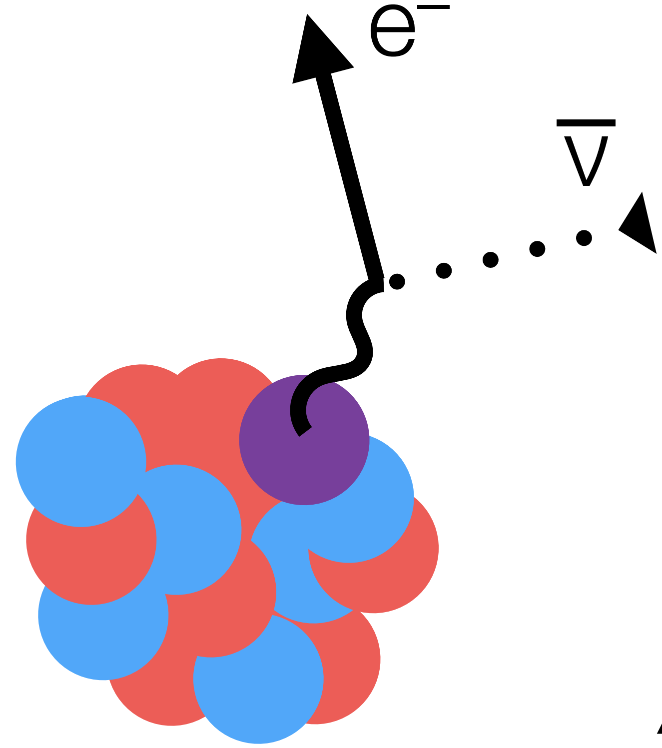
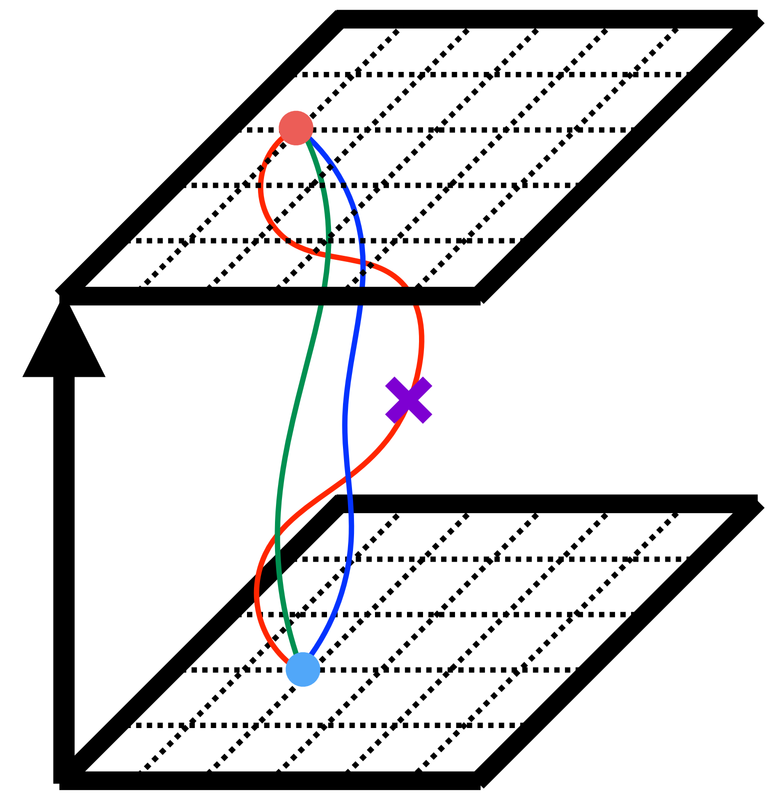
The Nucleon Axial Coupling from QCD





Evan Berkowitz
Institut für Kernphysik
Institute for Advanced Simulation
Forschungszentrum Jülich

