Bringing Light into the Darkness

— *About Dark Matter Scattering off Light Nuclei*

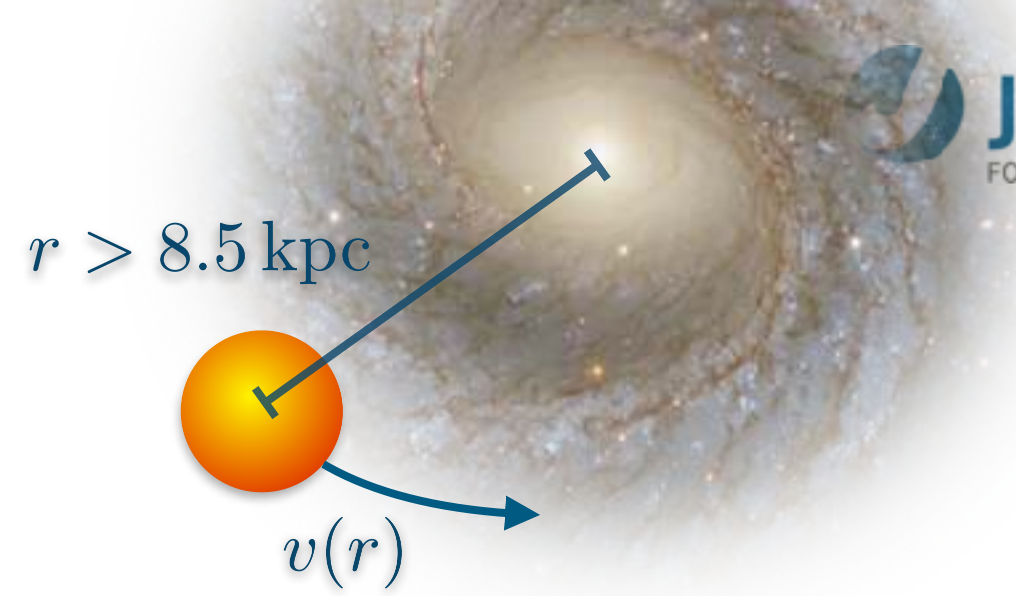
In Collaboration with **Jordy de Vries** and **Andreas Nogga**

[arxiv:1704.01150]

13. July 2016 | **Christopher Körber** | IKP-3/IAS-4 | Seminar@TP2 — Ruhr-Universität Bochum
PhD Advisor: **Tom Luu**

Dark Matter

— So far only Gravitational Evidence



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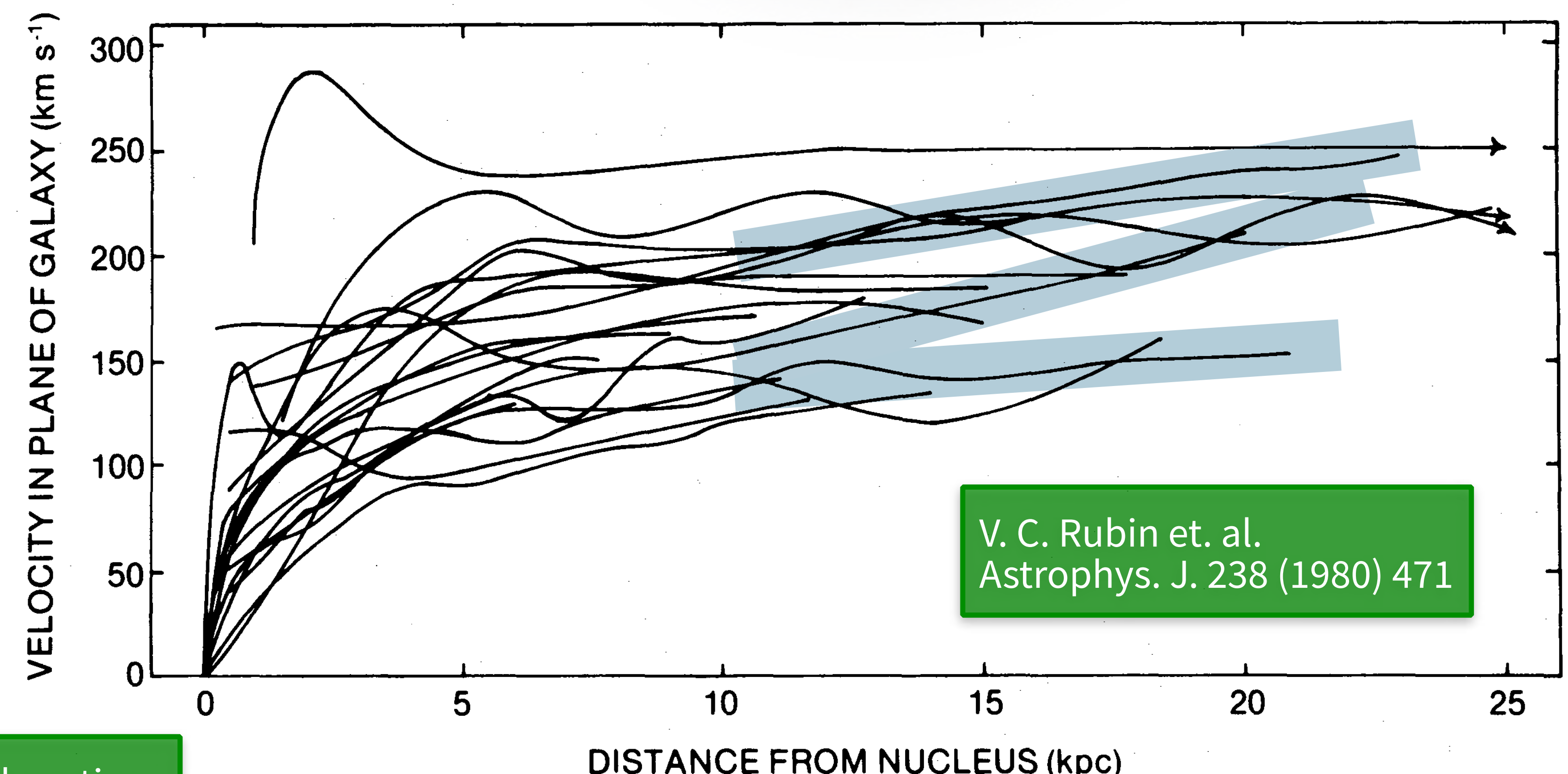
Rotation Curves: Milky Way

- Newtonian prediction

$$v(r) \propto \frac{1}{\sqrt{r}}$$

- Experimental observations

$$v(r) \propto r \Rightarrow \frac{M_{DM}}{M_M} \approx 20$$



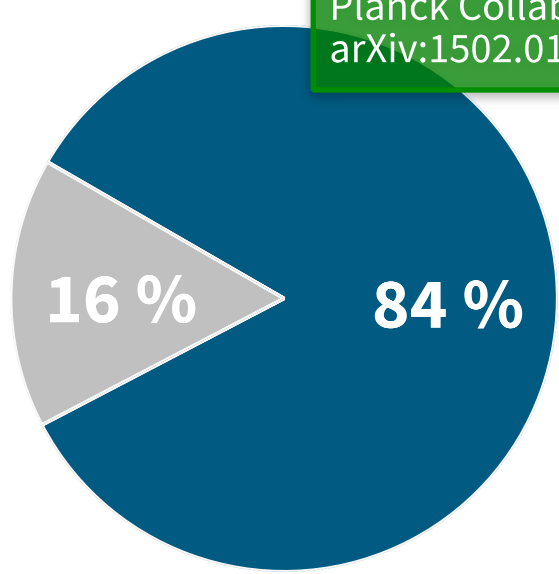
V. C. Rubin et. al.
Astrophys. J. 238 (1980) 471

