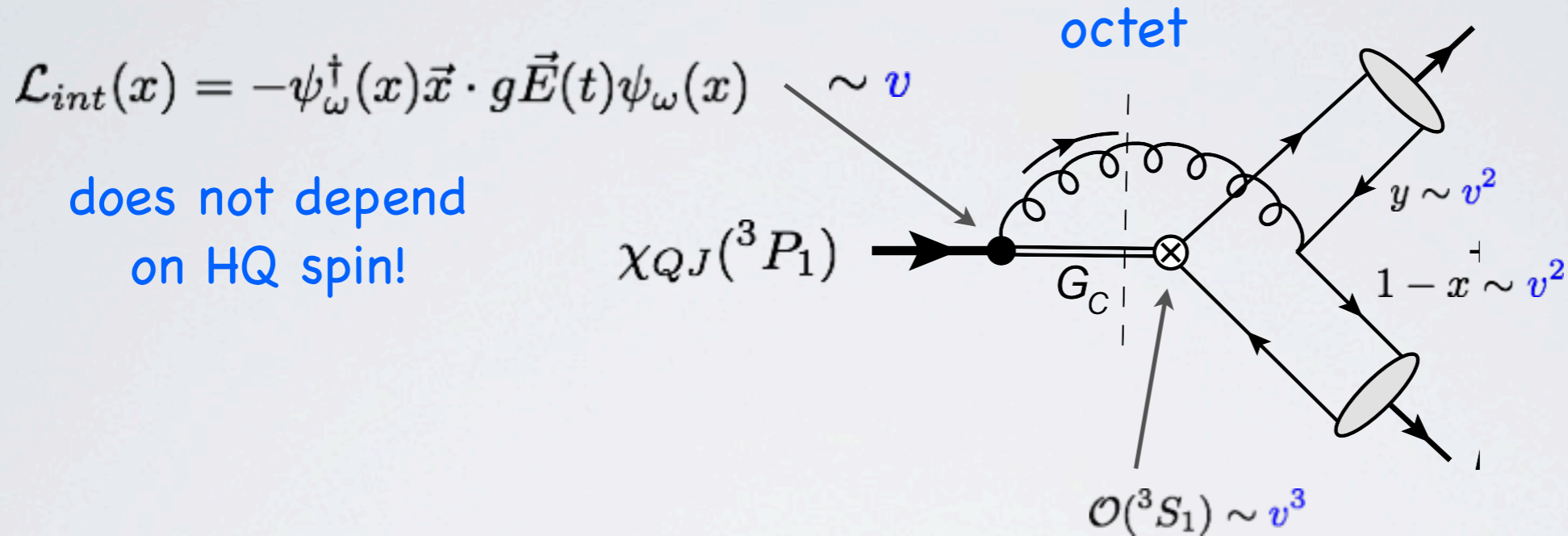
Soft-overlap matrix element in the Coulomb limit

Coulomb: $v^2 \gg \Lambda/m_Q$ $\langle K_{\perp}^* K | \alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8 | \chi_{cJ} \rangle \sim v^4 (\Lambda/m_Q)^3$



Soft-overlap matrix element in the Coulomb limit

$$\langle K_{\perp}^* K | \alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8 | \chi_{cJ} \rangle \sim v^4 (\Lambda/m_Q)^3$$



