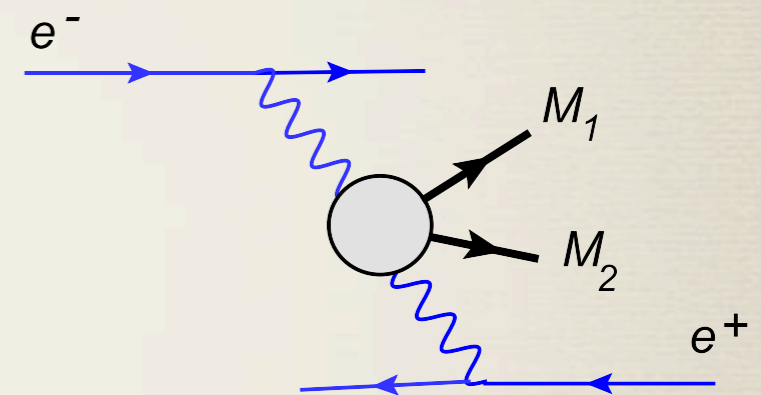


Production of tensor glueball  
in reaction  $\gamma\gamma \rightarrow G_2\pi^0$

Nikolay Kivel



based on [arXiv:1712.04285](https://arxiv.org/abs/1712.04285)



# Outline

● Introduction. Glueballs: theory and experiment

● The process  $\gamma+\gamma \rightarrow G(2^{++})+\pi^0$  as an opportunity to study tensor glueball

● Discussion: questions, suggestions, critics, skepticism, etc.



# Interpretation of $q\bar{q}$ -states in the quark model

$$J^{PC}: 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{--}, 2^{-+}, 2^{++}, \dots$$

A modified reproduction of the table from the 2006 Particle Data Book

$n^{2s+1}L_J$	$J^{PC}$	$I = 1$ $u\bar{d}\dots$	$I = \frac{1}{2}$ $u\bar{s}\dots$	$I = 0$ $f$	$I = 0$ $f'$	$\theta_q$	$\theta_l$
$1^1S_0$	$0^{-+}$	$\pi$	$K$	$\eta$	$\eta'$	$-11.5^\circ$	$-24.6^\circ$
$1^3S_1$	$1^{--}$	$\rho$	$K^*$	$\omega$	$\phi$	$38.7^\circ$	$36.0^\circ$
$1^1P_1$	$1^{+-}$	$b_1(1235)$	$K_{1B}$	$h_1(1170)$	$h_1(1380)$		
$1^3P_0$	$0^{++}$	$a_0(1450)$	$K_0^*(1430)$	$f_0(1370)$	$f_0(1710)$		
$1^3P_1$	$1^{++}$	$a_1(1260)$	$K_{1A}$	$f_1(1285)$	$f_1(1420)$		
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	$f_2(1270)$	$f_2'(1525)$	$29.6^\circ$	$28.0^\circ$
$1^1D_2$	$2^{-+}$	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$		
$1^3D_1$	$1^{--}$	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$			
$1^3D_2$	$2^{--}$		$K_2(1820)$				
$2^1S_0$	$0^{-+}$	$\pi(1300)$	$K(1460)$	$\eta(1295)$	$\eta(1475)$	$-22.4^\circ$	$-22.6^\circ$
$2^3S_1$	$1^{--}$	$\rho(1450)$	$K^*(1410)$	$\omega(1420)$	$\phi(1680)$		

$$|1\rangle = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s})$$

$$|8\rangle = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$f = \cos\theta |1\rangle + \sin\theta |8\rangle$$

$$f' = \cos\theta |8\rangle - \sin\theta |1\rangle$$

the mixing angle for the nonets:

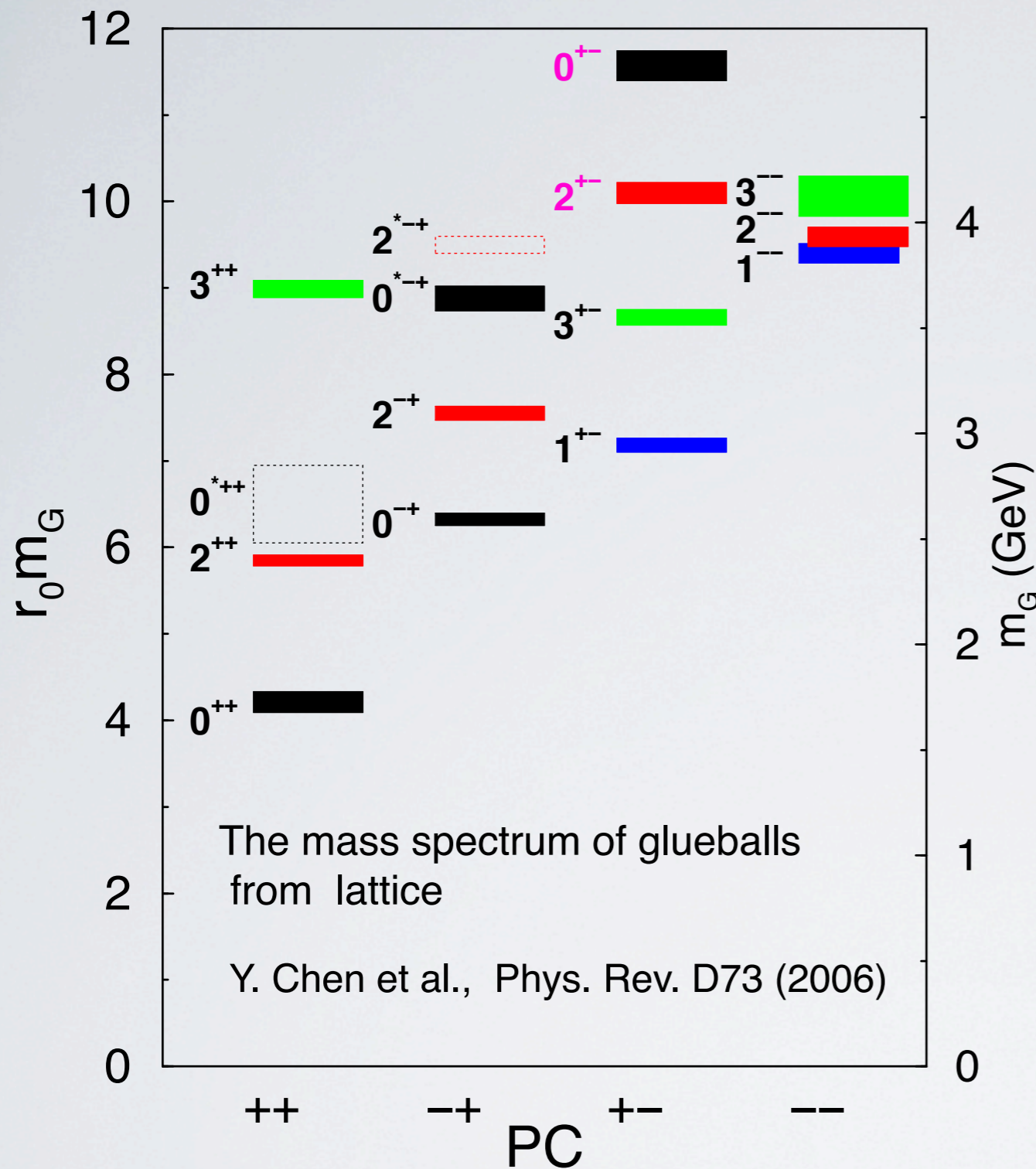
$$\tan\theta_l = \frac{4m_K - m_a - 3m_{f'}}{2\sqrt{2}(m_a - m_K)}$$

$$\tan^2\theta_q = \frac{4m_K - m_a - 3m_{f'}}{-4m_K + m_a + 3m_{f'}}$$

Measuring the masses and decay rates of mesons can be used to identify the quark content of a particular meson

Crede, Mayer, 2009

# Glueballs gg-state



$J^{PC}$	$M_G$ (GeV/ $c^2$ )
$0^{++}$	1.710(.050)(.080)
$2^{++}$	2.390(.030)(.120)
$0^{-+}$	2.560(.035)(.120)
$1^{+-}$	2.980(.030)(.140)
$2^{-+}$	3.040(.040)(.150)
$3^{+-}$	3.600(.040)(.170)
$3^{++}$	3.670(.050)(.180)
$1^{--}$	3.830(.040)(.190)
$2^{--}$	4.010(.045)(.200)
$3^{--}$	4.200(.045)(.200)
$2^{+-}$	4.230(.050)(.200)
$0^{+-}$	4.780(.060)(.230)

The lightest glueballs have  $J^{PC}$  quantum numbers of normal mesons and would appear as an  $SU(3)$  singlet state. If they are near a nonet of the same  $J^{PC}$  quantum numbers, they will appear as an extra f-like state. While the fact that there is an extra state is suggestive, the decay rates and production mechanisms are also needed to unravel the quark content of the observed mesons.