29 June 2017
Ruhr-Universität Bochum

1701.07559
1704.01114

JÜLICH
FORSCHUNGSZENTRUM

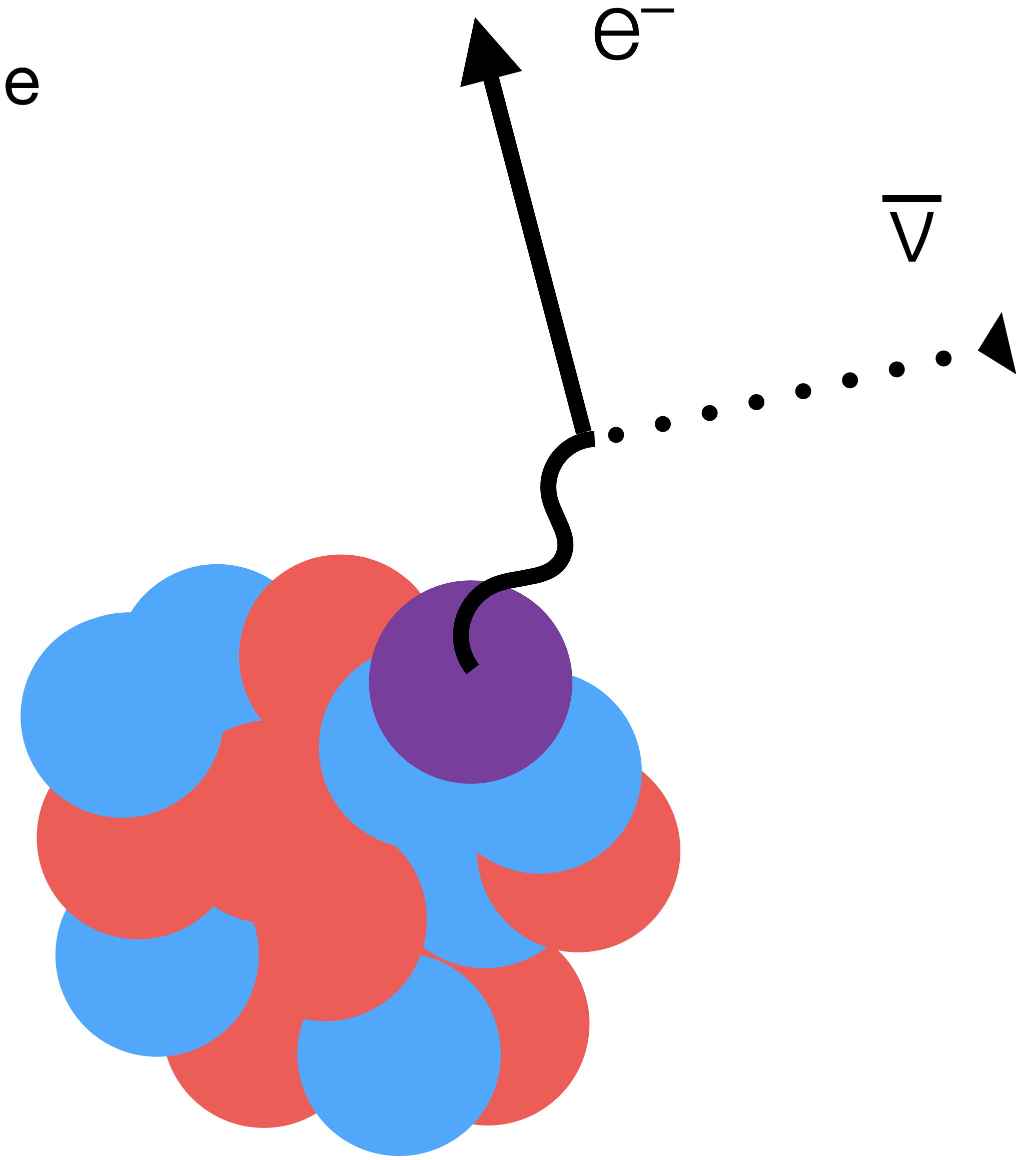
CalLat

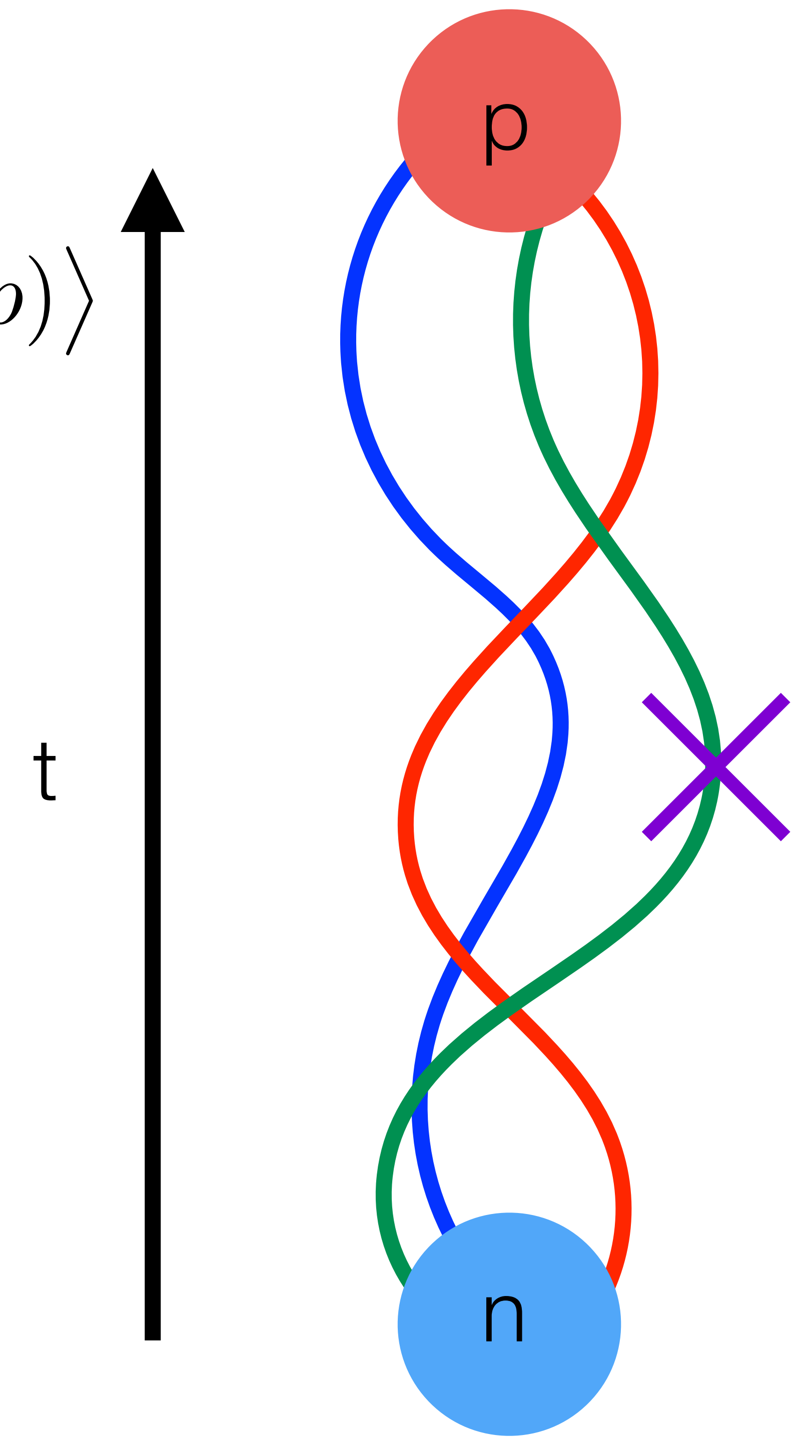
Berkeley LBL	David Brantley, Henry Monge Camacho, Chia Cheng (Jason) Chang, Ken McElvain, André Walker-Loud
RBRC	Enrico Rinaldi
FZJ	EB
JLab	Bálint Joó
Liverpool Plymouth	Nicolas Garron
LLNL	Pavlos Vranas
