The Nucleon Mass in Chiral Perturbation Theory Beyond one Loop

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RUB

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Quantum Chromodynamics (QCD)
≙ strong interaction
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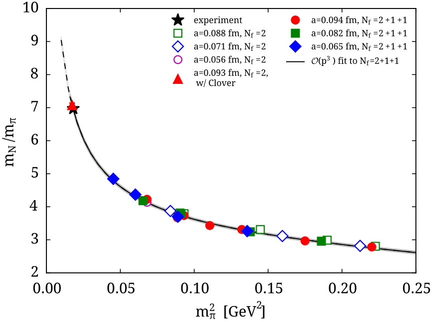


Lattice calculation, Abdhel-Rehim et. al, 2015

calculations in baryon ChPT

- BChPT power counting ✗
- HBChPT power counting ✓, Lorentz covariance ✗
- IR power counting ✓, Lorentz covariance ✓
  → problems due to analytic properties of the loop integrals
- EOMS power counting ✓, Lorentz covariance ✓

Introduction

nucleon mass up to Order $\mathcal{O}(q^3)$ (Scherer and Schindler 2012)

in terms of one loop integrals:

$$m_N = m - 4c_1 M^2 + \frac{3g_A^2}{8F^2\,(p\cdot p)}\Big\{\not p\left(m^2-(p\cdot p)\right)T_\pi + \left(-\not p\left((p\cdot p)+m^2\right)-2m\,(p\cdot p)\right)T_N$$
$$+\left(\not p\left(-\left(2m^2+M^2\right)(p\cdot p)+(p\cdot p)^2+m^4-m^2M^2\right)-2mM^2\,(p\cdot p)\right)T_{\pi N}\Big\}$$

renormalization of the integrals leads to:

$$m_N = m - 4c_{1_r}M^2 + \frac{3g_{A_r}^2 M^2}{32\pi^2 F_r^2}m - \frac{3g_{A_r}^2 M^3}{32\pi F_r^2} + \mathcal{O}(q^4)$$

nucleon mass up to order $\mathcal{O}(q^6)$ with IR by Schindler 2007

goal: calculate nucleon mass up to order $\mathcal{O}(q^6)$ with EOMS

The Lagrangian and Feynman Rules
The Lagragian

- chiral Lagrangian up to chiral order $\mathcal{O}(q^4)$ in SU(2) case

$$\mathcal{L} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\not{D} - m + \frac{g_A}{2}\not{u}\gamma_5\right)\Psi$$

$$\mathcal{L}_{\pi}^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U\left(\partial^\mu U\right)^\dagger\right)$$

$$M = \mathcal{O}(q^1)$$

$$\partial_\mu U = \mathcal{O}(q^1)$$

$$\partial_\mu \Psi = \mathcal{O}(q^1)$$

- with nucleon and pion field:

$$\Psi = (p, n)^t$$

$$U = \mathbb{1} + \frac{i}{F}\vec{\tau}\vec{\pi} - \frac{1}{2F^2}\vec{\pi}^2 - i\alpha\frac{1}{F^3}\vec{\pi}^2\vec{\tau}\vec{\pi} + (8\alpha - 1)\frac{1}{8F^4}\vec{\pi}^4$$

$$u = \mathbb{1}_2 + \frac{i}{2F}\vec{\tau}\vec{\pi} - \frac{8}{F^2}(\vec{\tau}\vec{\pi})^2 + \frac{i(8\alpha - 1)}{16F^3}(\vec{\tau}\vec{\pi})^3 + \frac{32\alpha - 5}{128F^4}(\vec{\tau}\cdot\vec{\pi})^4$$

The Lagrangian and Feynman Rules
The Lagrangian

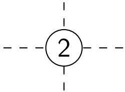
expand the Lagrangian in pion fields (using for example trace rules)

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} =& -\bar{\Psi} m \Psi + i\bar{\Psi}\not{\partial}\Psi \\
&+ \frac{g_A}{2F}\bar{\Psi}\gamma_5 \vec{\tau}\cdot\not{\partial}\vec{\pi}\Psi + \frac{1}{4F^2}\bar{\Psi}\vec{\pi}\cdot\left(\vec{\tau}\times\not{\partial}\vec{\pi}\right)\Psi \\
&- \frac{g_A}{4F^3}\bar{\Psi}\gamma_5\left(2\alpha(\vec{\pi}\cdot\vec{\pi})(\vec{\tau}\cdot\not{\partial}\vec{\pi}) + (4\alpha-1)(\vec{\pi}\cdot\vec{\tau})(\vec{\pi}\cdot\not{\partial}\vec{\pi})\right)\Psi \\
&- \frac{1}{16F^4}\bar{\Psi}(8\alpha-1)(\vec{\pi}\cdot\vec{\pi})\vec{\pi}\cdot\left(\vec{\tau}\times\not{\partial}\vec{\pi}\right)\Psi + \mathcal{O}(\pi^5)
\end{aligned}
$$

The Lagrangian and Feynman Rules

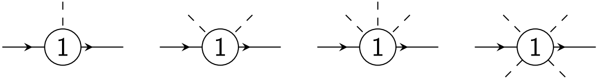
Feynman Rules

$\mathcal{L}_{\pi}^{(2)}$:

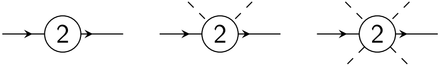


$\mathcal{L}_{\pi}^{(4)}$:

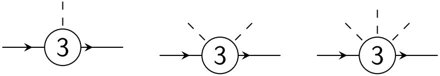
$\mathcal{L}_{\pi N}^{(1)}$:



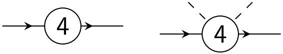
$\mathcal{L}_{\pi N}^{(2)}$:



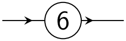
$\mathcal{L}_{\pi N}^{(3)}$:



$\mathcal{L}_{\pi N}^{(4)}$:

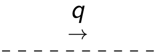


$\mathcal{L}_{\pi N}^{(6)}$:

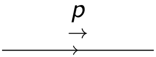


The Lagrangian and Feynman Rules

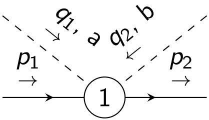
Feynman Rules



$$\hat{=} \quad \frac{i}{q^2 - M^2 + i\varepsilon}$$



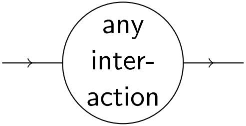
$$\hat{=} \quad \frac{i}{\not{p} - m + i\varepsilon} = \frac{i\left(\not{p} + m\right)}{p \cdot p - m^2 + i\varepsilon}$$

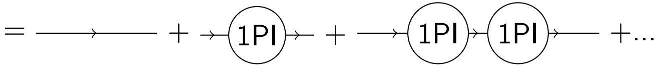


$$\hat{=} (2\pi)^4 \delta^4(p_1 - p_2 + q_1 + q_2) \frac{1}{4F^2} (\not{q}_1 - \not{q}_2) \varepsilon_{abc} \tau_c$$

Basics in QFT
Nucleon Mass

$$
{}_p\langle 0| T\{\Psi_H(x)\bar{\Psi}_H(y)\}|0\rangle_p =
$$





$$
= \frac{i}{\not{p} - m + i\varepsilon} + \frac{i}{\not{p} - m + i\varepsilon}(-i\Sigma)\frac{i}{\not{p} - m + i\varepsilon} + \ldots
$$

$$
= \frac{i}{\not{p} - m + i\varepsilon}\left[1 + \frac{\Sigma}{\not{p} - m + i\varepsilon} + \left(\frac{\Sigma}{\not{p} - m + i\varepsilon}\right)^2 + \ldots\right]
$$

$$
= \frac{i}{\not{p} - m - \Sigma + i\varepsilon} = \frac{i\left(\not{p} + m + \Sigma\right)}{p \cdot p - (m + \Sigma)^2 + i\varepsilon}
$$

Basics in QFT
Nucleon Mass

Definition

The **nucleon mass** is a pole in the two-point nucleon function.

$$\left[p \cdot p - (m + \Sigma)^2 + i\varepsilon\right]_{p \cdot p = m_N^2} = 0$$

$$\Rightarrow m_N = m + \Sigma$$

Basics in QFT
Power Counting

$$\frac{q}{m} < 1 \quad \text{and} \quad \frac{q}{m_N} < 1 \qquad\qquad M = \mathcal{O}(q)$$
$$|\vec{p}| = \mathcal{O}(q)$$

For integrals over pion and nucleon propagators one can show

- integrals in d dimensions count as $\mathcal{O}(q^d)$
- pion propagators count as $\mathcal{O}(q^{-2})$
- nucleon propagators count as $\mathcal{O}(q^{-1})$

for example

$$m \int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{(l \cdot l - M^2 + i\varepsilon)\,((l-p)\cdot(l-p) - m^2 + i\varepsilon)} = \mathcal{O}(q^{d-3})$$

Basics in QFT
Power Counting

for diagrams one can show that vertices of chiral order k count like

$$\delta^d(q)q^k \rightarrow t^{k-d}\delta^d(q)q^k$$

So a diagram with N_I^π internal pion lines N_I^N internal nucleon lines and N_{V_k} vertices of chiral order k count in d-dimensions as

$$FD \rightarrow t^D FD$$

$$D = d + (d-2)N_I^\pi + (d-1)N_I^N + \sum_k N_{V_k}(k-d)$$

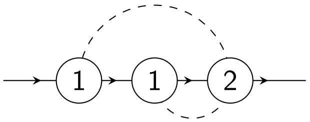
with

$$N_L - 1 = N_I - N_{V_k}$$

$$\Rightarrow D = dN_L - 2N_I^\pi - N_I^N + \sum_k kN_{V_k}$$

Basics in QFT
Power Counting

For example (in four dimensions):



is of order

$2 \cdot 4(\text{loops}) + 2 \cdot 1 + 2(\text{vertices}) - 2 \cdot 2(\text{pion lines}) - 2 \cdot 1(\text{nucleon lines}) = 6$