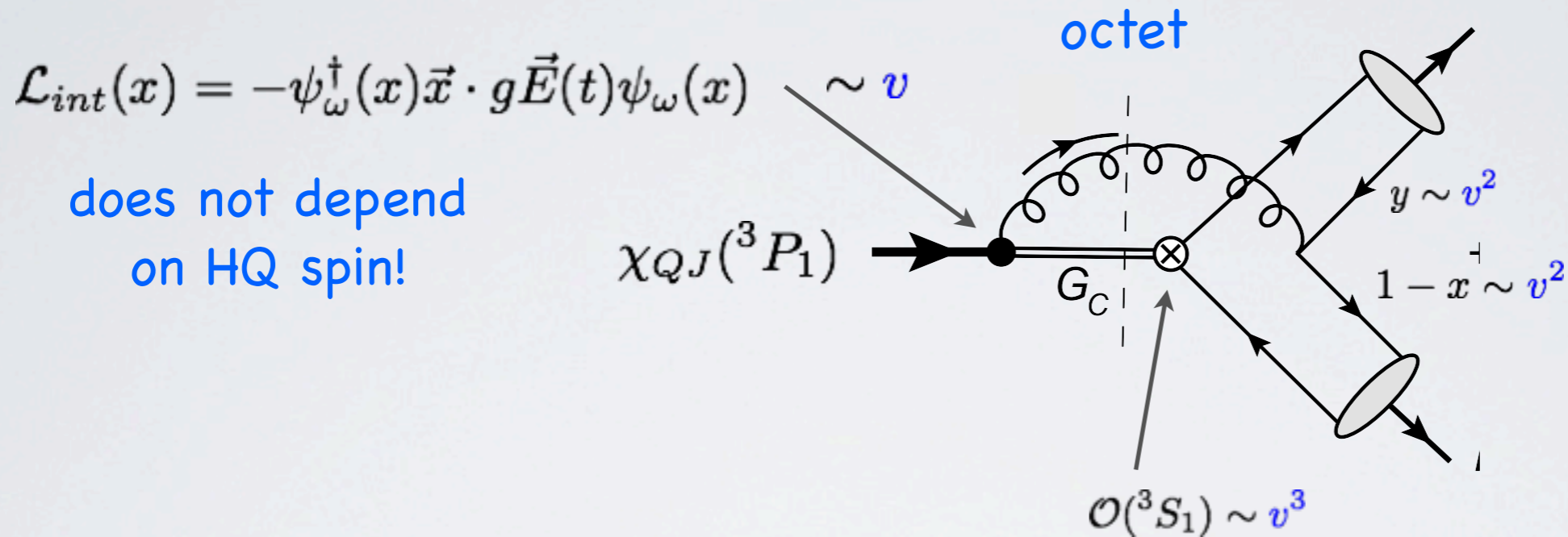
$$\langle K_{\perp}^* K | \dots | \chi_{cJ} \rangle \sim \alpha_s(m_Q v^2) \alpha_s(m_Q) \int d^3 \Delta \tilde{R}_{21}^c(\Delta) \text{tr} [\mathcal{P}_J \gamma_{\alpha}] V_{us}(\Delta)$$

$$\times \int_{1-\delta}^1 dx \int_0^{\delta} dy G_c(E - \Delta^2/m_Q, \bar{x}, y) [(1 + \ln y)(m_s - m_q) \phi'_{K_{\perp}^*}(1)]$$

$$\delta \rightarrow \infty \quad y \sim v^2 \quad 1 - x \sim v^2$$

Soft-overlap matrix element in the Coulomb limit

$$\langle K_{\perp}^* K | \alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8 | \chi_{cJ} \rangle \sim v^4 (\Lambda/m_Q)^3$$