Cosmological Parameters

- ΛCDM model

Planck Collaboration
arXiv:1502.01589v3

Baryonic Matter

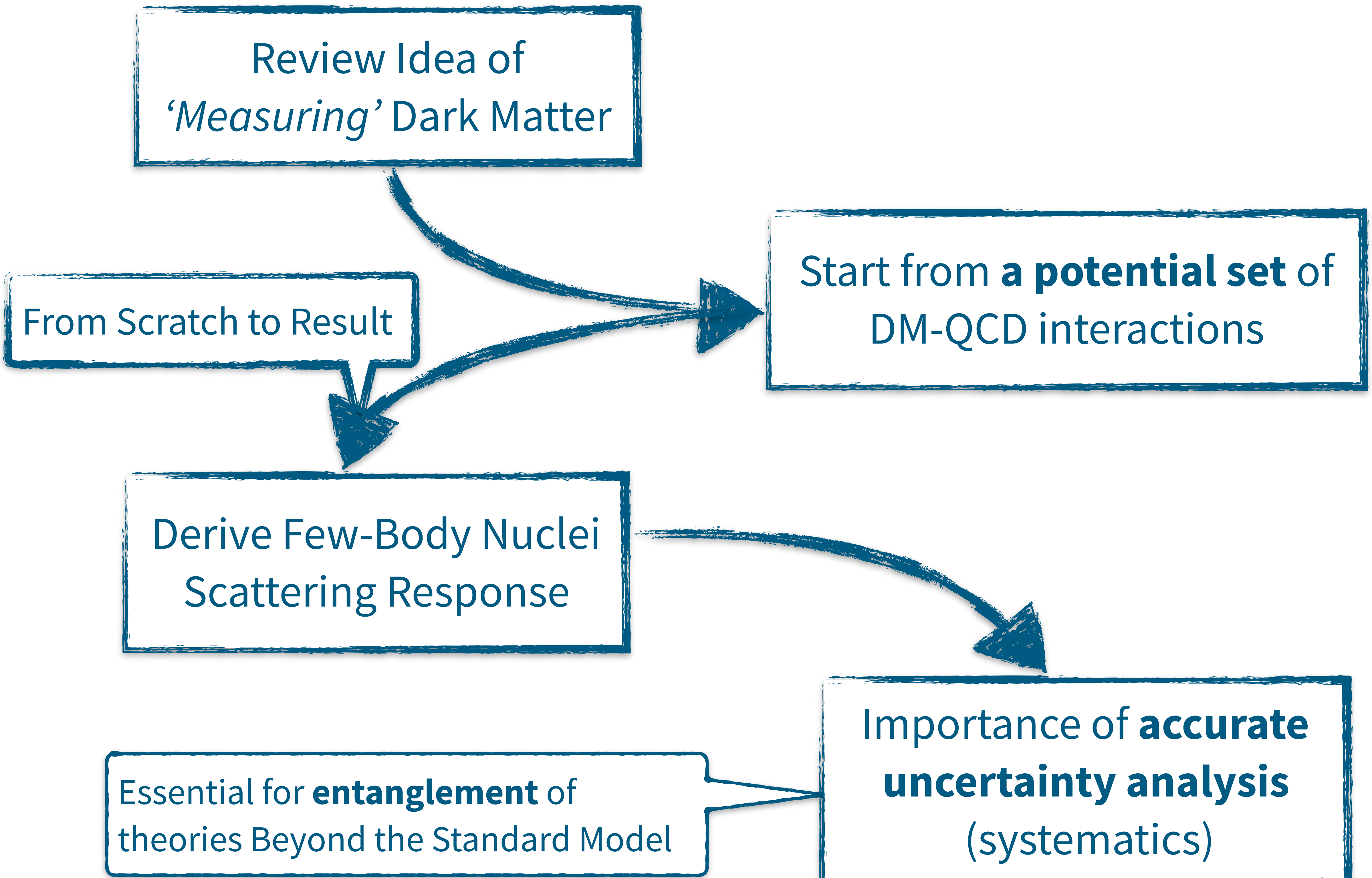


Dark Matter

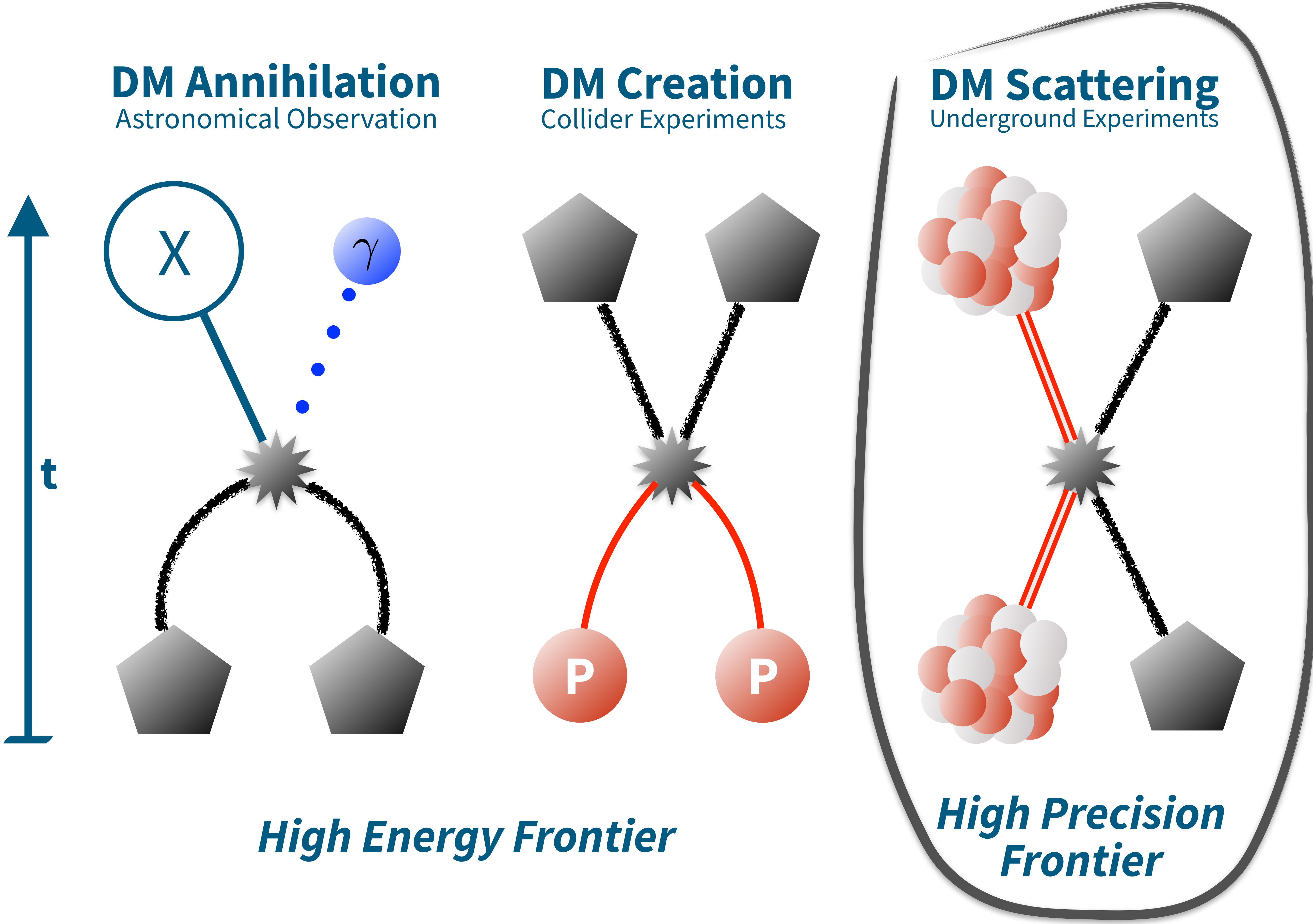
How to resolve this?

- **Assumption:** Existence of DM particle candidate
- **Connect experiments** on fundamental scales to theories **Beyond the Standard Model**