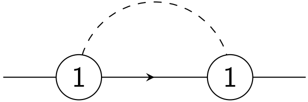
Diagrams
contact terms

$\mathcal{O}(q^2)$

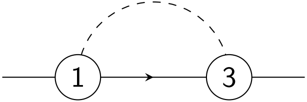
$\mathcal{O}(q^4)$

$\mathcal{O}(q^6)$

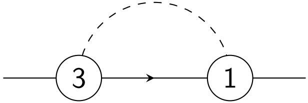
Diagrams
one loop



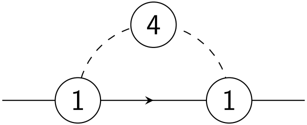
(a) $\mathcal{O}(q^3)$



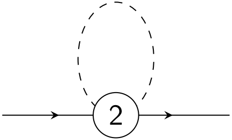
$(b)\mathcal{O}(q^5)$



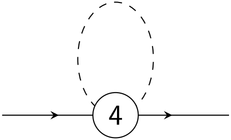
(c) $\mathcal{O}(q^5)$



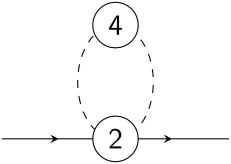
(d) $\mathcal{O}(q^5)$



(e) $\mathcal{O}(q^4)$

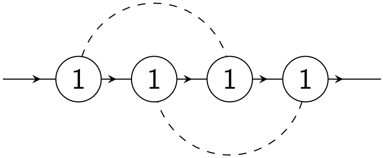


(f) $\mathcal{O}(q^6)$

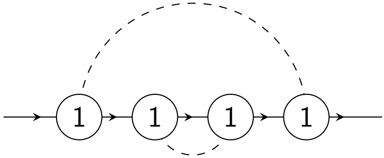


(g) $\mathcal{O}(q^6)$

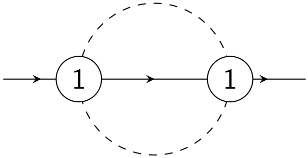
Diagrams
two loop



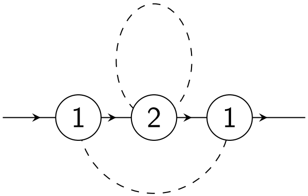
(a) $\mathcal{O}(q^5)$



(b) $\mathcal{O}(q^5)$



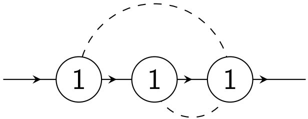
(c) $\mathcal{O}(q^5)$



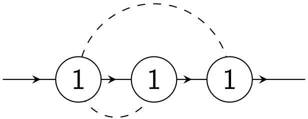
(j) $\mathcal{O}(q^6)$

Diagrams

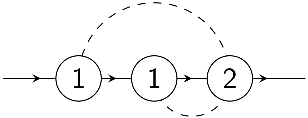
two loop



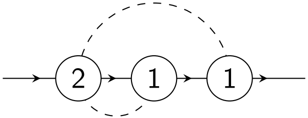
(d) $\mathcal{O}(q^5)$



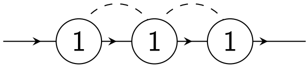
(e) $\mathcal{O}(q^5)$



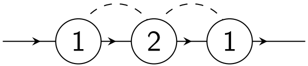
(f) $\mathcal{O}(q^6)$



(g) $\mathcal{O}(q^6)$



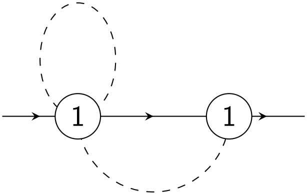
(h) $\mathcal{O}(q^5)$



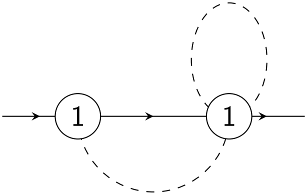
(i) $\mathcal{O}(q^6)$

Diagrams

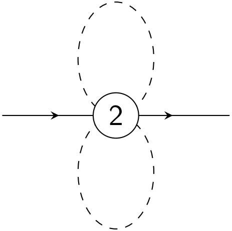
two loop



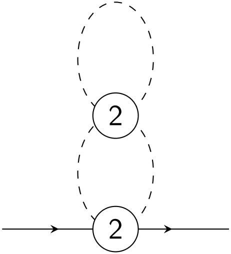
(k) $\mathcal{O}(q^5)$



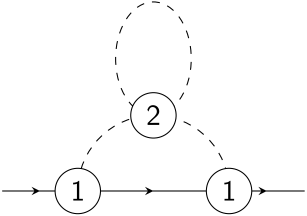
(l) $\mathcal{O}(q^5)$



(m) $\mathcal{O}(q^6)$

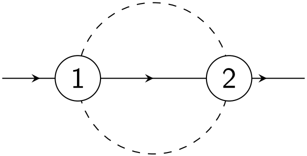


(n) $\mathcal{O}(q^6)$

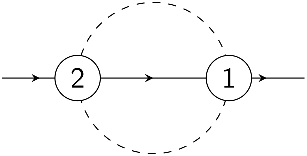


(o) $\mathcal{O}(q^5)$

Diagrams
two loop further



[f] $\mathcal{O}(q^6)$



[g] $\mathcal{O}(q^6)$

Calculate the Diagrams

Contact Interaction

$$\Sigma_c = -4c_1 M^2 - 2\left(e_{115} + e_{116} + 8e_{38}\right) M^4 + \hat{g}_1 M^6.$$

For the simplification of the calculations make a shift

$$m \to m + \Sigma_c \Rightarrow m_N - m = \Sigma_I = \mathcal{O}(q^3).$$

This implies

$$\lim_{p \to m_N} p \cdot p = m^2 + m_N^2 - m^2 = m^2 + \mathcal{O}(mq^3)$$

$$\lim_{p \to m_N} \bar{u}(\vec{p}) \not{p} u(\vec{p}) = \bar{u}(\vec{p}) \left(\not{p} - m_N + m + m_N - m\right) u(\vec{p}) = \bar{u}(\vec{p})\, m\, u(\vec{p})$$

For diagrams, which are of minimal order $\mathcal{O}(q^4)$ terms the replacments

$$\not{p} \to m \text{ and } p \cdot p \to m^2$$

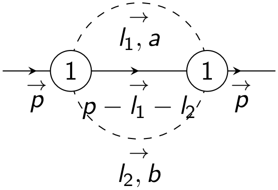
are valid, so that terms like $\not{p} - m\mathbb{1}_d$ and $p \cdot p - m^2$ vanish.

Calculate the Diagrams
Preface

1. The Feynman Rules are applied to get the matematical expression.
2. Suitable zeros are added to reduce the tensor rank.
3. All integrals are reduced to scalar integrals in $d + \ldots$ dimensions (Davydychev 1991, Tarasov 1996).
4. The integrals are expressed with a short set of “Master Integrals” (using the program TARCER based on Tarasov's algorithm).

example:

$-i\Sigma_c^{(2)} =$



Calculate the Diagrams
apply Feynman Rules

RUB

- apply FR
- use $\{\gamma_\mu, \gamma_5\} = 0$, $\gamma_5\gamma_5 = \mathbb{1}_d$, $\tau_a\tau_b = \delta_{ab}\mathbb{1}_2 + i\varepsilon_{abc}\tau_c$ to simplify the expressions
- simplify and order the expression using

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\mathbb{1}_d$$

$$\Rightarrow \not{p}\not{p} = (p \cdot p)\mathbb{1}_d \text{ and } \not{q}_2\not{q}_1 = -\not{q}_1\not{q}_2 + 2\left(q_1 \cdot q_2\right)\mathbb{1}_d$$