Crede, Mayer, 2009



# Identifying glueballs gg-state

## Conclusions from theory

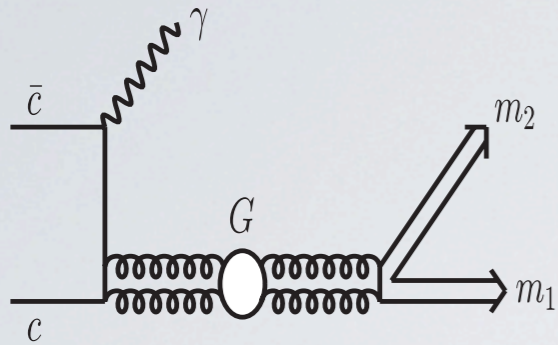
There is a general agreement in that the lightest gluonic state has quantum numbers  $J^{PC} = 0^{++}$ . One state is located around  $1.4-1.7 \text{ GeV}$

The next heavier states are expected with quantum numbers  $J^{PC}=2^{++}$  and with masses  $\gtrsim 2 \text{ GeV}$ . Experimental analyses have been difficult so far in this mass region.

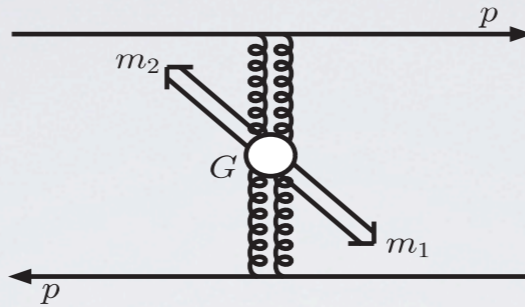
Therefore, the search for the scalar gluonic states looks particularly promising despite the experimental and theoretical uncertainties. W. Ochs, J.Phys. G40 (2013)



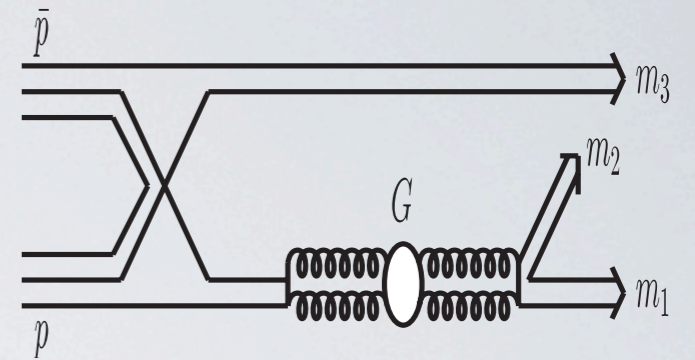
# Production of glueballs in gluon-rich processes



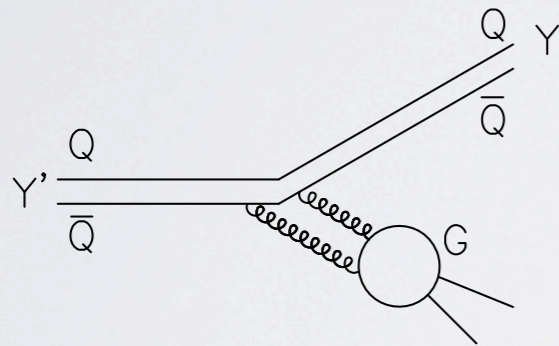
Radiative  $J/\psi$  or  $Y$  decays



Central production of mesons:  
“double Pomeron exchange”



$p\bar{p}$  annihilation



Decay of excited heavy quarkonium  
 $Y^{(n)}$  to ground state  $Y$

## Glueball in $\gamma\gamma$ collisions

a glueball couples to photons only through loop processes and then it is suppressed in  $\gamma\gamma$  reactions

W. Ochs, J.Phys. G40 (2013)



# Experimental evidence for tensor glueballs

## Experiment

BES III, Ablikim et al, PRD 93(2016)

TABLE I. Mass, width,  $\mathcal{B}(J/\psi \rightarrow \gamma X \rightarrow \gamma\phi\phi)$  (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

Resonance	M(MeV/c <sup>2</sup> )	$\Gamma$ (MeV/c <sup>2</sup> )	B.F.( $\times 10^{-4}$ )	Sig.	PDG
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	$9.5 \sigma$	✓
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	$6.4 \sigma$	✓
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	$11 \sigma$	✓

first observed in  $\pi^- + p \rightarrow \phi\phi n$

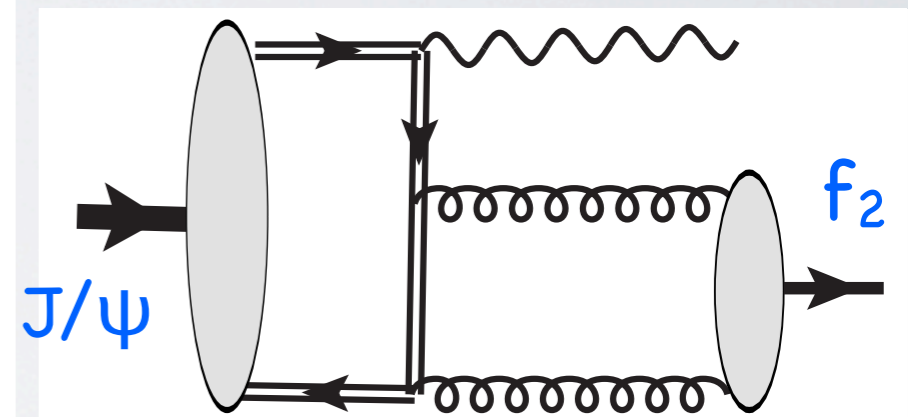
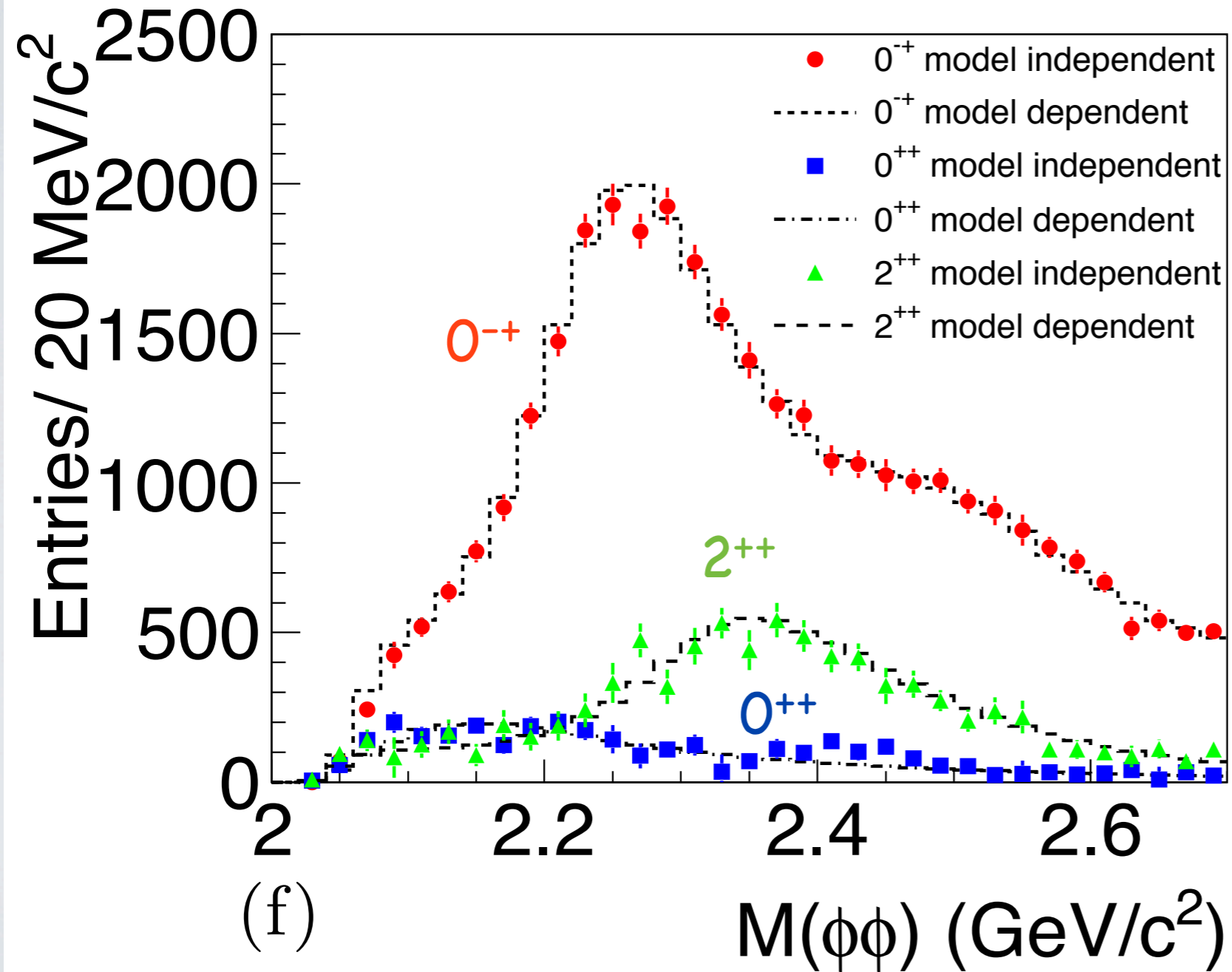
Etkin et al, PRL(1978), PLB(1985), PLB(1988)