The Next ~40 Minutes

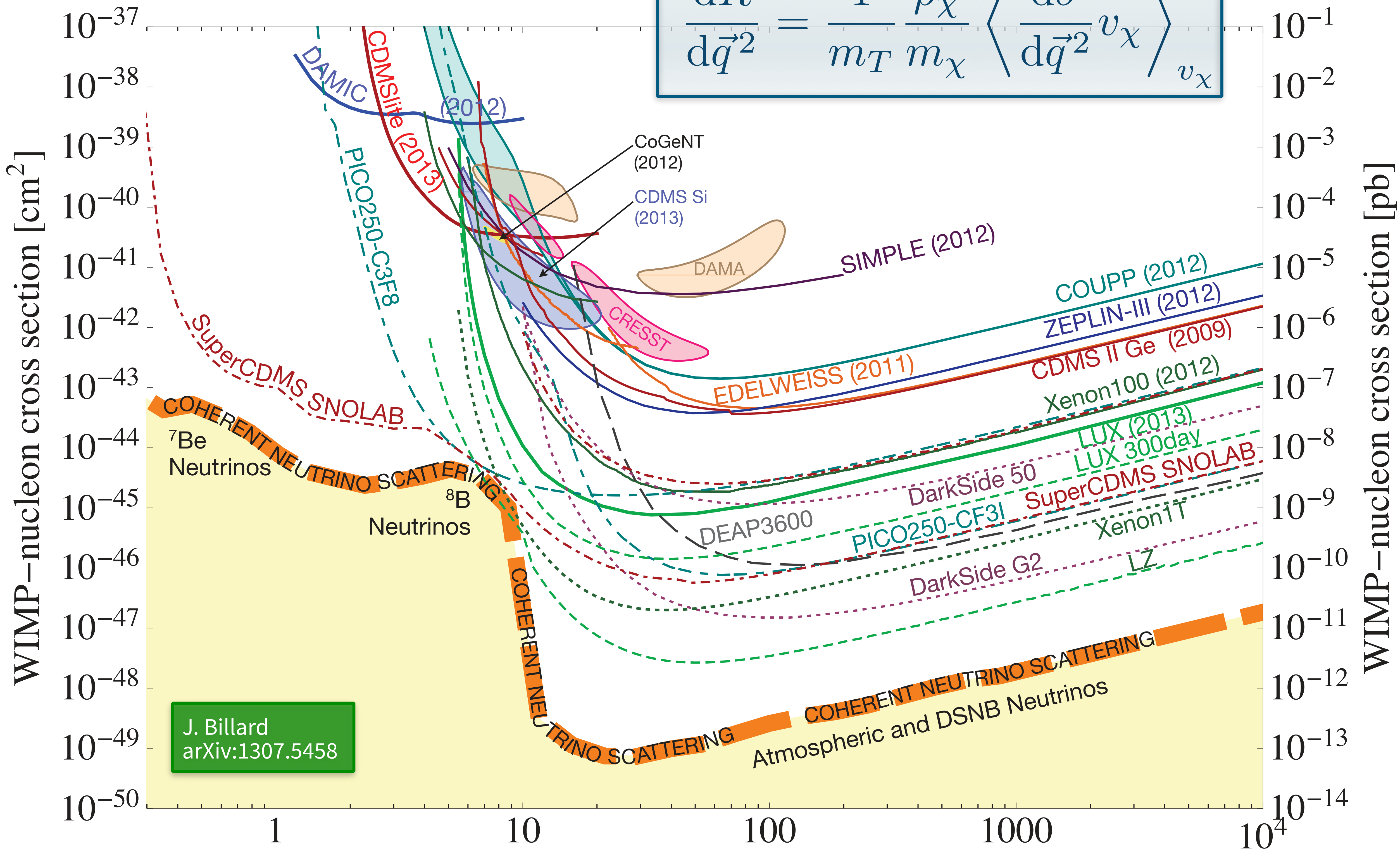


Experiments on Fundamental Scale



Direct Detection Bounds

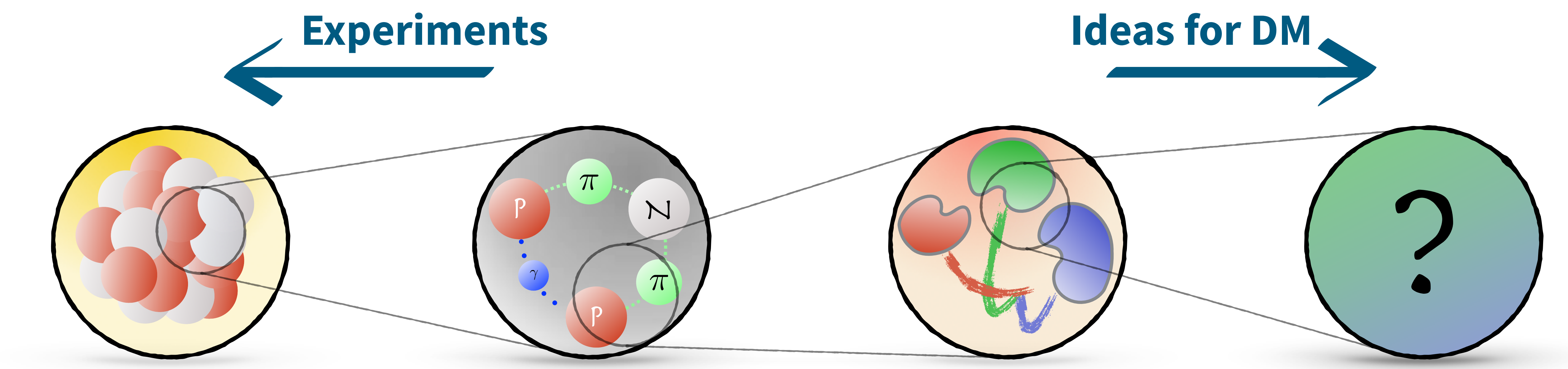
$$\frac{\mathrm{d}R}{\mathrm{d}\vec{q}^{\,2}} = \frac{1}{m_T}\frac{\rho_\chi}{m_\chi}\left\langle \frac{\mathrm{d}\sigma}{\mathrm{d}\vec{q}^{\,2}} v_\chi \right\rangle_{v_\chi}$$



Proposed experiments for light nuclei (^4He)

W. Guo and D. N. McKinsey
arXiv:1302.0534

From Theory to Experiment

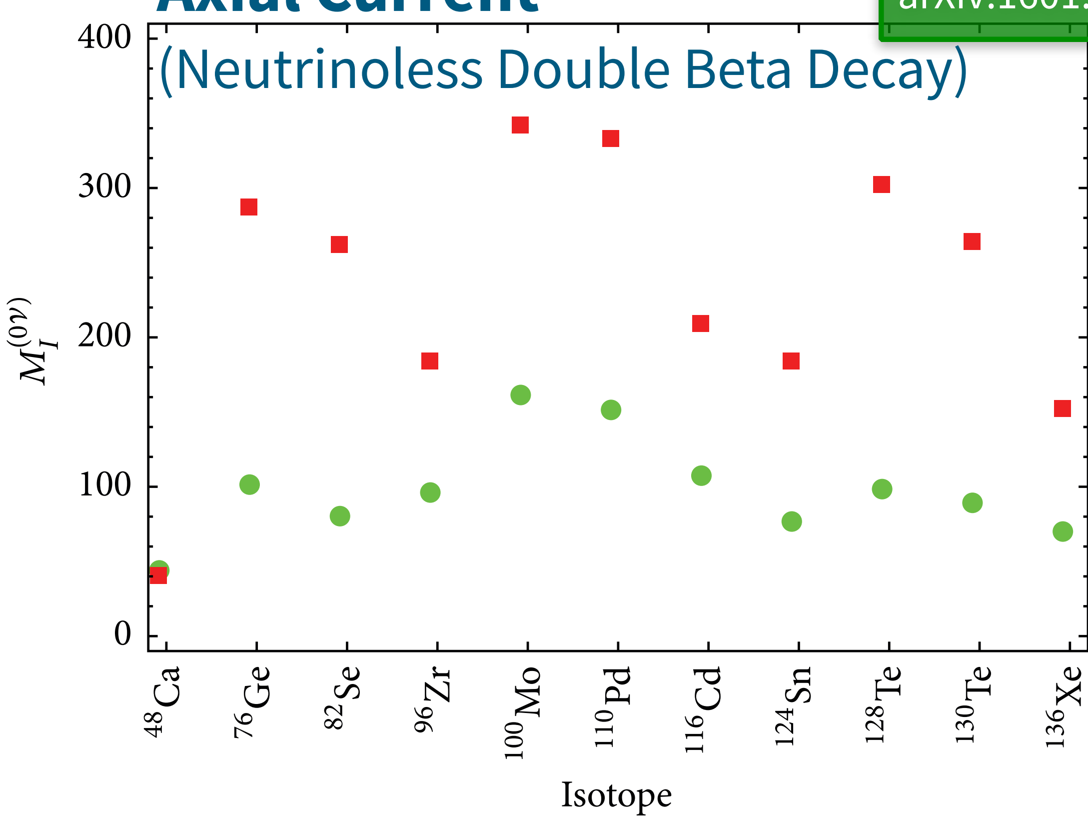


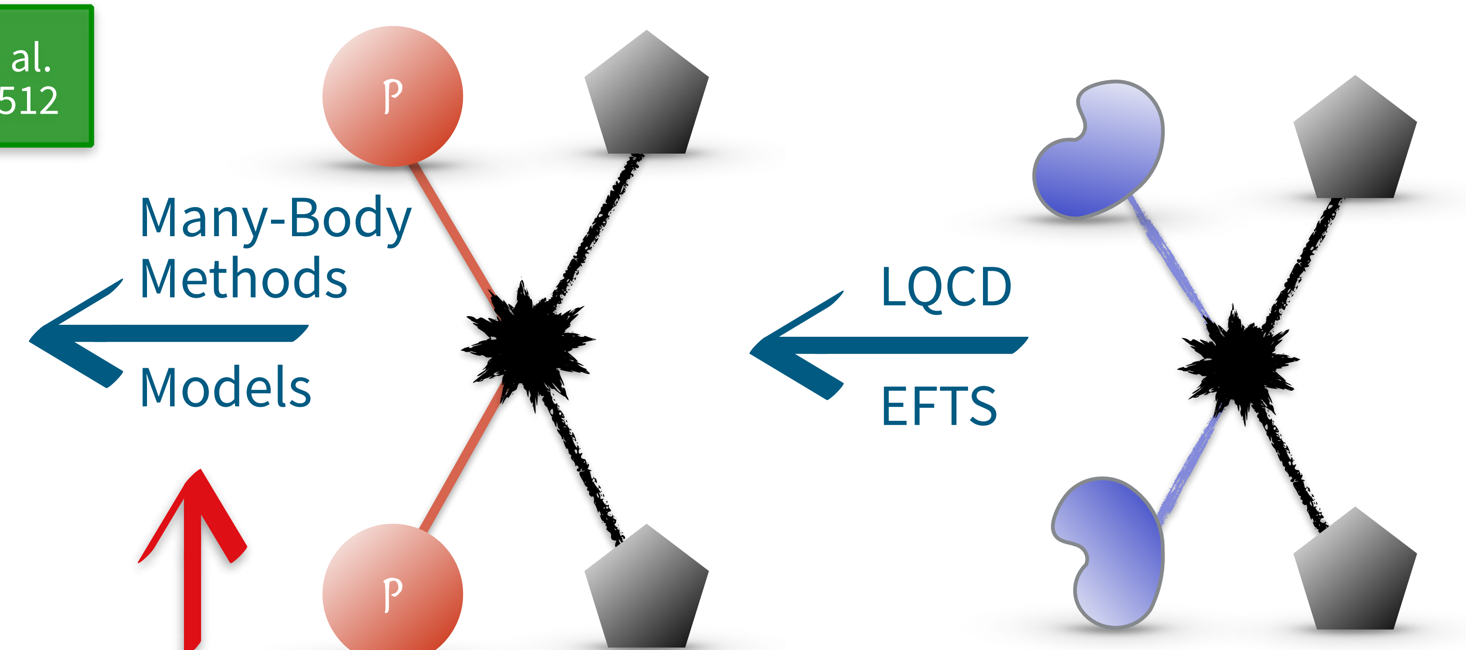
Nuclear Level	Hadronic Level	Standard Model Level	Beyond SM Level
Nuclear cores	Protons, Neutron, Pions, …	Quarks, Gluons, Leptons, …	Dark Matter?