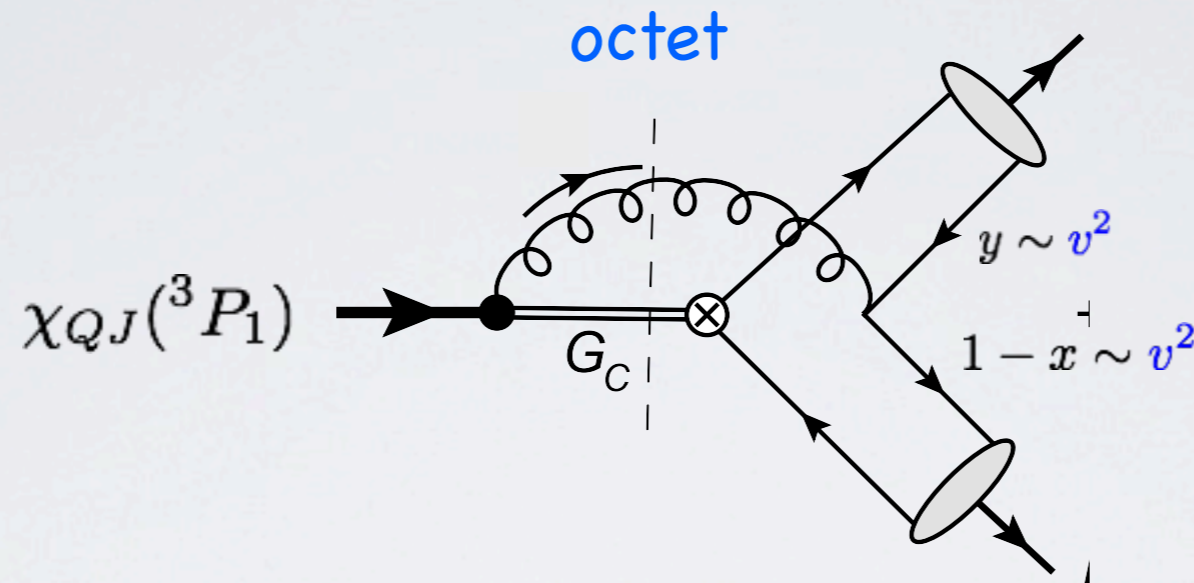
$$\langle K_{\perp}^* K | \dots | \chi_{cJ} \rangle \sim \alpha_s(m_Q v^2) \alpha_s(m_Q) \int d^3 \Delta \tilde{R}_{21}^c(\Delta) \Delta$$

$$\times (m_s - m_q) \phi'_{K_{\perp}^*}(1) \int_{1-\delta}^1 dx \int_0^{\delta} dy \frac{(1 + \ln y)}{[E - m_Q(y + \bar{x}) - \Delta^2/m_Q + i\epsilon]^2}$$

$$\delta \rightarrow \infty \quad y \sim v^2 \quad 1-x \sim v^2 \quad \text{IR finite!}$$

Soft-overlap matrix element in the Coulomb limit

$$\langle K_{\perp}^* K | \alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8 | \chi_{cJ} \rangle \sim v^4 (\Lambda/m_Q)^3$$



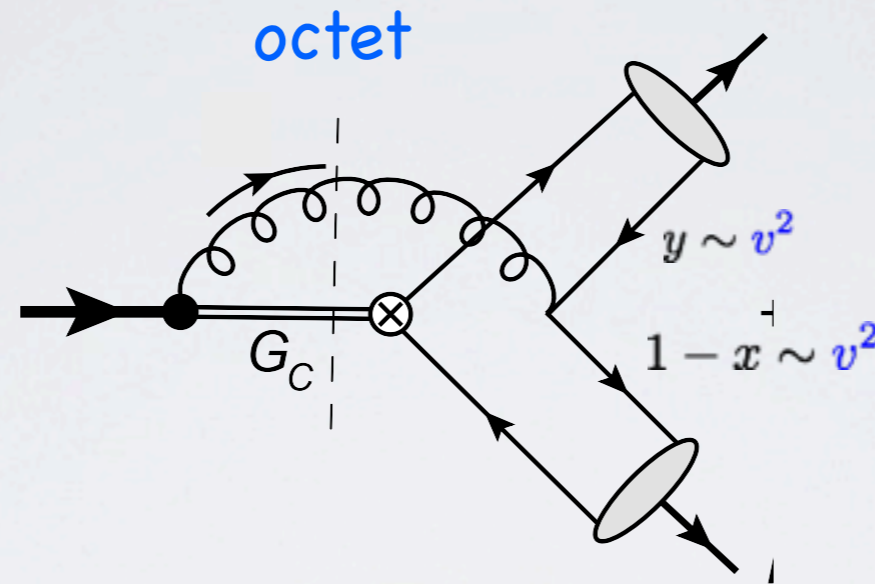
$$\langle K_{\perp}^* K | \dots | \chi_{cJ} \rangle \sim \alpha_s(m_Q v^2) \alpha_s(m_Q) \underline{(-1)^J 2^{J/2}} \int d^3 \Delta \tilde{R}_{21}^c(\Delta) \Delta$$