Lattice: Chen et al, PRL 111(2013)  $f_2(2340)$  might be glueball

# Experimental evidence for tensor glueballs

Experiment

BES III, Ablikim et al, PRD 93(2016)



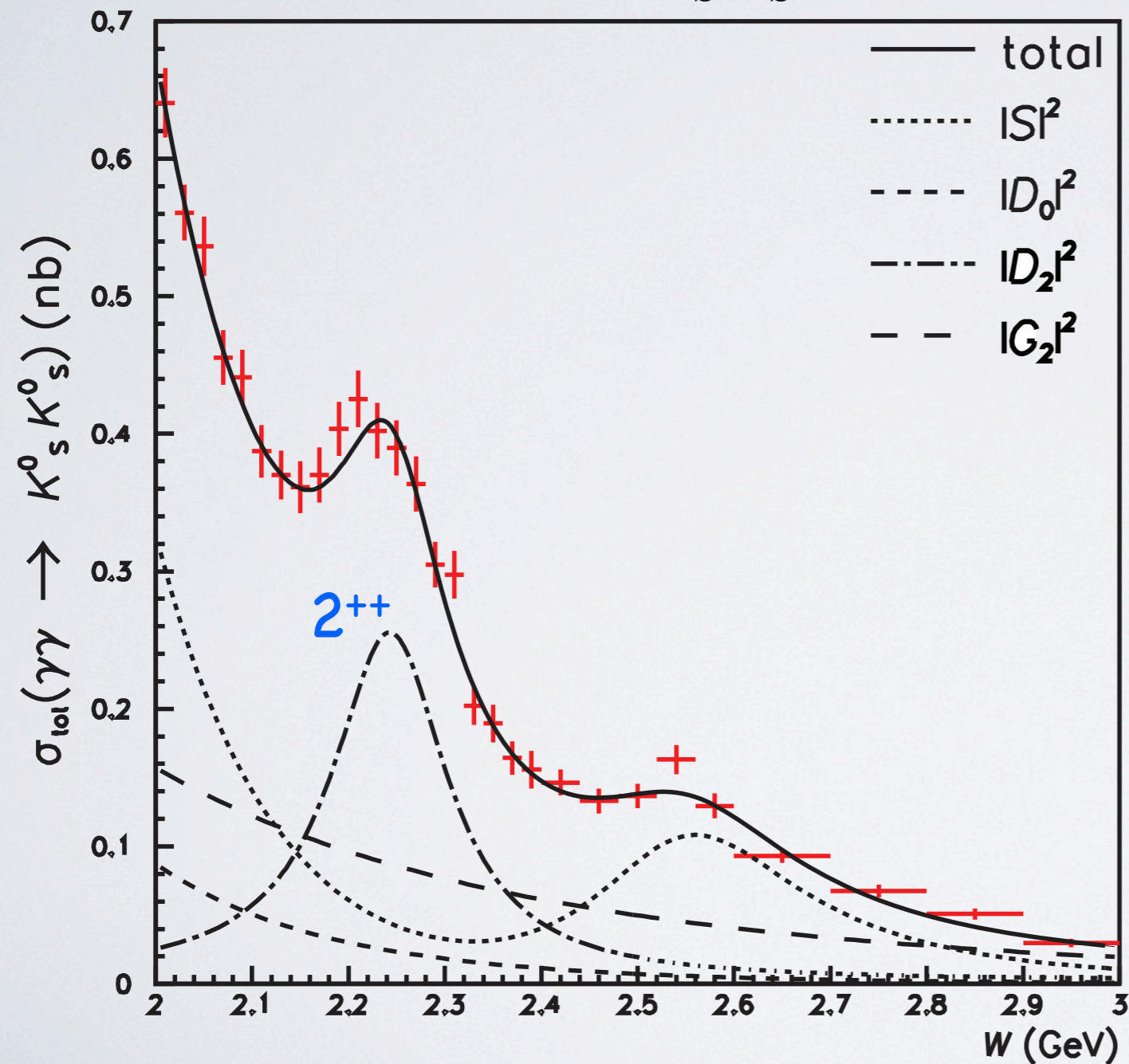


# Experimental evidence for tensor glueballs

Experiment

BELLE, Uehara et al, PTEP (2013)

$$\gamma\gamma \rightarrow K_S^0 K_S^0$$



$f_2(2300)$

$$M = 2297 \pm 28 \text{ MeV}$$

$$\Gamma = 149 \pm 40 \text{ MeV}$$

Probably this indicates that this meson is  $q\bar{q}$ -state or have large  $q\bar{q}$ -component



# Can we learn smth about glueballs in hard exclusive reactions?

## Advantages

the amplitude sensitive to the wave functions  
(distribution amplitudes)

strong coupling to gluonic component of WF must be  
observed

mixing with quarks is well understood (QCD evolution)

**special case** spin-2: there is gluonic DA which does not  
mix with quarks (QCD evolution)

## Disadvantages

mixing still can be problematic for interpretation if  
hadron is  $q\bar{q}$  and  $gg$  state (depends on the concrete  
process)

small cross sections at large hard scale  $Q^2$

**which reactions can be suggested?**







## coupling to gluons: qq-state

$n^{2s+1}L_J$	$J^{PC}$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	$I = 0$	$\theta_q$	$\theta_l$
		$u\bar{d}\dots$	$u\bar{s}\dots$	$f$	$f'$		
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	<u><math>f_2(1270)</math></u>	$f_2'(1525)$	$29.6^\circ$	$28.0^\circ$

Quark DA:  $|\bar{q}(+)q(-)(^1S_0)\rangle$

$$\langle f_2(p, \lambda = 0) | \bar{\psi}(z) \not{z} \psi(0) | 0 \rangle \Big|_{z_- = z_\perp = 0} \sim f_q \int_0^1 dx e^{ixz_+ p_-} \phi_2(x, \mu)$$

Gluon DA:  $|g(\pm)g(\mp)(^1S_0)\rangle$

$$\langle f_2(p, \lambda = 0) | z^\alpha z^\beta G_{\alpha\mu}^a(z) G_{\beta\mu}^a(0) | 0 \rangle \Big|_{z_- = z_\perp = 0} \sim f_g^S \int_0^1 dx e^{ixp_- z_+} \phi_g^S(x)$$



## coupling to gluons: qq-state

$n^{2s+1}L_J$	$J^{PC}$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	$I = 0$	$\theta_q$	$\theta_l$
		$u\bar{d}\cdots$	$u\bar{s}\cdots$	$f$	$f'$		
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	<u><math>f_2(1270)</math></u>	$f_2'(1525)$	$29.6^\circ$	$28.0^\circ$

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$$\langle f_2(p, \lambda = 0) | \bar{\psi}(z) \not{z} \psi(0) | 0 \rangle |_{z_- = z_\perp = 0} \sim f_q \int_0^1 dx e^{ixz+p-} \phi_2(x, \mu)$$

normalization constant  $\frac{1}{2} \langle f_2(P, \lambda) | \bar{q} [\gamma_\mu i \overleftrightarrow{D}_\nu + \gamma_\nu i \overleftrightarrow{D}_\mu] q | 0 \rangle = f_q m^2 e_{\mu\nu}^{(\lambda)*}$

$$f_u(1\text{GeV}) = f_d(1\text{GeV}) = 101(10)\text{MeV}$$

$$f_s(1\text{GeV}) \approx 0$$

Aliev, Shifman 1982 (QCD SR, TM dom.)

Terazawa, 1990/ Suzuki 1993 (TM dom.)