Nuclear Matrix Element Axial Current

S. Dell'Oro et. al. arXiv:1601.07512

(Neutrinoless Double Beta Decay)





Source of largest **theoretical uncertainties**

DM Theory Building: most general theory fulfilling desired symmetries

A Dark-Matter EFT

Dark Matter Interactions

— From Quarks to Nucleons

Effective Approaches
- **M. Hoferichter, A. Schwenk, …** arXiv:1605.08043
- **F. L. Fitzpatrick, W. Haxton, …** arXiv:1203.3542
- **V. Cirigliano, M. L. Graesser, …** arXiv:1205.2695

Possible Dark Matter Interaction

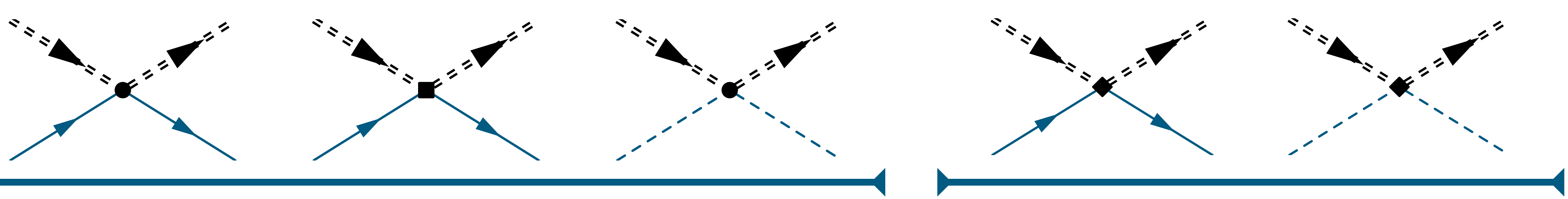
- Scalar interaction of DM with quarks and gluons

$$\mathscr{L}_{DM}^{(QCD)} = \bar{\chi}\chi \left[\sum_{f=u,d,s} \bar{c}_f m_f \, \bar{q}_f q_f + \bar{c}_g \alpha_s G_{\mu\nu}^{a} G^{\mu\nu\, a} \right]$$

Propagation of Scales

- Effective chiral Lagrangian based on underlaying interactions
- Depends on chiral as well as lattice input (sigma term, mass splitting, …)
- Gluon and strange quark structures can be grouped together (similar terms)

$$\mathscr{L}_{DM}^{(N\pi)} = \bar{\chi}\chi \left[c_q^{(ic)} \bar{N}N + c_q^{(iv)} \bar{N}\tau^3 N + c_q^{(\pi)} \boldsymbol{\pi}^2 \quad + \quad c_g^{(ic)} \bar{N}N + c_g^{(\pi)} \left((\partial_\mu \boldsymbol{\pi})^2 - 2m_\pi^2 \pi^2 \right) \right]$$



Quark structures **Gluon structures**

Dark Matter Interactions

— From QCD to Chiral Lagrangian

Idea: Compare to existing framework of regular Chiral Perturbation Theory (ChPT)

A Primer for Chiral Perturbation Theory
S. Scherer and M. R. Schindler

$$\mathscr{L}_{DM}^{(QCD)} = (\bar{c}_u m_u \, \bar{\chi}\chi) \, \bar{u}u + (\bar{c}_d m_d \, \bar{\chi}\chi) \, \bar{d}d + \cdots$$

- DM interaction as mass term

$$\chi_{\mathcal{M}} = 2B_0 \mathcal{M} \mapsto 2B_0 \left[\mathcal{M} - \text{diag}\left(\bar{c}_u m_u \, \bar{\chi}\chi, \, \bar{c}_d m_d \, \bar{\chi}\chi \right) \right]$$

$$\mathscr{L}^{(N\pi)} \mapsto \mathscr{L}_{DM}^{(N\pi)} = \frac{F_0^2}{4} \text{Tr} \left[U^\dagger \chi_{\mathcal{M}} + U \chi_{\mathcal{M}}^\dagger \right] + c_1 \text{Tr} \left[\chi_{\mathcal{M}}^+ \right] \bar{N}N + \cdots$$

- Expand interaction
- Iso-scalar interaction gets correction from one loop

$$U = u^2 = e^{i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/F_0} \qquad \chi_{\mathcal{M}}^+ = u^\dagger \chi_{\mathcal{M}} u^\dagger + h.c.$$

$$\mathscr{L}_{DM}^{(N\pi)} = \bar{\chi}\chi \left[c_q^{(\pi)} \boldsymbol{\pi}^2 + c_q^{(ic)} \bar{N}N \right] + \cdots$$

$$c_q^{(ic)} = \frac{\sigma_{\pi N}}{2} \bar{c}_{u,d} \qquad c_q^{(\pi)} = \frac{m_\pi^2}{4} \bar{c}_{u,d} \qquad \bar{c}_{u,d} = \bar{c}_u (1-\epsilon) + \bar{c}_d (1+\epsilon) \qquad \epsilon = \frac{m_d - m_u}{m_d + m_u} \simeq 0.37$$

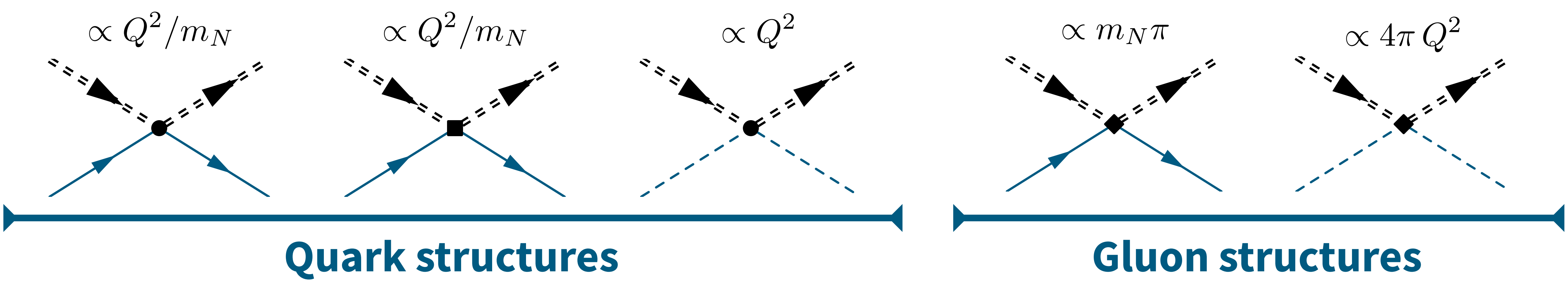
One-Body vs. Two-Body interactions

- DM **two-pion** exchange and **contact** interactions have **same 'quark-scaling'**
- Size of **nucleon sigma term** indicates relative strength of **one-body vs. two-body**