$$\times (m_s - m_q) \phi'_{K_{\perp}^*}(1) \int_0^{\infty} dx \int_0^{\infty} dy \frac{(1 + \ln y)}{[E - m_Q(y + x) - \Delta^2/m_Q + i\epsilon]^2}$$

$$y \sim v^2 \quad 1 - x \sim v^2$$

Soft-overlap matrix element in the Coulomb limit

$$\langle K_{\perp}^* K | \alpha_s C_h(\bar{Q}Q)_8(\bar{q}q)_8 | \chi_{cJ} \rangle \sim v^4 (\Lambda/m_Q)^3$$



The imaginary part is given by the region $0 < \Delta^2/m^2 < E$. Small Δ -- large distance between the HQ's. Hence the Im part is the 100% soft effect.

$$\langle K_{\perp}^* K | \dots | \chi_{cJ} \rangle \sim \alpha_s(m_Q v^2) \alpha_s(m_Q) \int d^3 \Delta \tilde{R}_{21}^c(\Delta) \Delta$$

$$\times (m_s - m_q) \phi'_{K_{\perp}^*}(1) \left(\frac{1}{2} \ln^2 [\Delta^2/m_Q^2 - E/m_Q - i0] + \ln [\Delta^2/m_Q^2 - E/m_Q - i0] \right)$$

imaginary part!

$$\tilde{R}_{21}^c(\Delta) = R'_{21}(0) \frac{16\pi\gamma_B\Delta}{(\Delta^2 + \gamma_B^2/4)^3} \quad \gamma_B = \frac{1}{2} m_Q \alpha_s C_F$$

Decay amplitude $A[\chi_{cJ} \rightarrow K_{\perp}^* K]$

$$A[\chi_{c1} \rightarrow K_{\perp}^* K] = R'_{21}(0) \phi_{K^*}^{\perp}(y) * \alpha_s^2 T_h^{J=1}(x, y) * \phi_K(y) \\ + \langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c1} \rangle$$

$$A[\chi_{c2} \rightarrow K_{\perp}^* K] = R'_{21}(0) \phi_{K^*}^{\perp}(y) * \alpha_s^2 T_h^{J=2}(x, y) * \phi_K(y) \\ + \langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c2} \rangle$$

HQ Spin Symmetry:

$$\langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c1} \rangle = -\frac{1}{\sqrt{2}} \langle K_{\perp}^* K | \alpha_s C_h (\bar{Q}Q)_8 (\bar{q}q)_8 | \chi_{c2} \rangle + \mathcal{O}(v^2)$$

$$A[\chi_{c1} \rightarrow K_{\perp}^* K] = -\frac{1}{\sqrt{2}} A[\chi_{c2} \rightarrow K_{\perp}^* K] \\ + R'_{21}(0) \phi_{K^*}^{\perp}(y) * \alpha_s^2 (T_h^{J=1} - \frac{1}{\sqrt{2}} T_h^{J=2})(x, y) * \phi_K(y)$$

well defined Spin-symmetry breaking corr's

Phenomenology $\chi_{cJ} \rightarrow K^* K$

$$R_{\text{exp}} = \frac{\text{Br}[\chi_{c2} \rightarrow \bar{K} K^* + c.c.]}{\text{Br}[\chi_{c1} \rightarrow \bar{K} K^* + c.c.]} \frac{\Gamma_{\text{tot}}[\chi_{c2}]}{\Gamma_{\text{tot}}[\chi_{c1}]} = 0.30 \pm 0.13,$$