Cheng, Koike, Yang 2010 (QCD SR, TM dom.)

for comparison

$$f_\pi = 130\text{MeV} \quad f_\rho = 221\text{MeV} \quad f_\omega = 198\text{MeV}$$



# coupling to gluons: qq-state

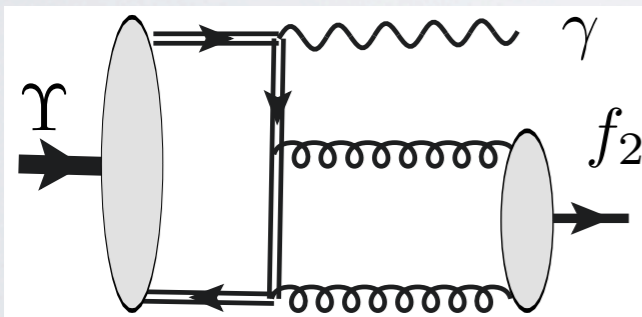
$n^{2s+1}L_J$	$J^{PC}$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	$I = 0$	$\theta_q$	$\theta_l$
		$u\bar{d}\dots$	$u\bar{s}\dots$	$f$	$f'$		
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	<u><math>f_2(1270)</math></u>	$f_2'(1525)$	$29.6^\circ$	$28.0^\circ$

**Gluon DA:**  $|g(\pm)g(\mp)(^1S_0)\rangle$

$$\langle f_2(P, \lambda) | z^\alpha z^\beta G_{\alpha\mu}^a(z) G_{\beta\mu}^a(0) | 0 \rangle \Big|_{z_- = z_\perp = 0} \sim f_g^S \int_0^1 dx e^{ixp-z_+} \phi_g^S(x)$$

**rich gluon process**

$$\Upsilon(1S) \rightarrow \gamma f_2 \quad M_\Upsilon = 9.5\text{GeV} \quad m_b \simeq 4.5\text{GeV}$$



$$\frac{Br[\Upsilon(1S) \rightarrow \gamma f_2]}{Br[\Upsilon(1S) \rightarrow e^+e^-]} = \frac{64\pi \alpha_s^2 (4m_b^2)}{3 \alpha} \left(1 - \frac{m_{f_2}^2}{M_\Upsilon^2}\right) \frac{[5f_g^S/4]^2}{m_b^2}$$

**simplest model**



$$\phi_g^S(x) = 30x^2(1-x)^2$$

$$f_g^S(\mu^2 = 4m_b^2) = (14.9 \pm 0.8) \text{ MeV}$$



# coupling to gluons: qq-state

$n^{2s+1}L_J$	$J^{PC}$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	$I = 0$	$\theta_q$	$\theta_l$
		$u\bar{d}\dots$	$u\bar{s}\dots$	$f$	$f'$		
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	<u><math>f_2(1270)</math></u>	$f_2'(1525)$	$29.6^\circ$	$28.0^\circ$

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$$\langle f_2(P, \lambda) | z^\alpha z^\beta G_{\alpha\mu}^a(z) G_{\beta\mu}^a(0) | 0 \rangle \Big|_{z_- = z_\perp = 0} \sim f_g^S \int_0^1 dx e^{ixp-z_+} \phi_g^S(x)$$

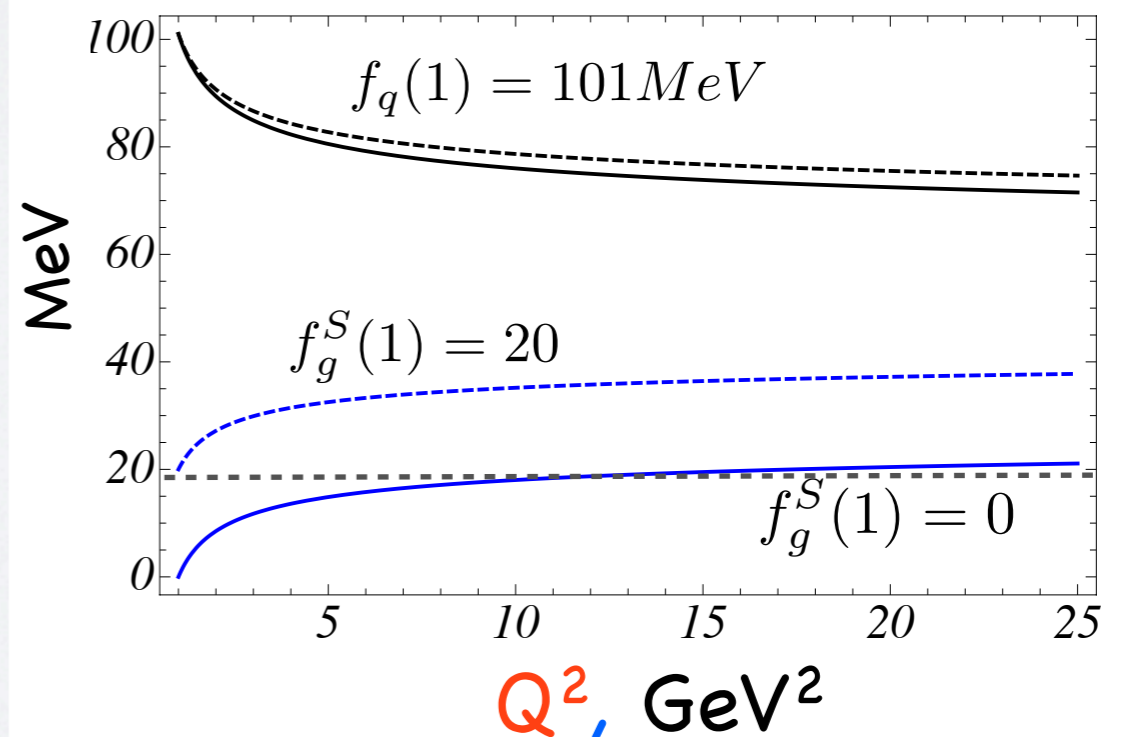
$$f_g^S(\mu^2 = 4m_b^2) = (14.9 \pm 0.8) \text{ MeV}$$

therefore this result compatible with

$$f_g^S(1 \text{ GeV}) \approx 0$$

i.e. the meson consists from  $q\bar{q}$   
at low normalization point

**QCD evolution mixes  $f_q$  and  $f_g^S$**





# Light-cone distribution amplitudes

describes the momentum-fraction distribution of partons at zero transverse separation in a 2-particle Fock state

only for tensor state  $2^{++}$   $|g(\pm)g(\pm)(^5S_2)\rangle$

$$\langle M(P, \lambda = 2) | z^\alpha z^\beta G_{\alpha\{\mu}(z) G_{\beta\nu}\}(0) | 0 \rangle |_{z_- = z_\perp = 0} = f_g^T e_{\{\mu\nu\}}^\perp \int_0^1 dx e^{ixp_- z_+} \phi_g^T(x)$$

such component does not mix with quarks!

$|\bar{q}q(^1D_2)\rangle$

$$\langle M(P, \lambda = 2) | \bar{\psi}(z) \overleftrightarrow{D}_{\{\perp\mu} \overleftrightarrow{D}_{\perp\nu} \psi(0) | 0 \rangle \sim \frac{\Lambda^2}{Q^2} \longrightarrow \sim \frac{m^2}{Q^2} \langle f_q \rangle + \dots$$

QCD EOM

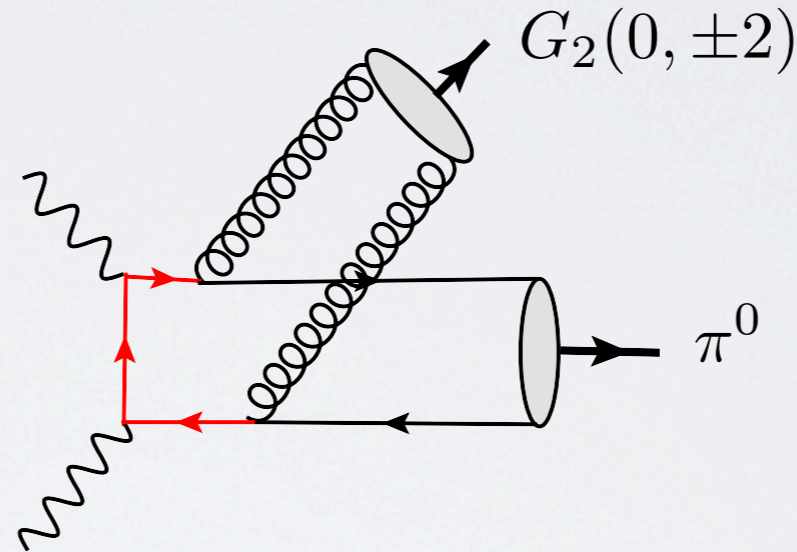
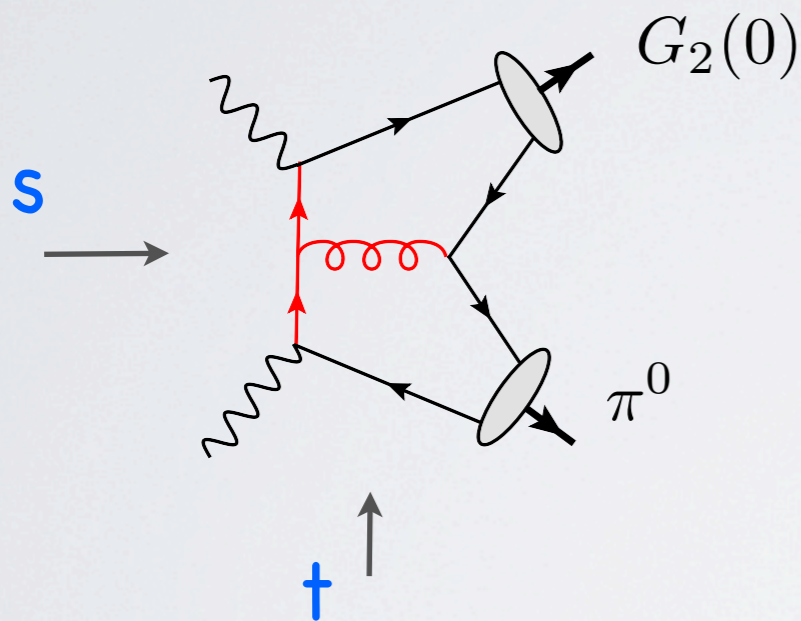
which reactions can be suggested?