Calculate the Diagrams
apply Feynman Rules – Example

$$-2i\Sigma_c^{(2)} =$$

$$\left(\frac{\not{l}_1 \otimes (\tau_c \varepsilon_{abc})}{4F^2} - \frac{\not{l}_2 \otimes (\tau_c \varepsilon_{abc})}{4F^2}\right)$$

$$\odot \left(\frac{i\mathbb{1}_d \otimes \mathbb{1}_2}{(l_1 \cdot l_1) + i\varepsilon - M^2} \odot \frac{i\mathbb{1}_d \otimes \mathbb{1}_2}{(l_2 \cdot l_2) + i\varepsilon - M^2} \odot \frac{i(m\mathbb{1}_d \otimes \mathbb{1}_2 + (-\not{l}_1 - \not{l}_2 + \not{p}) \otimes \mathbb{1}_2)}{((-l_1 - l_2 + p) \cdot (-l_1 - l_2 + p)) + i\varepsilon - m^2}\right)$$

$$\odot \left(\frac{-\not{l}_1 \otimes (\tau_d \varepsilon_{abd})}{4F^2} - \frac{-\not{l}_2 \otimes (\tau_d \varepsilon_{abd})}{4F^2}\right)$$

$$-2i\Sigma_c^{(2)} =$$

$$\frac{3i}{8F^4 \left(l_1 \cdot l_1 - M^2 + i\varepsilon\right) \left(l_2 \cdot l_2 - M^2 + i\varepsilon\right) \left((l_1 + l_2 - p) \cdot (l_1 + l_2 - p) - m^2 + i\varepsilon\right)}$$

$$\Big[\mathbb{1}_d m(-2\,(l_1 \cdot l_2) + (l_1 \cdot l_1) + (l_2 \cdot l_2)) - \not{p}(-2\,(l_1 \cdot l_2) + (l_1 \cdot l_1) + (l_2 \cdot l_2))$$

$$-\not{l}_1(2\,(l_1 \cdot l_2) - 2\,(l_1 \cdot p) + (l_1 \cdot l_1) + 2\,(l_2 \cdot p) - 3\,(l_2 \cdot l_2))$$

$$+\not{l}_2(-2\,(l_1 \cdot l_2) - 2\,(l_1 \cdot p) + 3\,(l_1 \cdot l_1) + 2\,(l_2 \cdot p) - (l_2 \cdot l_2))\Big] \otimes \mathbb{1}_2$$

Calculate the Diagrams
add Zeros

- idea: reduce tensor rank by adding suitable zeros

$$\frac{l \cdot l}{l \cdot l - M^2 + i\varepsilon} = \frac{l \cdot l - M^2 + M^2}{l \cdot l - M^2 + i\varepsilon} = 1 + \frac{M^2}{l \cdot l - M^2 + i\varepsilon}$$

- (in the case of one loop diagrams) rewrite in the denominator

$$q \cdot q - m^2 + i0 \to P[q, m]$$

and in the numerator

$$l \cdot p \to -\frac{1}{2}\left[(l-p)\cdot(l-p) - (l \cdot l + p \cdot p)\right]$$

$$(l-p)\cdot(l-p) \to P[l-p, M] + m^2$$

$$l \cdot l \to P[l, M] + M^2$$

- individual procedure – for example:
  - rewrite $l \cdot p$ only when also $P[l, m]$ appears in the denominator
  - rewrite $(l \cdot p)^2$ only once for $P[l, m]$ to the power of one in the denominator

Calculate the Diagrams
add Zeros

further simplification:

- scale-less integrals vanish in dimensional renormalization

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} l^{\nu_1} l^{\nu_2} \to 0$$

- use T-notation
- cancel odd integrals

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\nu}{(p-l)\cdot(p-l)-m^2+i\varepsilon} = 0$$

- omit terms like $p \cdot p - m^2$

Calculate the Diagrams

add Zeros – Notation

$$T^{(1),\nu_1\ldots\nu_n}_{p,M,m}(d;\alpha_1,\alpha_2) =$$

$$= \int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^{\nu_1}\ldots l^{\nu_n}}{(l\cdot l - M^2 + i\varepsilon)^{\alpha_1}\left((l-p)\cdot(l-p) - m^2 + i\varepsilon\right)^{\alpha_2}}$$

$$T^{(2),\nu^1_1\ldots\nu^1_{n_1}\nu^2_1\ldots\nu^2_{n_2}}_{p,m_1,m_2,m_3,m_4,m_5}(d;\beta_1,\beta_2,\beta_3,\beta_4,\beta_5) =$$

$$= \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \int \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{l_1^{\nu^1_1}\ldots l_2^{\nu^1_{n_1}} l_2^{\nu^2_1}\ldots l_2^{\nu^2_{n_2}}}{(l_1\cdot l_1 - m_1^2 + i\varepsilon)^{\beta_1}(l_2\cdot l_2 - m_2^2 + i\varepsilon)^{\beta_2}}$$

$$\frac{1}{\left((l_1-p)\cdot(l_1-p) - m_3^2 + i\varepsilon\right)^{\beta_3}\left((l_2-p)\cdot(l_2-p) - m_4^2 + i\varepsilon\right)^{\beta_4}}$$

$$\frac{1}{\left((l_1+l_2-p)\cdot(l_1+l_2-p) - m_5^2 + i\varepsilon\right)^{\beta_5}}$$

Calculate the Diagrams
add Zeros – Example

RUB

$$
\begin{aligned}
&-2i\Sigma_c^{(2)} = \\
&-\frac{3ig_{\mu_1\nu_1}\left(p^{\mu_1}(\mathbb{1}_d m - \not{p}) - 2M^2\gamma^{\mu_1}\right) T^{(2),\nu_1^1}_{p,M,M,m,m,m}(1,1,0,0,1)}{4F^4} \\
&-\frac{3ig_{\mu_1\nu_1}\left(p^{\mu_1}(\mathbb{1}_d m - \not{p}) - 2M^2\gamma^{\mu_1}\right) T^{(2),\nu_1^2}_{p,M,M,m,m,m}(1,1,0,0,1)}{4F^4} \\
&+\frac{3iM^2(\mathbb{1}_d m - \not{p}) T^{(2)}_{p,M,M,m,m,m}(1,1,0,0,1)}{2F^4} + \frac{3i(\mathbb{1}_d m - \not{p}) T^{(2)}_{p,M,M,m,m,m}(0,1,0,0,1)}{4F^4} \\
&+\frac{3i(\mathbb{1}_d m - \not{p}) T^{(2)}_{p,M,M,m,m,m}(1,0,0,0,1)}{4F^4} - \frac{3i(\mathbb{1}_d m - \not{p}) T^{(2)}_{p,M,M,m,m,m}(1,1,0,0,0)}{8F^4} \\
&-\frac{3i\gamma^{\mu_1}p^{\mu_2}\left(g_{\mu_1\nu_2}g_{\mu_2\nu_1} + g_{\mu_1\nu_1}g_{\mu_2\nu_2}\right) T^{(2),\nu_1^1\nu_2^2}_{p,M,M,m,m,m}(1,1,0,0,1)}{2F^4} \\
&+\frac{3i\gamma^{\mu_1}g_{\mu_1\nu_1} T^{(2),\nu_1^2}_{p,M,M,m,m,m}(0,1,0,0,1)}{2F^4} + \frac{3i\gamma^{\mu_1}g_{\mu_1\nu_1} T^{(2),\nu_1^1}_{p,M,M,m,m,m}(1,0,0,0,1)}{2F^4} \\
&-\frac{3i\gamma^{\mu_1}g_{\mu_1\nu_1} T^{(2),\nu_1^1}_{p,M,M,m,m,m}(1,1,0,0,0)}{8F^4} - \frac{3i\gamma^{\mu_1}g_{\mu_1\nu_1} T^{(2),\nu_1^2}_{p,M,M,m,m,m}(1,1,0,0,0)}{8F^4}
\end{aligned}
$$