NERSC	Thorsten Kurth
UNC	Amy Nicholson
nVidia	Kate Clark
Glasgow	Chris Bouchard
Rutgers	Chris Monahan
William & Mary	Kostas Orginos

The Nucleon Axial Coupling

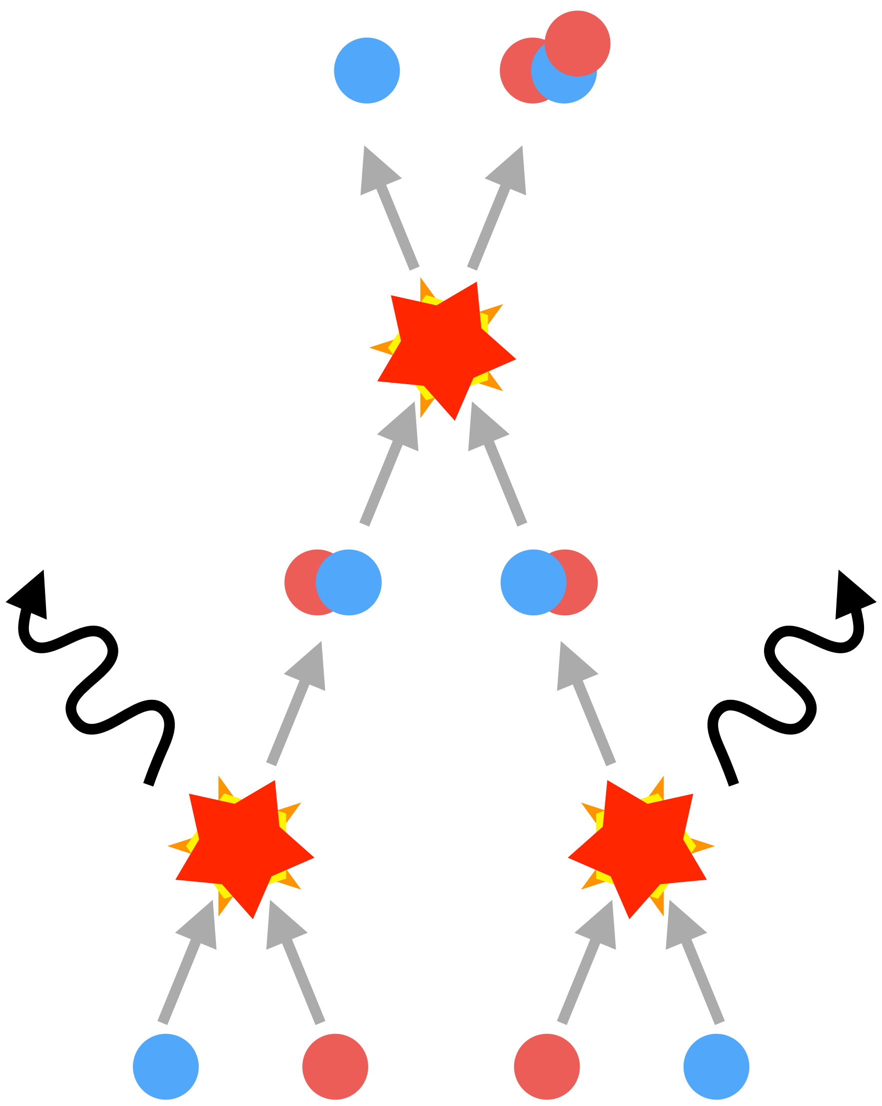
$$\langle N(p) \mid A^a_\mu \mid N(p) \rangle = \langle N(p) \mid \bar{\psi}\gamma_\mu\gamma_5\tau^a\psi \mid N(p) \rangle$$
$$= g_A \, \bar{n}(p)\gamma_\mu\gamma_5\tau^a n(p)$$