# One more way to study tensor glueball: $\gamma\gamma \rightarrow \pi^0 G(2^{++})$

wide angle scattering  $s \sim -t \sim -u \gg \Lambda^2$

$$\frac{d\sigma_{\gamma\gamma}[\pi^0 G(2^{++})]}{d\cos\theta} = \frac{1}{64\pi} \frac{s+m^2}{s^2} (|\overline{A_{++}}|^2 + |\overline{A_{+-}}|^2)$$



all terms are of order  $\alpha_s$

BEFORE:

$$\gamma\gamma \rightarrow G_0 \pi^0$$

$$\gamma\gamma \rightarrow G_{0,2} \pi^0$$

Atkinson, Sucher and Tsokos, Phys. Lett. 137B (1984)

Wakely and Carlson, Phys. Rev. D 45 (1992)

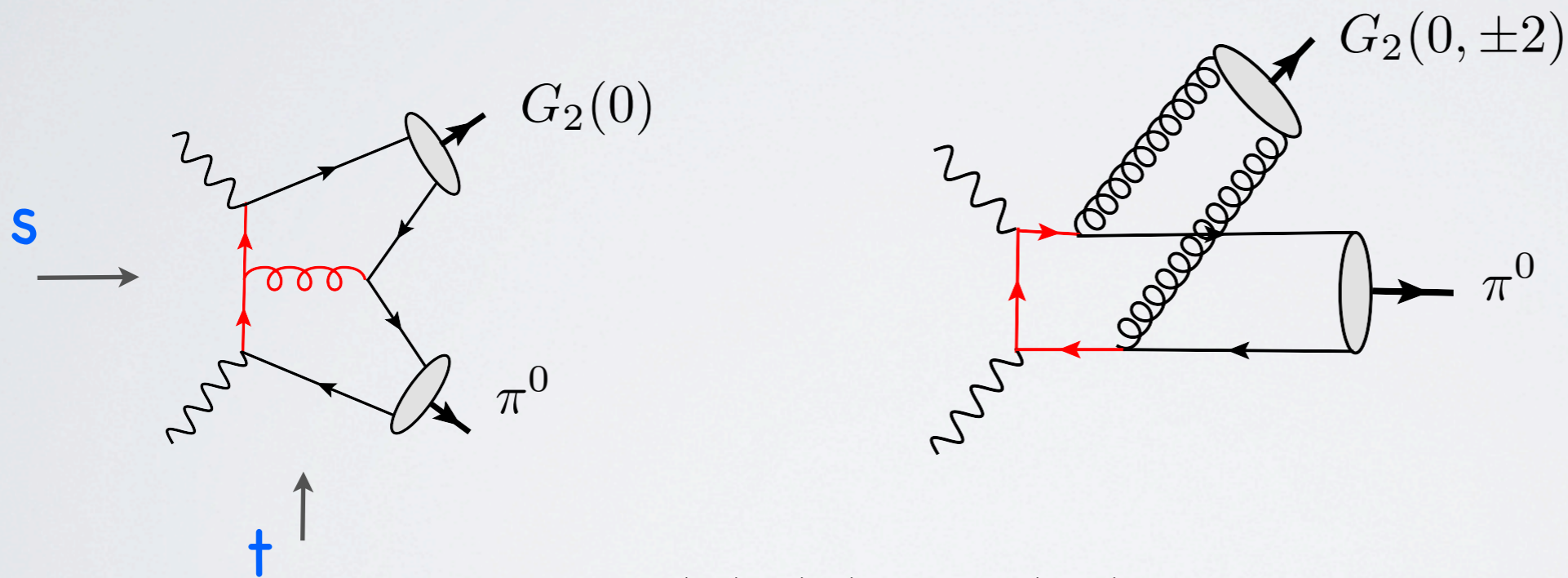
Ichola and Parisi, Z. Phys. C 66 (1995) 653



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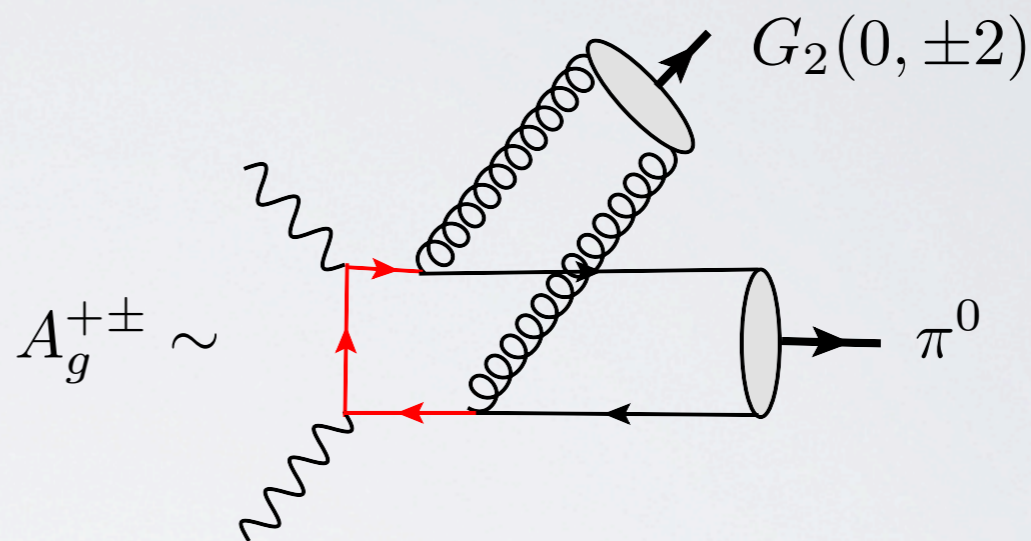
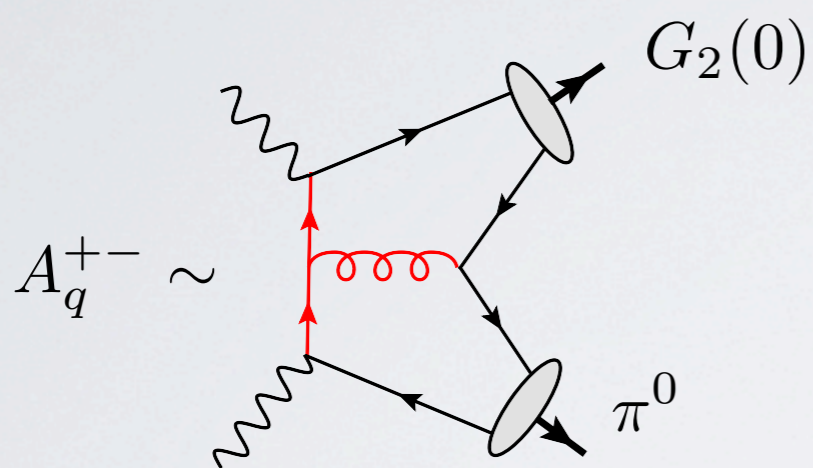
$A_{\pm\pm} : \gamma(\pm)\gamma(\pm) \rightarrow G_2(\pm 2)$  tensor gluon DA

$A_{\pm\mp} : \gamma(\pm)\gamma(\mp) \rightarrow G(0)$  quark & gluon DAs



# Amplitude and cross section

$$\frac{d\sigma_{\gamma\gamma}[\pi^0 G_2]}{d\cos\theta} = \frac{1}{64\pi} \frac{s+m^2}{s^2} (|A_g^{++}|^2 + |A_q^{+-} + A_g^{+-}|^2)$$



$$s \rightarrow \infty \quad \frac{d\sigma_{\gamma\gamma}[\pi^0 G(2^{++})]}{d\cos\theta} \sim \frac{1}{s} \left( \left| \frac{f_\pi f_g^T}{s} g_{++}(\theta) \right|^2 + \left| \frac{f_\pi f_g^S}{s} g_{+-}(\theta) + \frac{f_\pi f_q}{s} f_{+-}(\theta) \right|^2 \right)$$

$$f_\pi = 131 \text{ MeV}$$

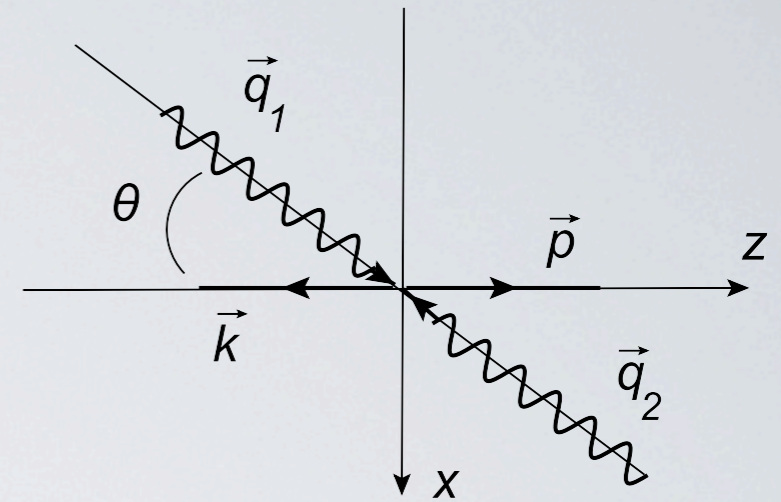
$$f_g^{T,S}, f_q \quad \text{unknown}$$



# Angular behaviour

$$A_g^{++} \sim \frac{f_\pi f_g^T}{s} \alpha \alpha_s I_g^{++}(\cos \theta),$$

$$I_g^{++}(\cos \theta) = \int_0^1 dy \frac{\phi_\pi(y)}{y\bar{y}} \int_0^1 dx \frac{\phi_g^T(x)}{x\bar{x}} \frac{(-2)}{(1 - \cos \theta)x\bar{y} + (1 + \cos \theta)y\bar{x}}$$