Calculate the Diagrams

scalar integrals

- factorize into one loop integrals for example

$$T^{\nu_1^1\ldots\nu_{n_1}^1\nu_1^2\ldots\nu_{n_2}^2}_{p,\{m_k\}_{k=1}^5}(d;\alpha_1,\alpha_2,0,0,0) = T^{\nu_1\ldots\nu_{n_1}}_{p,m_1,0}(d;\alpha_1,0)\, T^{\nu_1\ldots\nu_{n_2}}_{p,m_2,0}(d;\alpha_2,0)$$

$$T^{\nu_1^1\ldots\nu_{n_1}^1\nu_1^2\ldots\nu_{n_2}^2}_{p,\{m_k\}_{k=1}^5}(d;0,\alpha_2,0,0,\alpha_5) = \int\frac{\mathrm{d}^d l_1}{(2\pi)^d}\int\frac{\mathrm{d}^d l_2}{(2\pi)^d}$$

$$\frac{1}{\left(l_1\cdot l_1 - m_5^2 + i\varepsilon\right)^{\alpha_5}\left(l_2\cdot l_2 - m_2^2 + i\varepsilon\right)^{\alpha_2}}\left(l_1^{\nu_1^1}\ldots l_1^{\nu_{n_1}^1} l_2^{\nu_1^2}\ldots l_2^{\nu_{n_2}^2}\right)\Big|_{l_1\to l_1-l_2+p}$$

- reduce tensor integrals to integrals in $d+\ldots$-dimensions
  1. Rewrite the tensor structure by using derivatives.
  2. Rewrite the propagators with the Schwinger trick/ in λ-representation
  3. Evaluate the integral(s) over the loop momentum/ momenta per Gaussian integration.
  4. Construct a operator and reverse the Gaussian integration and the λ-parametrization for the scalar integral.
  5. Apply the operator to the scalar integral.

Calculate the Diagrams
scalar integrals

$$T^{(1),\nu_1\ldots\nu_n}_{p,m_1,m_2}(d;\alpha_1,\alpha_2) = \int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^{\nu_1}\ldots l^{\nu_n}}{\left((l\cdot l) - m_1^2 + i\varepsilon\right)^{\alpha_1}\left((l-p)\cdot(l-p) - m_2^2 + i\varepsilon\right)^{\alpha_2}}$$

- first two steps yield

$$(-i)^n \prod_{k=1}^{n} \frac{\partial}{\partial a_{\nu_k}}\Big|_{a=0} \frac{(-i)^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_0^\infty \int_0^\infty \mathrm{d}\lambda_1 \mathrm{d}\lambda_2\, \lambda_1^{\alpha_1-1}\lambda_2^{\alpha_2-1}\, G^{(1)}$$

- Gauss integration

$$G^{(1)} = \frac{1}{(2\pi)^d} i \left(\frac{\pi}{i}\right)^{\frac{d}{2}} \frac{1}{(D(\lambda))^{\frac{d}{2}}} \exp\left(i\frac{Q(\lambda,a)}{D(\lambda)}\right)$$

$$\exp\left(i\lambda_1\left(-m_1^2 + i\varepsilon\right) + i\lambda_2\left(p^2 - m_2^2 + i\varepsilon\right) - i\frac{\lambda_2^2 p^2}{\lambda_1+\lambda_2}\right)$$

$$D(\lambda) = \lambda_1 + \lambda_2 \quad Q(\lambda,a,b) = (p\cdot a)\,Q_1 + a^2 Q_{11}$$

$$Q_1 = \lambda_2 \quad Q_{11} = -\frac{1}{4}$$

Calculate the Diagrams

scalar integrals

$$G^{(1)} = \frac{1}{(2\pi)^d} i \left(\frac{\pi}{i}\right)^{\frac{d}{2}} \frac{1}{(D(\lambda))^{\frac{d}{2}}} \exp\left(i \frac{Q(\lambda, a)}{D(\lambda)}\right)$$

$$\exp\left(i\lambda_1\left(-m_1^2 + i\varepsilon\right) + i\lambda_2\left(p^2 - m_2^2 + i\varepsilon\right) - i\frac{\lambda_2^2 p^2}{\lambda_1 + \lambda_2}\right)$$

$$\frac{1}{D(\lambda)} \frac{1}{(2\pi)^d} \left(\frac{\pi}{i}\right)^{\frac{d}{2}} \frac{1}{(D(\lambda))^{\frac{d}{2}}} = (2\pi)^2 \frac{i}{\pi} \frac{1}{(2\pi)^{d+2}} \left(\frac{\pi}{i}\right)^{\frac{d+2}{2}} \frac{1}{(D(\lambda))^{\frac{d+2}{2}}}.$$