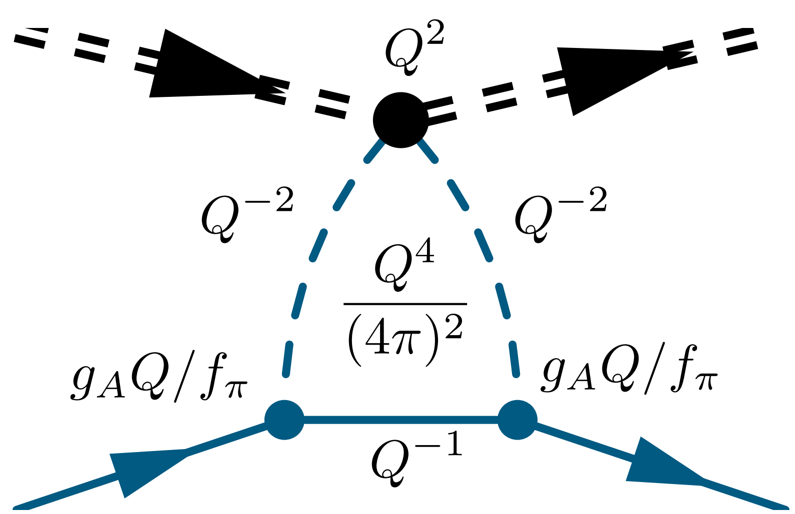
Power Counting

— Operator Counting

$\propto g_A^2 Q^3 / m_N^2$

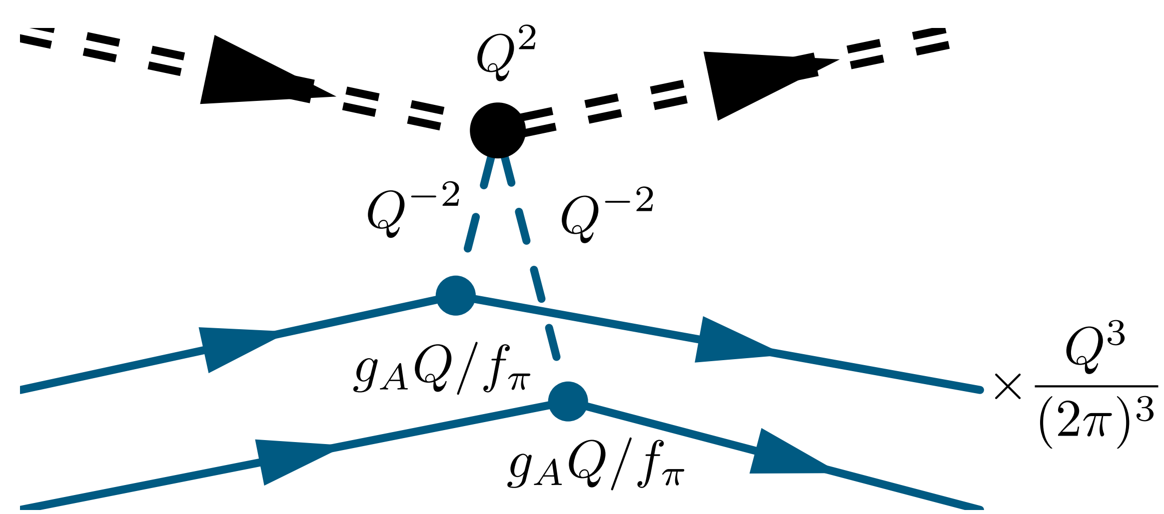


Iso-Scalar Loop



$$\left(\frac{g_A Q}{f_\pi}\right)^2 Q^2 \frac{Q^4}{(4\pi)^2} \frac{1}{Q^5} \approx g_A^2 \frac{Q^3}{m_N^2}$$

Two-Pion Exchange



$$\left(\frac{g_A Q}{f_\pi}\right)^2 Q^2 \frac{Q^3}{(2\pi)^3} \frac{1}{Q^4} \approx \frac{2g_A^2}{\pi} \frac{Q^3}{m_N^2}$$

From Nucleons to Nuclei

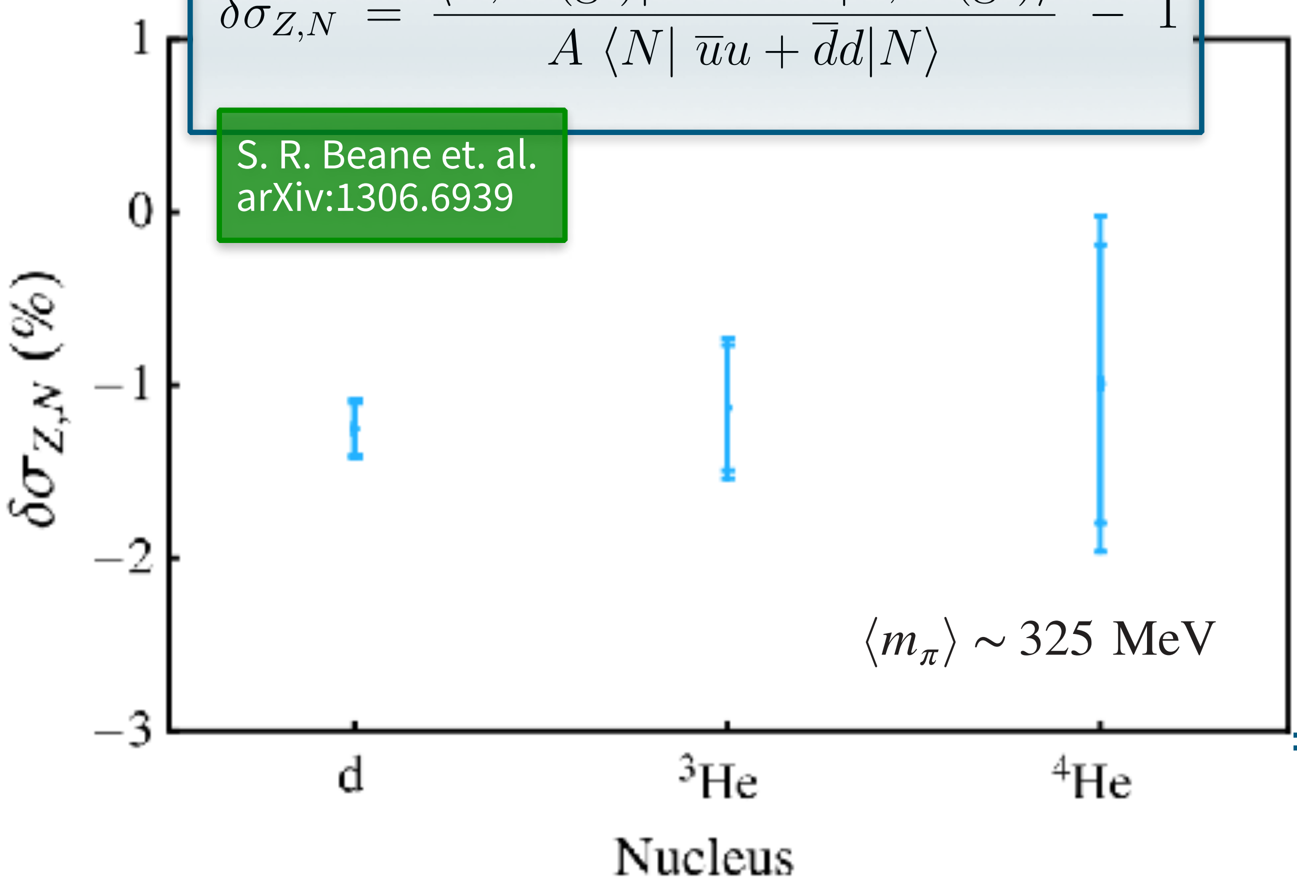
Dark Matter Interactions
— From Nucleons to Nuclei

Challenges
- Possible cancellations of leading structures (*fine-tuning*)
- Relevance of two-body effects and meson exchange

Lattice Results

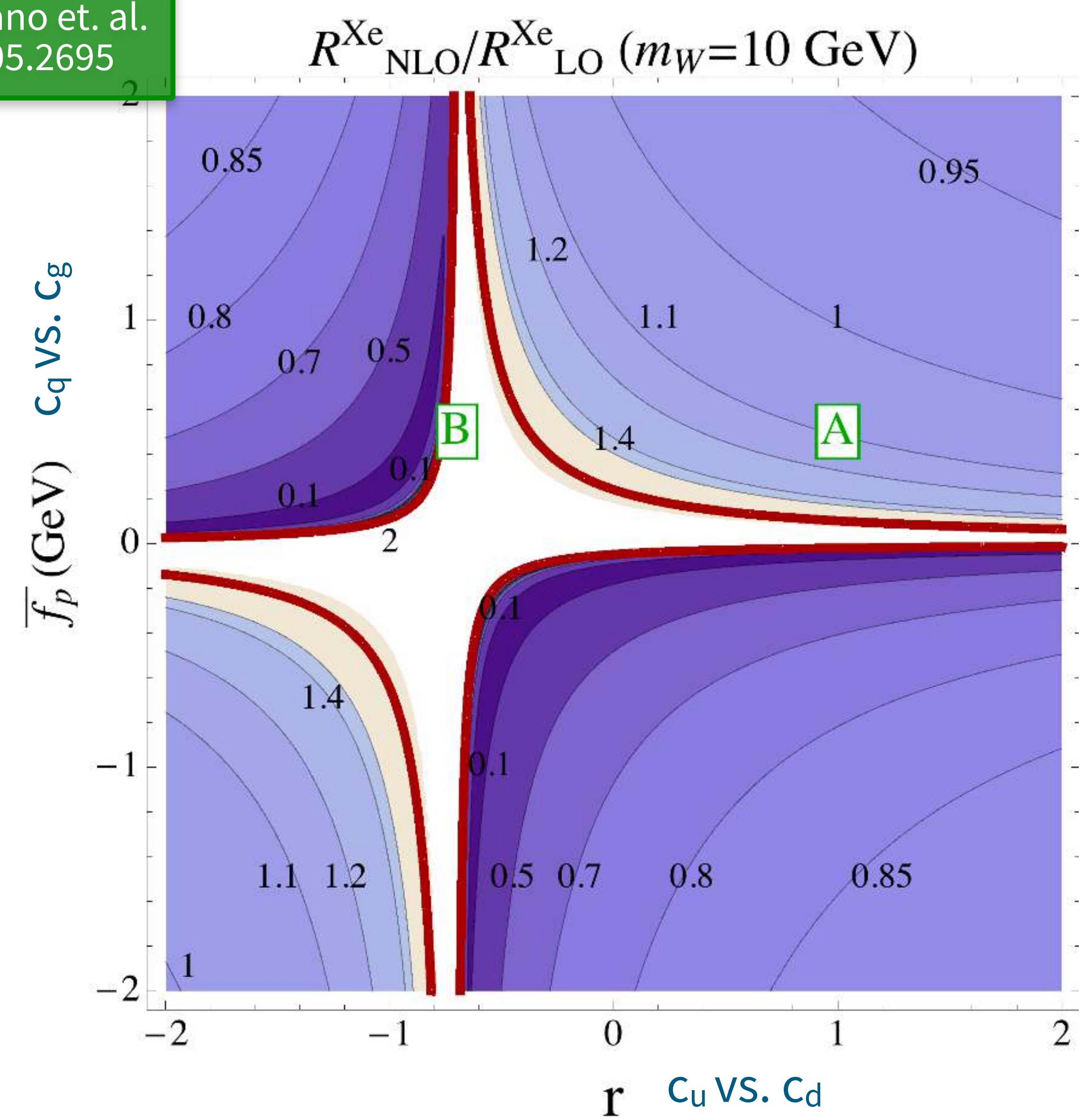
$$\delta\sigma_{Z,N} = \frac{\langle Z, N(\mathrm{gs})|\ \overline{u}u + \overline{d}d|Z, N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u + \overline{d}d|N\rangle} - 1$$