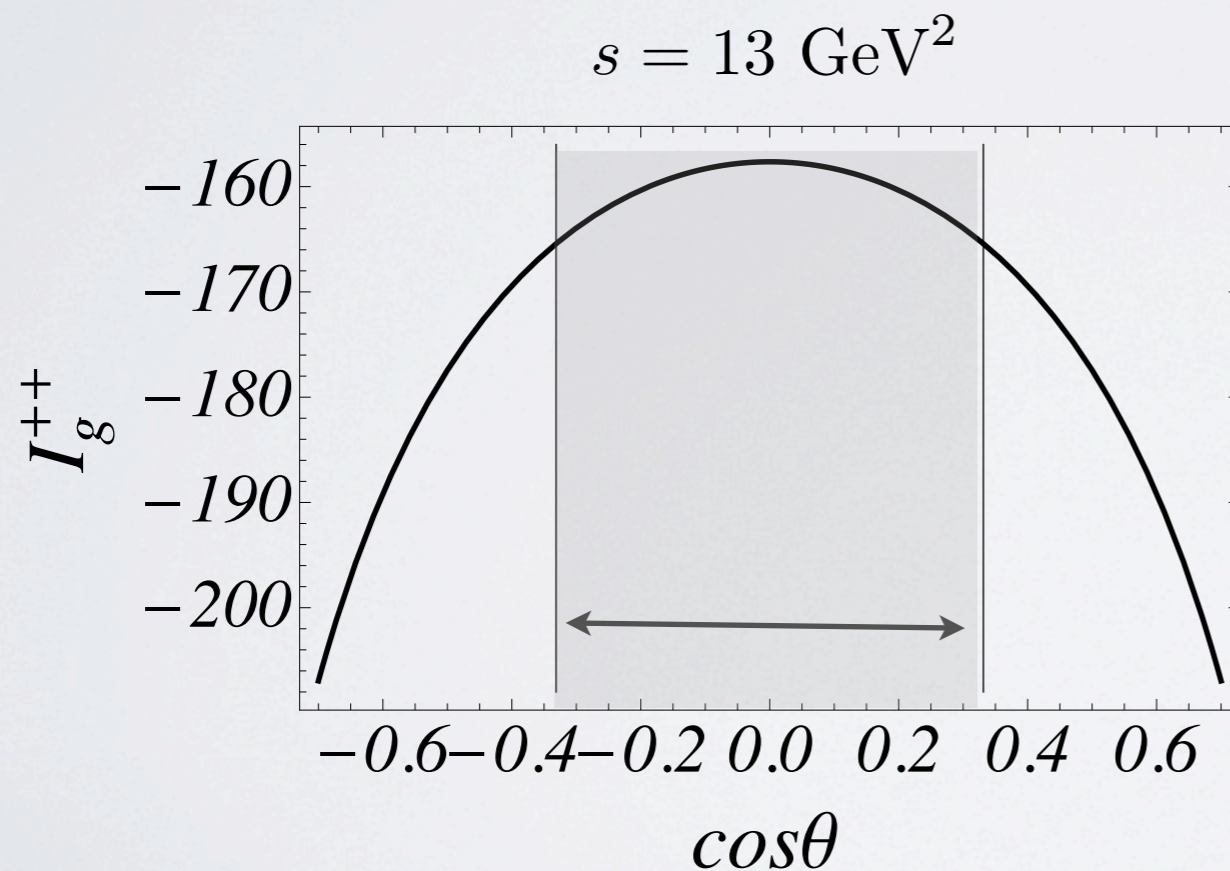
$$\bar{x} \equiv 1 - x$$

models for the DAs

$$\phi_g^T(x) = 30x^2\bar{x}^2$$

$$\phi_\pi(y) \simeq 6y\bar{y} + 6a_2(\mu)y\bar{y}C_2^{3/2}(2y - 1)$$

$$a_2(\mu = 1\text{GeV}) = 0.20$$



$$s \sim -t \sim -u \gg \Lambda^2 \quad \longleftrightarrow \quad |u|, |t| \geq 2.5 \text{ GeV}^2$$

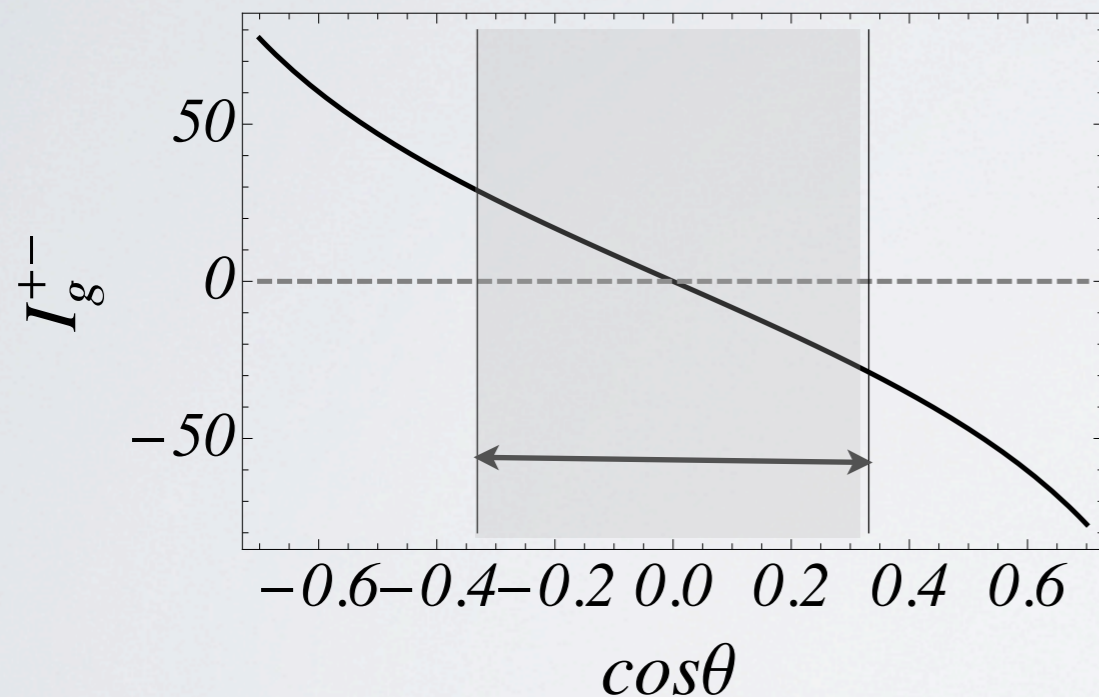


# Angular behaviour

$$A_g^{+-} \sim \frac{f_\pi f_g^S}{s} \alpha \alpha_s I_g^{+-}(\cos\theta)$$

$$\bar{x} \equiv 1 - x$$

$$I_g^{+-}(\cos\theta) = \int_0^1 dy \frac{\phi_\pi(y)}{y\bar{y}} \int_0^1 dx \frac{\phi_g^S(x)}{x\bar{x}} \frac{-\cos\theta}{(1-\cos\theta)x\bar{y} + (1+\cos\theta)y\bar{x}}$$



## models for the DAs

$$\phi_g^S(x) = 30x^2\bar{x}^2$$

$$\phi_\pi(y) \simeq 6y\bar{y} + 6a_2(\mu)y\bar{y}C_2^{3/2}(2y-1)$$

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$$s \sim -t \sim -u \gg \Lambda^2 \quad |u|, |t| \geq 2.5\text{GeV}^2$$