$$\frac{1}{D(\lambda)} \hat{=} i4\pi^2 \boldsymbol{d}^+$$

$$\lambda_i \hat{=} i\frac{\partial}{\partial m_i^2} \quad \text{(numerator)}$$

Calculate the Diagrams

scalar integrals

one can build a operator

$$T^{\nu_1\ldots\nu_n}_{p,m_1,m_2}\left(d;\alpha_1,\alpha_2\right)=\boldsymbol{T}^{\nu_1\ldots\nu_n}\left(p,\{\partial/\partial m^2\},\boldsymbol{d}^+\right)T_{p,m_1,m_2}\left(d;\alpha_1,\alpha_2\right)$$

$$\boldsymbol{T}^{\nu_1\ldots\nu_n}\left(p,\{\partial/\partial m^2\},\boldsymbol{d}^+\right)=(-i)^n\prod_{k=1}^{n}\frac{\partial}{\partial a_{\nu_k}}\exp\left(iQ\left(\lambda,a\right)\rho\right)\Bigg|_{\substack{a=0\\ \lambda_j=i\frac{\partial}{\partial m_j^2}\\ \rho=i4\pi\boldsymbol{d}^+}}$$

similar in the two loop case

$$T^{\nu_1^1\ldots\nu_{n_1}^1\nu_1^2\ldots\nu_{n_2}^2}_{p,\{m_k\}_{k=1}^5}\left(d;\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5\right)$$

$$=\boldsymbol{T}^{\nu_1^1\ldots\nu_{n_1}^1\nu_1^2\ldots\nu_{n_2}^2}\left(p,\{\partial/\partial m^2\},\boldsymbol{d}^+\right)T_{p,\{m_k\}_{k=1}^5}\left(d;\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5\right)$$

$$\boldsymbol{T}^{\nu_1^1\ldots\nu_{n_1}^1\nu_1^2\ldots\nu_{n_2}^2}\left(p,\{\partial/\partial m^2\},\boldsymbol{d}^+\right)$$

$$=(-i)^{n_1+n_2}\prod_{k_1=1}^{n_1}\prod_{k_2=1}^{n_2}\frac{\partial}{\partial a_{\nu_{k_1}}}\frac{\partial}{\partial b_{\nu_{k_2}}}\exp\left(iQ\left(\lambda,a,b\right)\rho\right)\Bigg|_{\substack{a=b=0\\ \lambda_j=i\frac{\partial}{\partial m_j^2}\\ \rho=-16\pi^2\boldsymbol{d}^+}}$$

Calculate the Diagrams
scalar integrals – Example

$$-2i\Sigma_c^{(2)} =$$

$$\frac{12i\pi^2\left(\not{p}\left((p\cdot p)+2M^2\right)-\mathbb{1}_d m\,(p\cdot p)\right)\,T^{(2)}_{p,m_1,m_2,m_3,m_4,m_5}(d+2;1,2,0,0,2)}{F^4}$$

$$+\frac{12i\pi^2\left(\not{p}\left((p\cdot p)+2M^2\right)-\mathbb{1}_d m\,(p\cdot p)\right)\,T^{(2)}_{p,m_1,m_2,m_3,m_4,m_5}(d+2;2,1,0,0,2)}{F^4}$$

$$+\frac{3iM^2(\mathbb{1}_d m-\not{p})T^{(2)}_{p,m_1,m_2,m_3,m_4,m_5}(d;1,1,0,0,1)}{2F^4}$$

$$-\frac{3iT(m_5,0)(d;1,0)(\not{p}(T^{(1)}_{m_1,0}(d;1,0)+T(m_2,0)(d;1,0))-\mathbb{1}_d m T^{(1)}_{m_2,0}(d;1,0))}{4F^4}$$

$$-\frac{3iT^{(1)}_{m_1,0}(d;1,0)((\mathbb{1}_d m-\not{p})T^{(1)}_{m_2,0}(d;1,0)-2\mathbb{1}_d m T^{(1)}_{m_5,0}(d;1,0))}{8F^4}$$

$$-\frac{1536i\pi^4\not{p}\,(p\cdot p)\,T^{(2)}_{p,m_1,m_2,m_3,m_4,m_5}(d+4;2,2,0,0,3)}{F^4}$$

$$+\frac{24i\pi^2\not{p}T^{(2)}_{p,m_1,m_2,m_3,m_4,m_5}(d+2;1,1,0,0,2)}{F^4}$$

Calculate the Diagrams

Master Integrals – Integration by Parts

RUB

$$
\begin{aligned}
0 &= \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \int \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{\partial}{\partial l_1^\mu} l_1^\mu P_{l_1,m_1}^{\alpha_1} P_{l_2,m_2}^{\alpha_2} P_{l_1-l_2,m_5}^{\alpha_5} \\
&= \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \int \frac{\mathrm{d}^d l_2}{(2\pi)^d} d P_{l_1,m_1}^{\alpha_1} P_{l_2,m_2}^{\alpha_2} P_{l_1+l_2-p,m_5}^{\alpha_5} - 2\alpha_1 \left(l_1 \cdot l_1\right) P_{l_1,m_1}^{\alpha_1+1} P_{l_2,m_2}^{\alpha_2} P_{l_1-l_2,m_5}^{\alpha_5} \\
&\quad -2\alpha_2 \left(l_1 \cdot l_2\right) P_{l_1,m_1}^{\alpha_1} P_{l_2,m_2}^{\alpha_2+1} P_{l_1-l_2,m_5}^{\alpha_5} - 2\alpha_5 \left(\left(l_1 \cdot l_1\right) - \left(l_1 \cdot l_2\right)\right) P_{l_1,m_1}^{\alpha_1} P_{l_2,m_2}^{\alpha_2} P_{l_1-l_2,m_5}^{\alpha_5+1}
\end{aligned}
$$

one can cancel the scalar products to obtain

$$
\begin{aligned}
&\Bigg[d - 2\alpha_1 \left(1 + m_1^2 \mathbf{1}^+\right) - 2\alpha_2 \left(1 + m_2^2 \mathbf{2}^+\right) \\
&-2\alpha_5 \mathbf{5}^+ \left(\mathbf{1}^- + m_1^2 + \frac{1}{2} \left(\mathbf{5}^- + m_5^2 - \mathbf{1}^- - m_1^2 - \mathbf{2}^- - m_2^2 \right) \right) \Bigg] \\
&\bar{T}^{(2)}_{p,\{m_k\}_{k=1}^5} \left(d; \alpha_1, \alpha_2, 0, 0, \alpha_5\right) = 0
\end{aligned}
$$