BESII, PRD 74, 2006

BESIII, PRD 96, 2017

Assume that HQSS breaking is negligible

$$A[\chi_{c1} \rightarrow K_{\perp}^* K] \simeq -\frac{1}{\sqrt{2}} A[\chi_{c2} \rightarrow K_{\perp}^* K]$$

then $R_{\text{th}} = \frac{\Gamma[\chi_{c2} \rightarrow \bar{K}^0 K^{*0} + c.c.]}{\Gamma[\chi_{c1} \rightarrow \bar{K}^0 K^{*0} + c.c.]} \simeq 0.55$

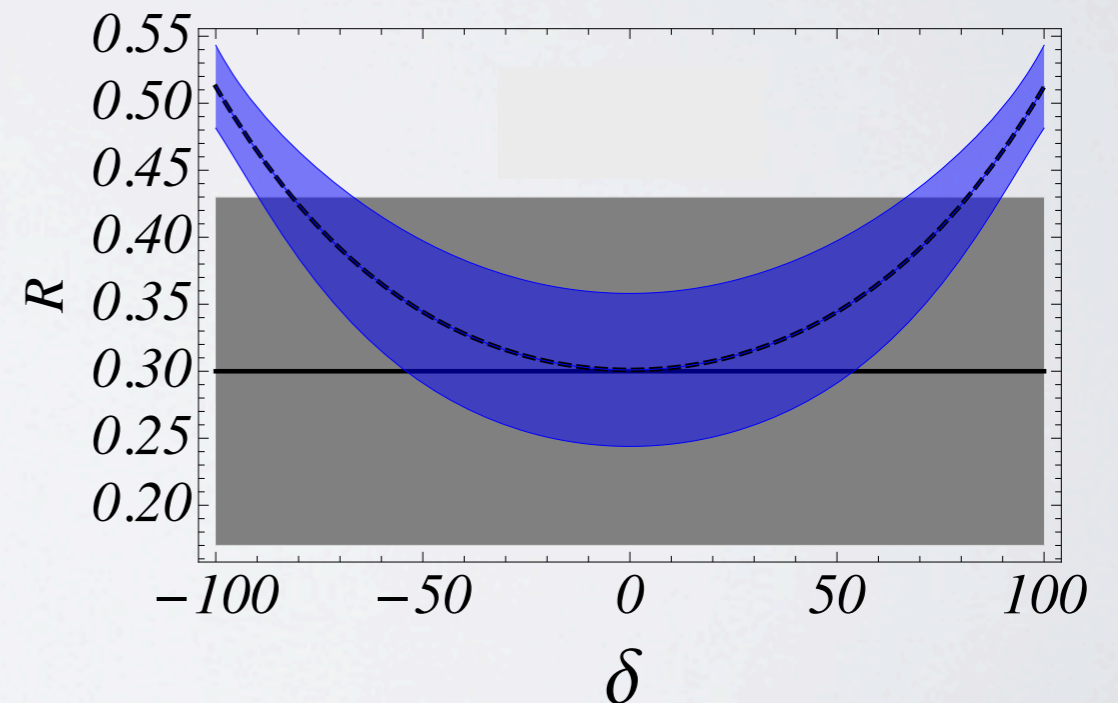
with HQSS breaking corrections:

$$|\mathcal{A}_2^{\perp}| = (7.0 \pm 1.5) \times 10^{-3}$$

$$\Delta A = (-1.6 \pm 0.2) \times 10^{-3}$$

$$\mathcal{A}_2^{\perp} = |\mathcal{A}_2^{\perp}| e^{i\delta} \quad \text{imaginary phase is unknown}$$

$$R = \frac{6}{10} 0.91 \left(\underbrace{1 - 2\sqrt{2} \cos \delta \frac{\Delta A}{|\mathcal{A}_2^{\perp}|}}_{+0.65 \cos \delta} + \underbrace{2 \frac{|\Delta A|^2}{|\mathcal{A}_2^{\perp}|^2}}_{+0.10} \right)^{-1}$$



Color-octet mechanism provide dominant effect

Thank you!