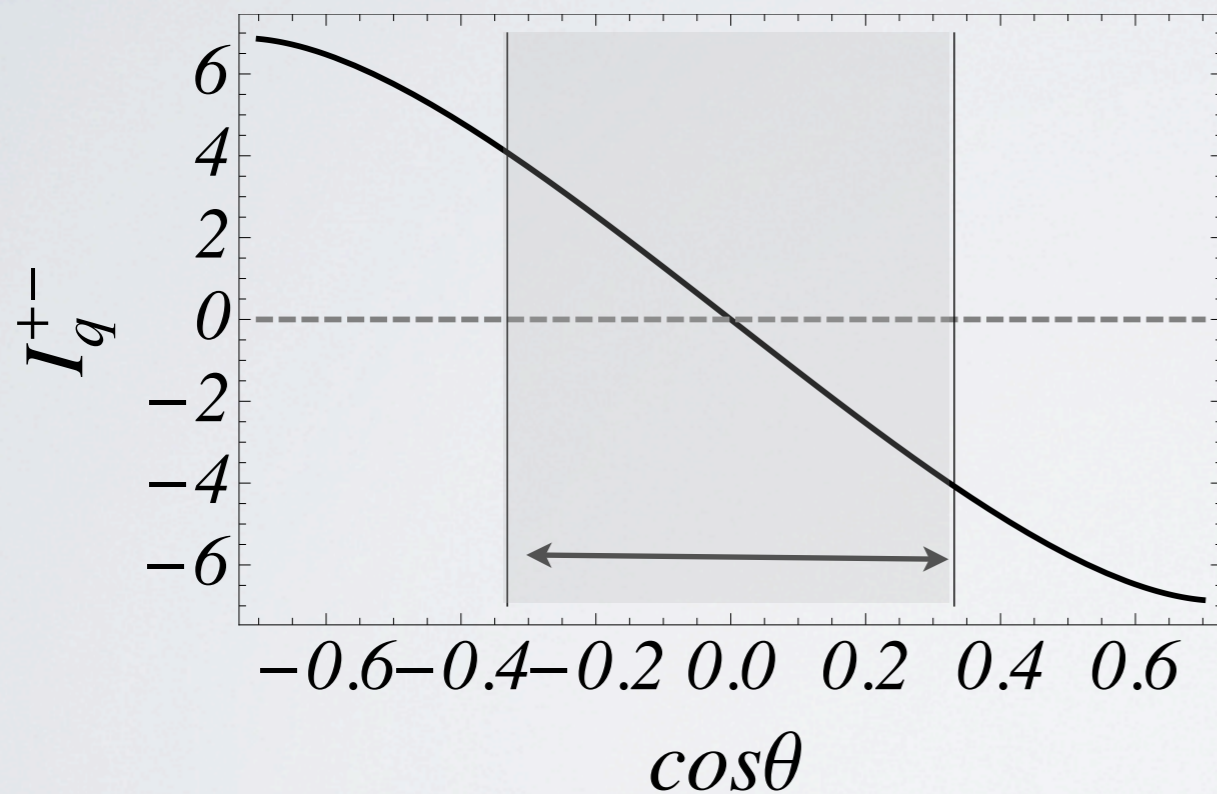
$$\left| I_g^{++} \right| \gg \left| I_g^{+-} \right|$$



# Angular behaviour

$$A_q^{+-} \sim \frac{f_\pi f_q}{s} \alpha \alpha_s I_q^{+-}(\cos \theta)$$

$$I_q^{+-}(\cos \theta) = \int_0^1 dy \frac{\phi_\pi(y)}{y\bar{y}} \int_0^1 dx \frac{\phi_2(x)}{x\bar{x}} \frac{\cos \theta (1 - \cos^2 \theta) (y - x) (\bar{x} - y)^2}{[(\bar{x} - y)^2 (1 - \cos^2 \theta) + 4x\bar{x}y\bar{y}]}$$



## models for the DAs

$$\phi_2(x) = 30x\bar{x}(2x - 1)$$

$$\phi_\pi(y) \simeq 6y\bar{y} + 6a_2(\mu)y\bar{y}C_2^{3/2}(2y - 1)$$

$$a_2(\mu = 1\text{GeV}) = 0.20$$

☞  $|I_g^{++}| \gg |I_g^{+-}| \gg |I_q^{+-}|$

at large angles  $G_2$  is dominantly produced in tensor polarization

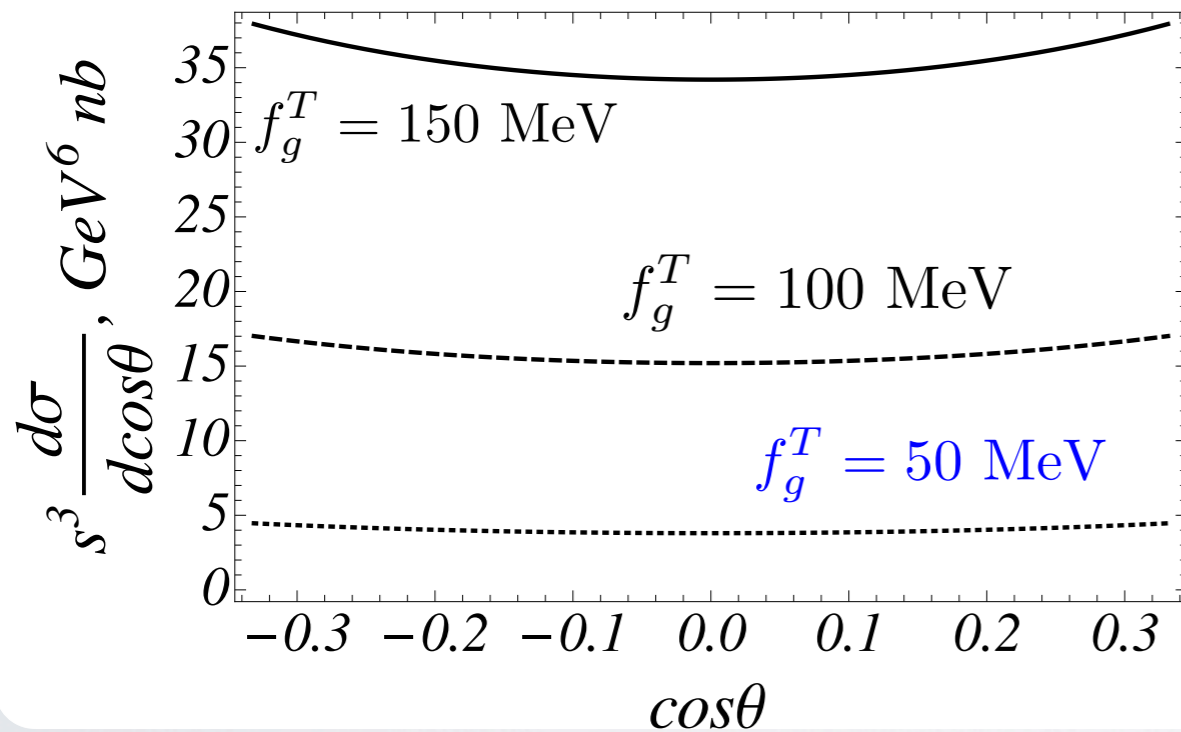


# Cross section

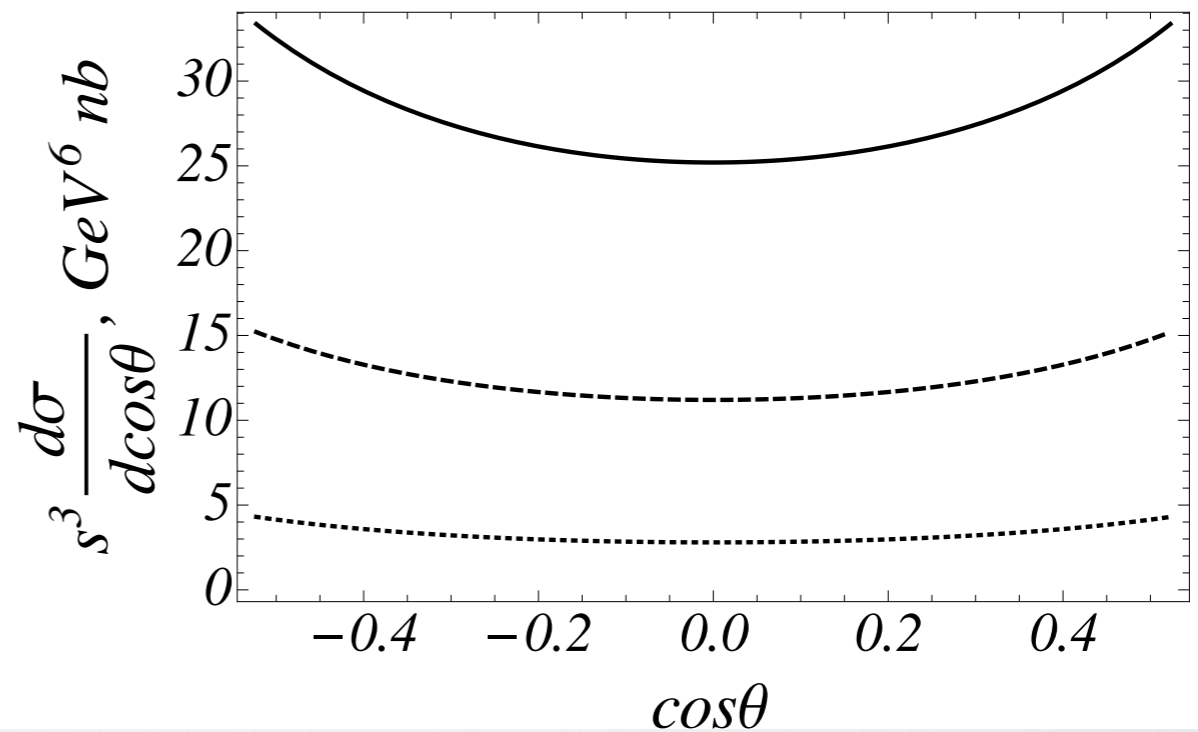
$$\frac{d\sigma_{\gamma\gamma}[\pi^0 G(2^{++})]}{d\cos\theta} = \frac{1}{64\pi} \frac{s+m^2}{s^2} (|\overline{A}_{++}|^2 + |\overline{A}_{+-}|^2)$$