S. R. Beane et. al.
arXiv:1306.6939



V. Cirigliano et. al.
arXiv:1205.2695

Integrated Recoil Rates



⇒ **need accurate and precise description**

Light Nuclei Methodology
— Deterministic Scattering Solutions

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Nuclear Forces:
- **Chiral Forces —** E. Epelbaum, H. Krebs, U. G. Meißner arXiv:1412.4623
- **AV18 —** R. B. Wiringa et. al. arXiv:nucl-th/9408016
- **CDB —** R. Machleidt arXiv:nucl-th/0006014

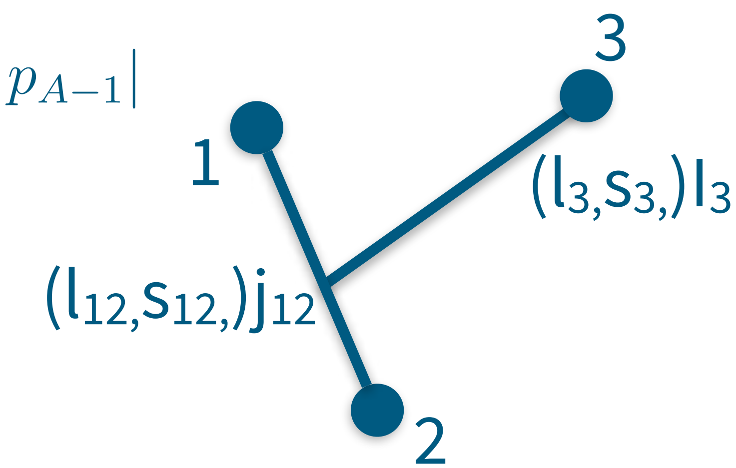
Obtain Nuclear Wave
- Solve Schrödinger or Faddeev equation (^{2}H, ^{3}H, ^{3}He)

$$\hat{H}\,|\psi, \boldsymbol{P}, J, m_\tau\rangle = E\,|\psi, \boldsymbol{P}, J, m_\tau\rangle$$

$$\hat{\mathbb{1}} = \sum_\alpha \prod_{i=1}^{A-1} \int \mathrm{d}p_i\, p_i^2\, |\alpha, p_1, \cdots, p_{A-1}\rangle \langle \alpha, p_1, \cdots, p_{A-1}|$$

LSJ-Coupling scheme

$$|\alpha\rangle\big|_{A=3} := |((l_{12}s_{12})j_{12}\,(l_3 s_3)I_3)\,j_3 m_{j_3}\rangle \otimes |(t_{12}t_3)\tau_3 m_{\tau_3}\rangle$$



Compute DM-Scattering Amplitude
- Describe DM-Interaction as momentum dependent current (perturbative)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\vec{q}^{\,2}} \propto \left|\sum_{\nu,t} \alpha_t \mathcal{F}_t^{(\nu)}(\vec{q}^{\,2})\right|^2 \qquad \mathcal{F}_t^{(\nu)}(\vec{q}^{\,2}) \propto \left\langle \psi, \vec{P}+\vec{q}, J, m_\tau \middle| \hat{J}_t^{(\nu)}(\vec{q}) \middle| \psi, \vec{P}, J, m_\tau \right\rangle$$

Leading response function normalized to 1 at q = 0 MeV

Relevant for light nuclei: q ~ few MeV

Uncertainty of Results
- Convergence of basis (numerical uncertainty < 1%)
- Nuclear uncertainty (analyze chiral behaviour)

Power Counting

Based on E. Epelbaum, H. Krebs, U.-G. Meißner arXiv:1412.0142

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Convergence of ChPT observables

- ChPT provides **systematic** expansion of observables in small parameter: $Q \sim \frac{m_\pi}{\Lambda}$

$|\mathcal{F}|^2$

$$X^{(\nu)} = X + \delta^{(\nu)} X = X + \sum_{i=1} Q^{\nu+i} \Delta X_i^{(\nu)}$$

Chiral Approx. of Observable — The *'Real'* Value of Observable — Chiral Uncertainty — Expansion Parameter — Expansion Coefficient

$$\nu < \nu' \;\Rightarrow\; X^\nu - X^{\nu'} = Q^{\nu+1}\left(\Delta X_1^{(\nu)} + \mathcal{O}(Q)\right)$$

The Estimate *(very conservative)*

- Estimate chiral uncertainty by approximating expansion coefficients

$$\delta^{(\nu)} X \leq Q^\nu \sum_{i=1} Q^i \left|\Delta X_i^{(\nu)}\right| \leq Q^\nu \left|\Delta X_{\max}^{(\nu)}\right| \sum_{i=1} Q^i \leq \frac{Q^{\nu+1}}{1-Q} \left|\Delta X_{\max}\right|$$

- Approximate maximal coefficient as maximum over chiral orders

$$\left|\Delta X_{\max}^{(\nu)}\right| \leq |\Delta X_{\max}| = \max_\nu \left(\left|\Delta X_{\max}^{(\nu)}\right|\right) = \max_\nu \left(\max_{\nu'>\nu}\left(\frac{\left|X^{(\nu)} - X^{(\nu')}\right|}{Q^{\nu+1}}\right)\right)$$

Our Results

Chiral Uncertainty Estimation

— Deuteron Quark-DM Amplitudes

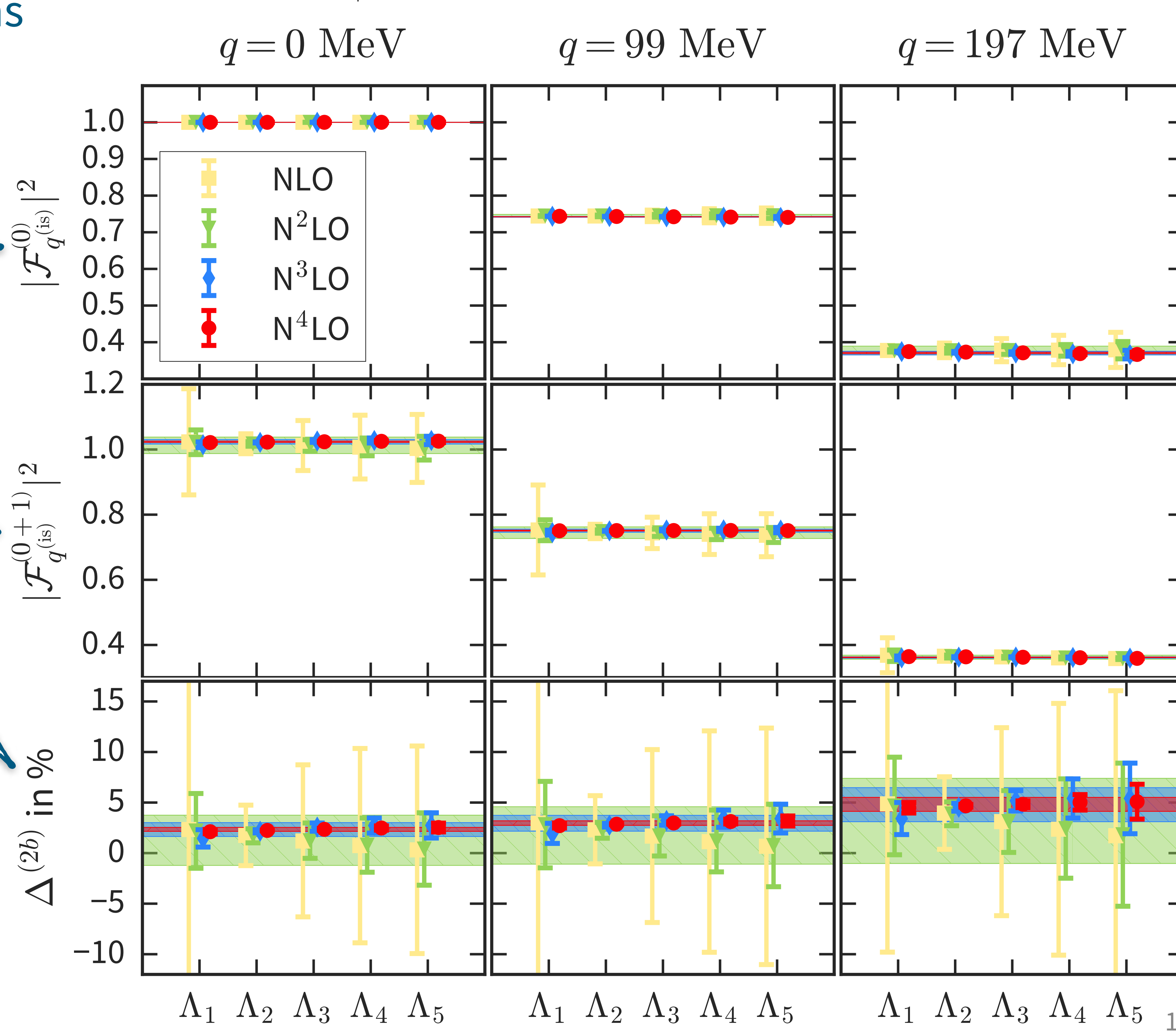
cut-off	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5
R	0.8 fm	0.9 fm	1.0 fm	1.1fm	1.2 fm
Λ_b	600 MeV	600 MeV	600 MeV	500 MeV	400 MeV