Calculate the Diagrams
Master Integrals – Dimension Reduction

$$T^{(1)}_{p,m_1,m_2}(d;\alpha_1,\alpha_2)$$
$$= \frac{(-i)^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \prod_{j=1}^{2} \int_0^\infty \mathrm{d}\lambda_j \, \lambda_j^{\alpha_j-1} \frac{1}{(2\pi)^d} i \left(\frac{\pi}{i}\right)^{\frac{d}{2}} \frac{1}{(D(\lambda))^{\frac{d}{2}}}$$
$$\exp\left(i\lambda_j \left(\hat{l}_j - m_j^2 + i\varepsilon\right) - i\frac{\lambda_2^2 p^2}{D(\lambda)}\right)$$

apply on both sides $D(\lambda) = \lambda_1 + \lambda_2$ in operator form

$$\boldsymbol{D}(\lambda) = i\partial_{m_1^2} + i\partial_{m_2^2}$$

gives

$$i4\pi\, T^{(1)}_{p,m_1,m_2}(d-2;\alpha_1,\alpha_2)$$
$$= i\alpha_1\, T^{(1)}_{p,m_1,m_2}(d;\alpha_1+1,\alpha_2) + i\alpha_2\, T^{(1)}_{p,m_1,m_2}(d;\alpha_1,\alpha_2+1)$$

Calculate the Diagrams
Master Integrals

- relations between integrals in same dimension
  integration by parts
- relations between integrals in different dimensions
  Schwinger representation and Gauss integration

$\Rightarrow$ recurrence relations to reduce integrals to a set of
Master Integrals

Calculate the Diagrams
Master Integrals – Example

RUB

$$
\begin{aligned}
&-i\Sigma_c^{(2)} = \\
&-\frac{1}{2}\frac{i}{2(d-2)(3d-4)F^4 m}\Big\{2\Big[\left(2\left(2d^2-7d+5\right)m^4\right. \\
&+\left(-7d^2+23d-16\right)m^2M^2+\left(d^2-5d+6\right)M^4\Big)\,T^{(2)}_{M,m,0,0,M}(d;1,1,0,0,1) \\
&-4M^2\left(M^2-m^2\right)\left((d-2)M^2-2(d-1)m^2\right)T^{(2)}_{M,m,0,0,M}(d;2,1,0,0,1)\Big] \\
&+(d-2)\left((d-1)m^2+(d-2)M^2\right)\left(T^{(1)}_{M,0}(d;1,0)\right)^2 \\
&+(2-d)\left(4(d-1)m^2+(d-2)M^2\right)T^{(1)}_{m,0}(d;1,0)\,T^{(1)}_{M,0}(d;1,0)\Big\}
\end{aligned}
$$

Calculate the Diagrams
Master Integrals – one loop

RUB

$$-i\Sigma_a^{(1)} = \frac{3g_A^2}{8F^2\,(p\cdot p)}\Bigg\{\not{p}\left(m^2-(p\cdot p)\right)T(M,0)(\{d\},1,0)$$
$$+\left(-\not{p}\left((p\cdot p)+m^2\right)-2m\,(p\cdot p)\right)T(m,0)(\{d\},1,0)$$
$$+\left(\not{p}\left(-\left(2m^2+M^2\right)(p\cdot p)+(p\cdot p)^2+m^4-m^2M^2\right)-2mM^2\,(p\cdot p)\right)T(M,m)(\{d\},1,1)\Bigg\}$$

$$-i\Sigma_b^{(1)} = -\,i\Sigma_c^{(1)} = \frac{3g_A mM^2}{F^2}\Big\{M^2(\mathrm{d18}-2\mathrm{d16})T(M,m)(\{d\},1,1)+(\mathrm{d18}-2\mathrm{d16})T(m,0)(\{d\},1,0)\Big\}$$

$$-i\Sigma_d^{(1)} = \frac{3g_A^2}{2F^4\left(4m^2-M^2\right)}\Bigg\{-2mM^4\Bigg[l_3\left(2(d-1)m^2-(d-2)M^2\right)$$
$$+l_4\left(M^2-4m^2\right)\Bigg]T(M,m)(\{d\},1,1)$$
$$+2m\left((d-2)l_3M^4+l_4\left(4m^2M^2-M^4\right)\right)T(m,0)(\{d\},1,0)$$
$$-(d-2)l_3mM^4T(M,0)(\{d\},1,0)\Bigg\}$$

Calculate the Diagrams
Master Integrals – one loop

$$-i\Sigma_e^{(1)} = \frac{1}{2}\frac{6M^2 T(M,0)(\{d\},1,0)\left(dm^2(2\mathrm{c}1-\mathrm{c}3)-\mathrm{c}2m^2\right)}{dF^2m^2}$$

$$-i\Sigma_f^{(1)} = -\frac{1}{2}\frac{24M^4 T(M,0)(\{d\},1,0)}{d(d+2)F^2}\Big\{d^2(2e_{14}+2e_{19}-e_{36}-4e_{38}) + 2d(2e_{14}+e_{15}+2e_{19}+e_{20}+e_{35}-e_{36}-4e_{38}) + 4e_{15}+6e_{16}+4e_{20}+4e_{35}\Big\}$$

$$-i\Sigma_g^{(1)} = -\frac{1}{2}\frac{6M^4(2\mathrm{c}1d(2l_4-(d-2)l_3)+(\mathrm{c}2+\mathrm{c}3d)(dl_3-2l_4))T(M,0)(\{d\},1,0)}{dF^4}$$