$$M_G = 2.3 \text{ GeV}$$

$$s = 13 \text{ GeV}^2$$



$$s = 16 \text{ GeV}^2$$



$$|t| \ \& \ |u| > 2.5 \text{ GeV}^2$$

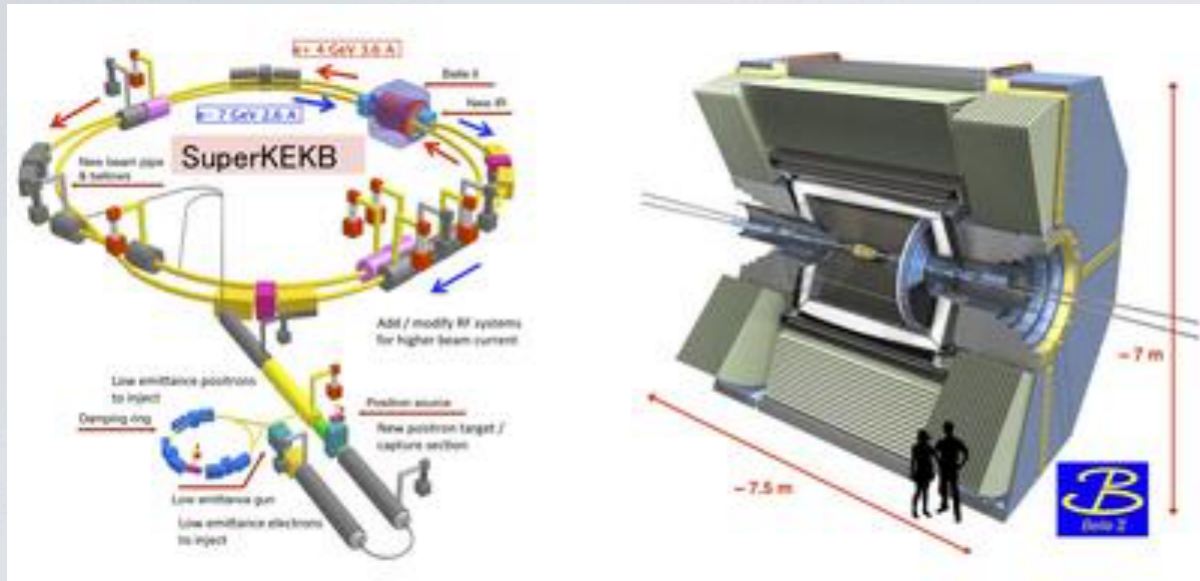
$$f_q(\mu = 1 \text{ GeV}) \simeq 10 - 100 \text{ MeV}$$

$$f_g(\mu = 1 \text{ GeV}) \simeq 100 \text{ MeV}$$

tensor channel is dominant



# Can one measure the cross section in BELLE II?



$e^+e^-$  asymmetric collider

KEKB

instantaneous luminosity  
of  $2.11 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ .

SuperKEKB

instantaneous luminosity  
of  $8 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$

larger by a factor 40

The ambitious goal is to accumulate an integrated luminosity of  $50 \text{ attob}^{-1}$  ( $10^{-18}$ ) by the mid of next decade, which is 50 times more data than the previous Belle detector acquired

A lot of work have been already done

$$W = \sqrt{s}$$

$\gamma\gamma \rightarrow \pi^- \pi^+$		$2.4 \text{ GeV} < W < 4.1 \text{ GeV}$	H. Nakazawa et al., <i>Phys.Lett. B615</i> (2005)
$\gamma\gamma \rightarrow K^+ K^-$			
$\gamma\gamma \rightarrow \pi^0 \pi^0$		$0.6 \text{ GeV} < W < 4.1 \text{ GeV}$	S. Uehara et al., <i>Phys. Rev. D 79</i> (2009)
$\gamma\gamma \rightarrow K_S^0 \bar{K}_S^0$		$1.0 \text{ GeV} \leq W \leq 4.0 \text{ GeV}$	S. Uehara et al., <i>PTEP 2013</i> (2013)
$\gamma\gamma \rightarrow \eta\eta$		$1.1 \text{ GeV} < W < 3.8 \text{ GeV}$	S. Uehara et al., <i>Phys. Rev. D 82</i> (2010)

also  $\gamma^* \gamma \rightarrow \pi^0$

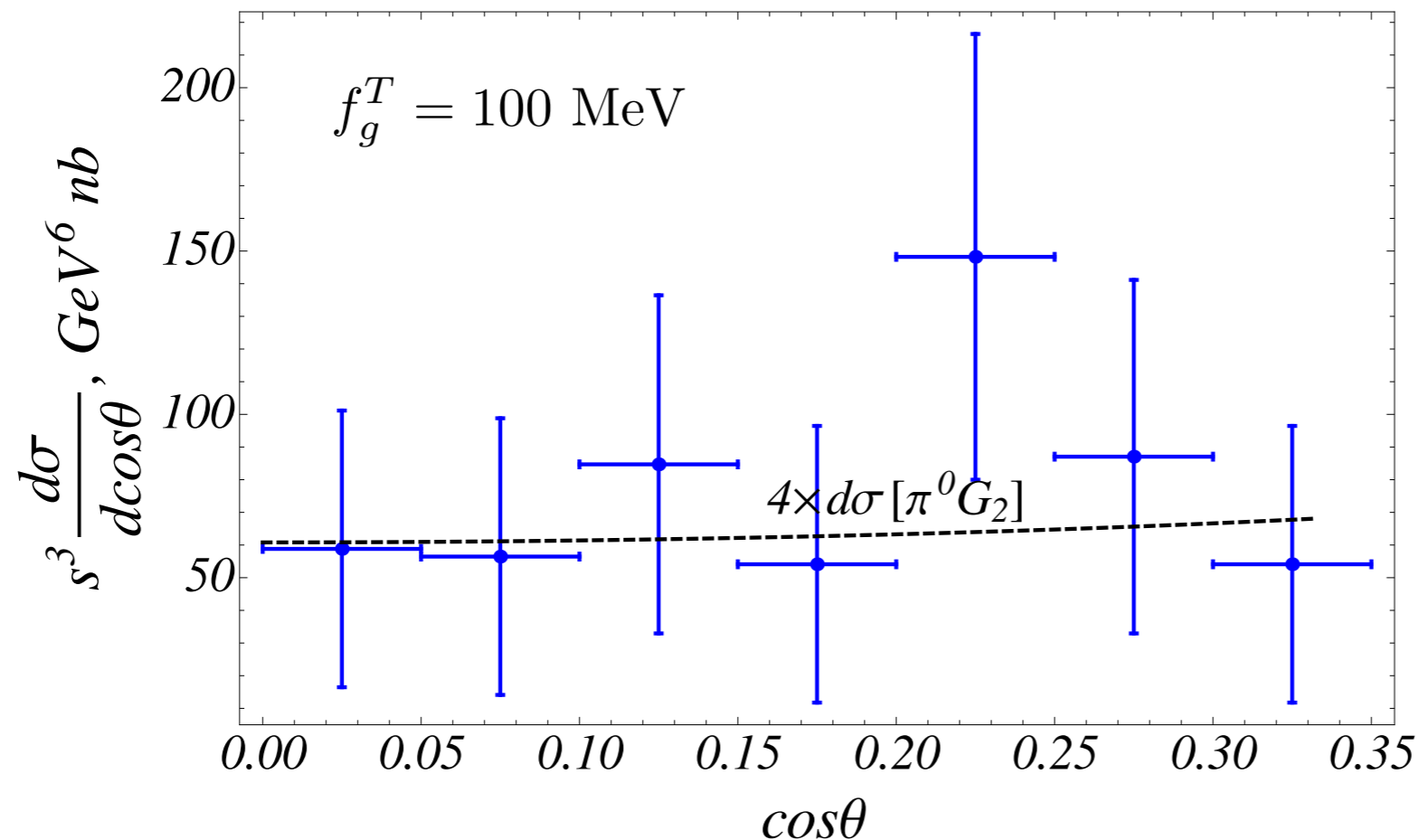


# Can one measure the glueball cross section in BELLE II?

$$\frac{d\sigma[\gamma\gamma \rightarrow \pi^0 G_2(2340) \rightarrow \pi^0 \phi\phi]}{d\cos\theta}$$

Comparison with BELLE data  $\gamma\gamma \rightarrow \pi^0\pi^0$

$$s = 13\text{GeV}^2 \quad |t| \ \& \ |u| > 2.5 \text{GeV}^2$$



## Conclusion

The glueball  
production can be  
measured at BELLE  
II experiment

*Thank you!*