- Results for $c_u = c_d$
- Isoscalar DM interactions
- **Fixed order current**

Leading Order Currents

Next to Leading Order Currents

2-Body vs. 1-Body Currents



- Results **consistent** over different cut-offs
- Second cut-off gives best results

Chiral Uncertainty Estimation

— Helion Quark-DM Amplitudes

- Results for $c_u = c_d$
- Isoscalar DM interactions
- **Fixed order current**

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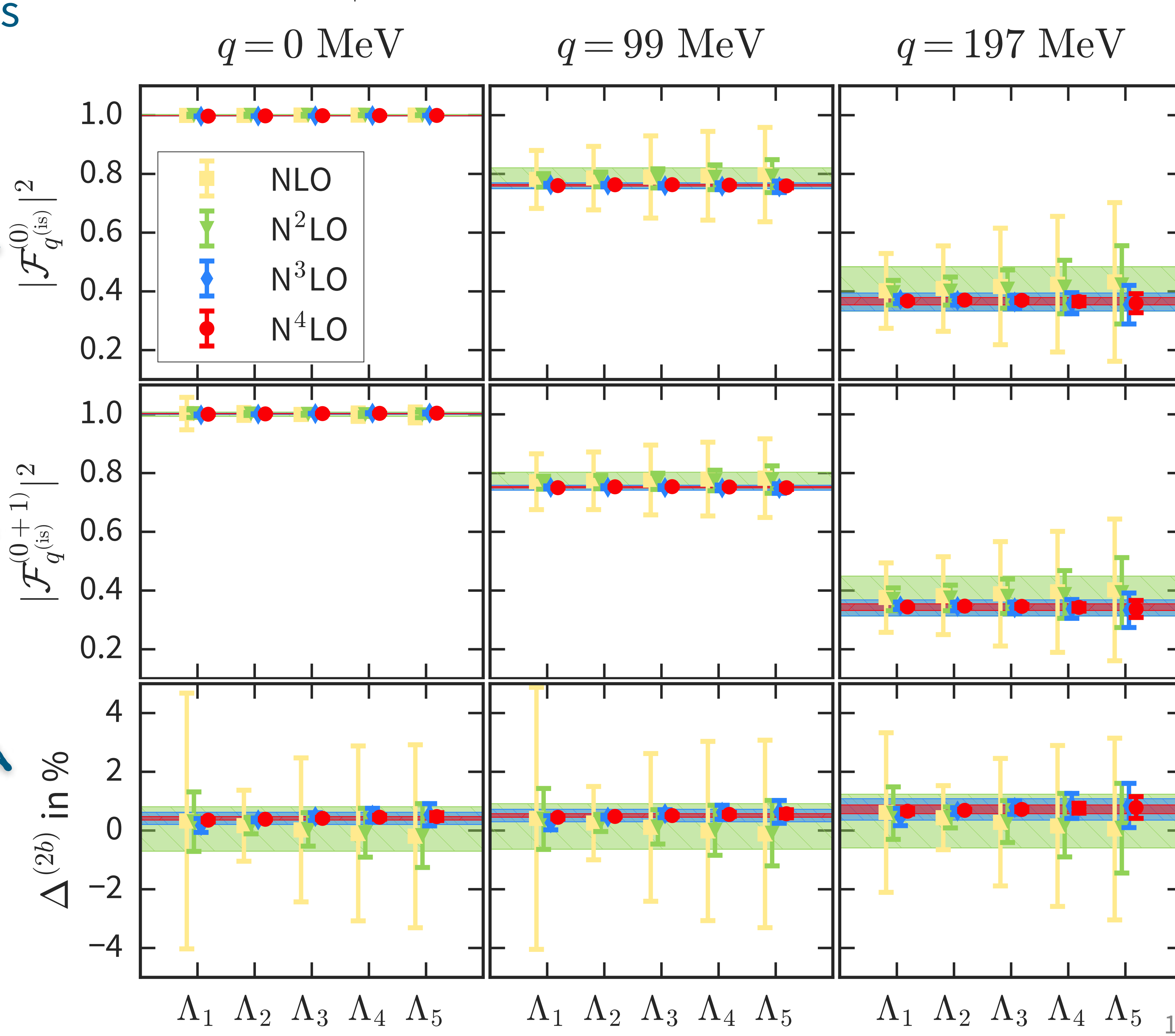
Leading Order Currents

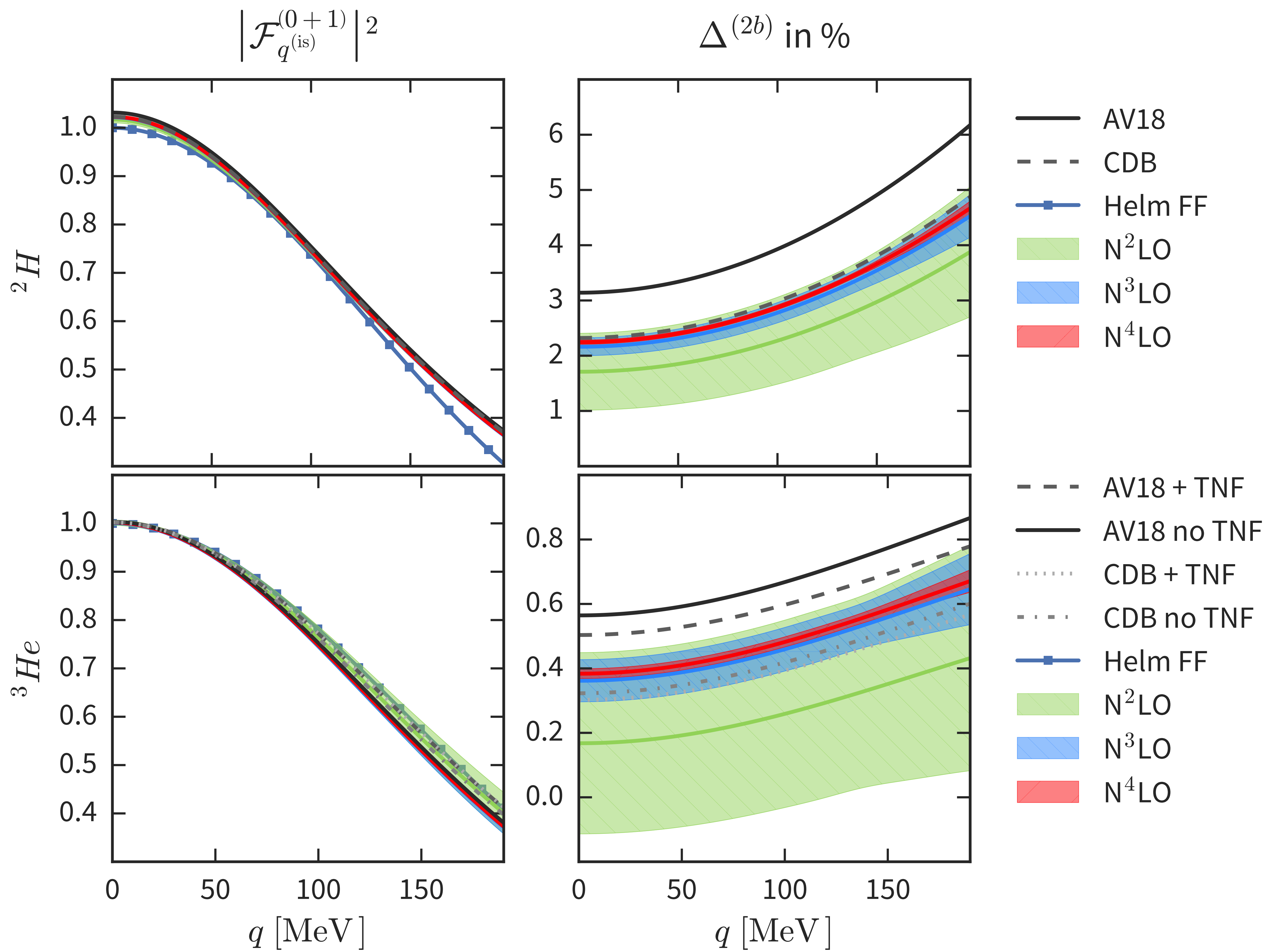
Next to Leading Order Currents

2-Body vs. 1-Body Currents



- Results **consistent** over different cut-offs
- Second cut-off gives best results