Calculate the Diagrams

Master Integrals – two loop

RUB

$$-i\Sigma_k^{(2)} = -\, i\Sigma_l^{(2)} = \frac{1}{2}\,\frac{6i(10\alpha-1)g_A^2 m T(M,0)(\{d\},1,0)\left(M^2 T^{(1)}_{M,m}(1,1) + T^{(1)}_{m,0}(1,0)\right)}{4F^4}$$

$$-i\Sigma_m^{(2)} = -\,\frac{1}{8}\,\frac{12iM^2(T^{(1)}_{M,0}(1,0))^2\left((2-20\alpha)c_2 m^2 + dm^2(5(8\alpha-1)c_1 - 20\alpha c_3 + 2c_3)\right)}{dF^4 m^2}$$

$$-i\Sigma_n^{(2)} = \frac{1}{4}\,\frac{3iM^2(T^{(1)}_{M,0}(1,0))^2\left(dm^2(2c_1(40\alpha+d-6) - c_3(40\alpha+d-4)) - c_2(40\alpha+d-4)m^2\right)}{dF^4 m^2}$$

$$-i\Sigma_o^{(2)} = -\,\frac{1}{2}\,\frac{6ig_A^2}{8F^4\left(4m^2-M^2\right)}$$

$$mT^{(1)}_{M,0}(1,0)\Bigg[M^2\Bigg(2\left((d-3)m^2 + m^2(80\alpha+d-7) - M^2(20\alpha+d-4)\right)$$

$$T^{(1)}_{M,m}(1,1) + (d-2)T^{(1)}_{M,0}(1,0)\Bigg) + 2\left((80\alpha-8)m^2 - M^2(20\alpha+d-4)\right)T^{(1)}_{m,0}(1,0)\Bigg]$$

Extended-on-mass-shell Renormalization

one loop results

$$T_\pi = T^{(1)}_{M,0}(d;1,0) \qquad R = \frac{2}{d-4} - [\ln(4\pi) + \gamma_E + 1]$$

$$T_N = T^{(1)}_{m,0}(d;1,0) \qquad \Omega = \frac{p\cdot p - m_1^2 - m_2^2}{2m_1 m_2}$$

$$T_{\pi N} = T^{(1)}_{M,m}(d;1,1) \qquad F(\Omega) = \sqrt{\Omega^2-1}\arccos(-\Omega) \qquad -1 \le \Omega \le 1$$

Dimensional Regularization leads to

$$\mu^{4-d} T_\pi = -i\frac{M^2}{16\pi^2}\left[R + \ln\left(\frac{M^2}{\mu^2}\right)\right] + O(4-d)$$

$$\mu^{4-d} T_N = -i\frac{m^2}{16\pi^2}\left[R + \ln\left(\frac{m^2}{\mu^2}\right)\right] + O(4-d)$$

$$\mu^{4-d} T_{\pi N}$$

$$= -i\frac{1}{16\pi^2}\left[R + \ln\left(\frac{m^2}{\mu^2}\right) - 1 + \frac{p\cdot p - m^2 - M^2}{p\cdot p}\ln\left(\frac{M}{m}\right) + \frac{2Mm}{p\cdot p}F(\Omega)\right]$$

$$+O(4-d)$$

Extended-on-mass-shell Renormalization

one loop results

Dimensional Regularization ($\mu = m$, omit $O(4-d)$) leads to

$$\mu^{4-d} T_\pi = -i \frac{M^2}{16\pi^2} \left[R + \ln\left(\frac{M^2}{m^2}\right) \right]$$

$$\mu^{4-d} T_N = -i \frac{m^2}{16\pi^2} R$$

$$\mu^{4-d} T_{\pi N} = -i \frac{1}{16\pi^2} \left[R - 1 + \frac{p \cdot p - m^2 - M^2}{p \cdot p} \ln\left(\frac{M}{m}\right) + \frac{2Mm}{p \cdot p} F(\Omega) \right]$$

using $\widetilde{\text{MS}}$ and let $d \to 4$

$$T_\pi = -i \frac{M^2}{16\pi^2} \ln\left(\frac{M^2}{m^2}\right)$$

$$T_{\pi N} = -i \frac{1}{16\pi^2} \left[-1 + \frac{p \cdot p - m^2 - M^2}{p \cdot p} \ln\left(\frac{M}{m}\right) + \frac{2Mm}{p \cdot p} F(\Omega) \right]$$

PC: $T_\pi = \mathcal{O}(q^2)$, $mT_N = \mathcal{O}(q^3)$ and $mT_{\pi N} = \mathcal{O}(q)$

Extended-on-mass-shell Renormalization

one loop results

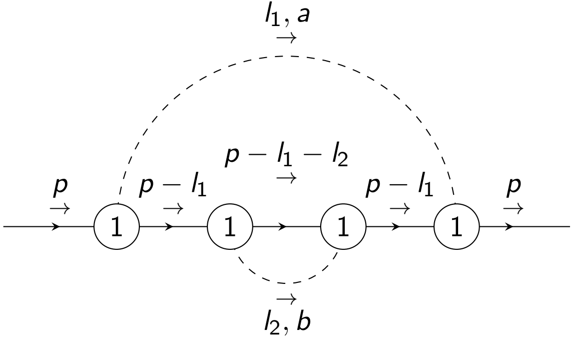
EOMS leads to

$$T_\pi = -i\frac{M^2}{16\pi^2}\ln\left(\frac{M^2}{m^2}\right)$$

$$T_N = 0$$

$$T_{\pi N} = -i\frac{1}{16\pi^2}\left[\frac{p\cdot p - m^2 - M^2}{p\cdot p}\ln\left(\frac{M}{m}\right) + \frac{2Mm}{p\cdot p}F(\Omega)\right]$$

Extended-on-mass-shell Renormalization
two loop idea



problematic cases

1. $l_1 = \mathcal{O}(q)$ and $l_2 >> q$
2. $l_1 >> q$ and $l_2 = \mathcal{O}(q)$
3. $l_1 >> q$ and $l_2 >> q$
4. $l_1 >> q$ and $l_2 >> q$

Conclusion and Outlook

- Using a naive power counting scheme all self-energy diagrams up to chiral order $\mathcal{O}(q^6)$ are constructed.
- The expressions of the diagrams contain tensor integrals.
- The Tensor Integrals are reduced by (adding zeros and) going to higher dimensions.
- All Integrals are reduced to a set of Master Integrals in d-dimension.
- The renormalized expression of diagrams are derived for one loop and "factorizing" two loop diagrams.

outlook

- renormalize all two loop integrals
- get a numeric expression for the nucleon mass depending on the pion mass

Thanks for your attention!

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