Summary Plot — Results for 2nd cut-off

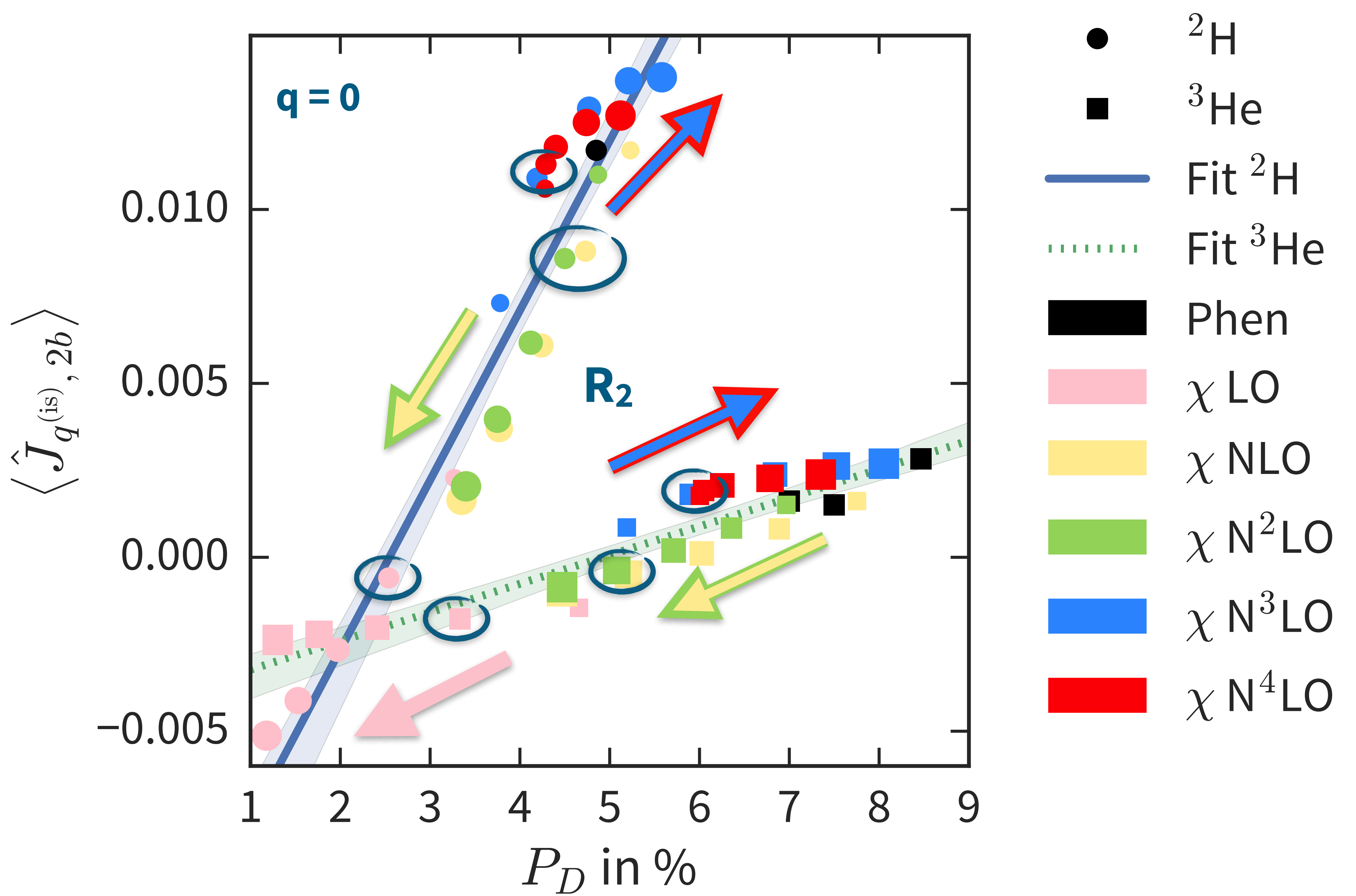


Uncertainty of Two-Body Results

$$\langle \hat{J} \rangle = \langle S|\hat{J}|S\rangle + 2\langle S|\hat{J}|D\rangle + \cdots$$

- Size of marker corresponds size of cut-off

➡ Smallest marker = R_1 ➡ largest marker = R_5



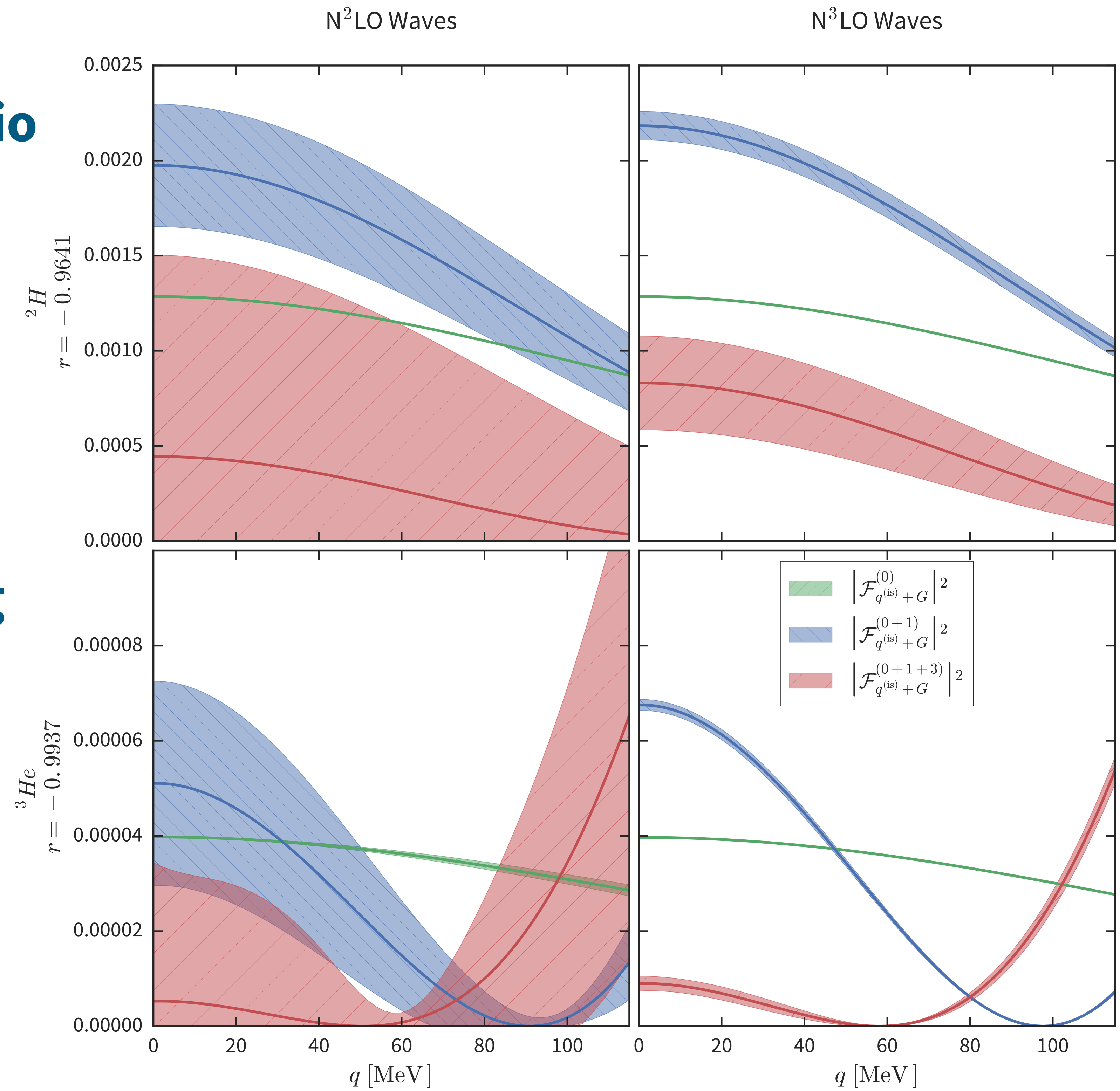
Fine-Tunig Scenarios

The Major Difference

$$\mathcal{F}(\vec{q}^2) = \left(\alpha_{q^{(\mathrm{is})}} + \alpha_G\right) \mathcal{F}^{(0)}_{q^{(\mathrm{is})}}(\vec{q}^2) + \alpha_{q^{(\mathrm{is})}} \left(\mathcal{F}^{(1)}_{q^{(\mathrm{is})},\,r}(\vec{q}^2) + \mathcal{F}^{(1)}_{q^{(\mathrm{is})},\,2b}(\vec{q}^2)\right) + \alpha_G \mathcal{F}^{(3)}_{G,\,2b} + \dots$$

Simulate Fine-Tunig Scenario

- Define $r = \dfrac{\alpha_G}{\alpha_{q^{(\mathrm{is})}}}$
- Choose **'*r*'** such that cross section gets 50% correction from NLO currents (for AV18)



Consequences of Fine-Tunig

- Current power counting mixed up
- Seemingly distinguishable results only for N^{3}LO waves

Intentions for the Future

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(Drawings are not in scale)

Soon… ***…a few months…*** ***…in the future***

^{4}He Analysis

Nuclear Lattice Benchmark (NLEFT)

Lattice Predictions

Spin Dependent DM Interactions

Neutrinoless Double Beta Decay

Possible Additions

- Bayesian Uncertainty Analysis
- Chiral Three-Body Forces
- N^2LO Current Structures