- Free neutron lifetime
- Nuclear force
- Nuclear β decay



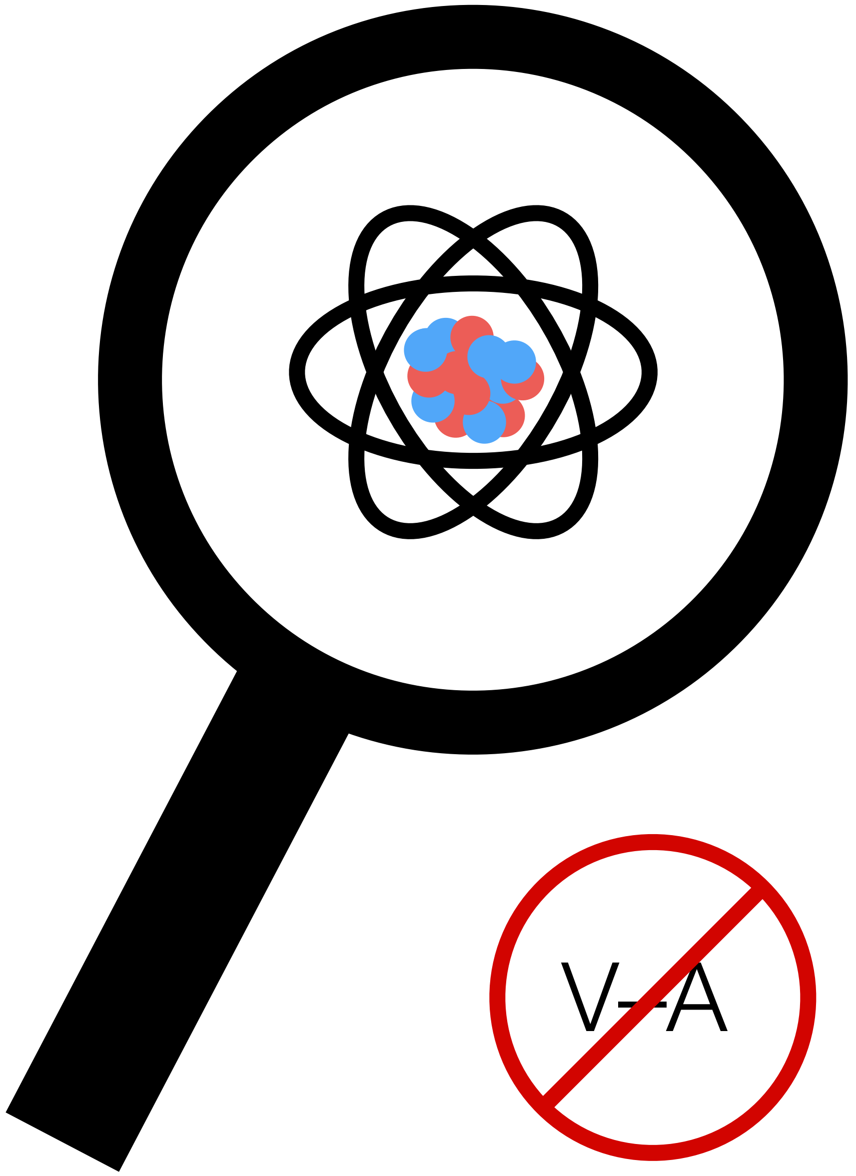


Applications



Big Bang Nucleosynthesis

Astrophysics



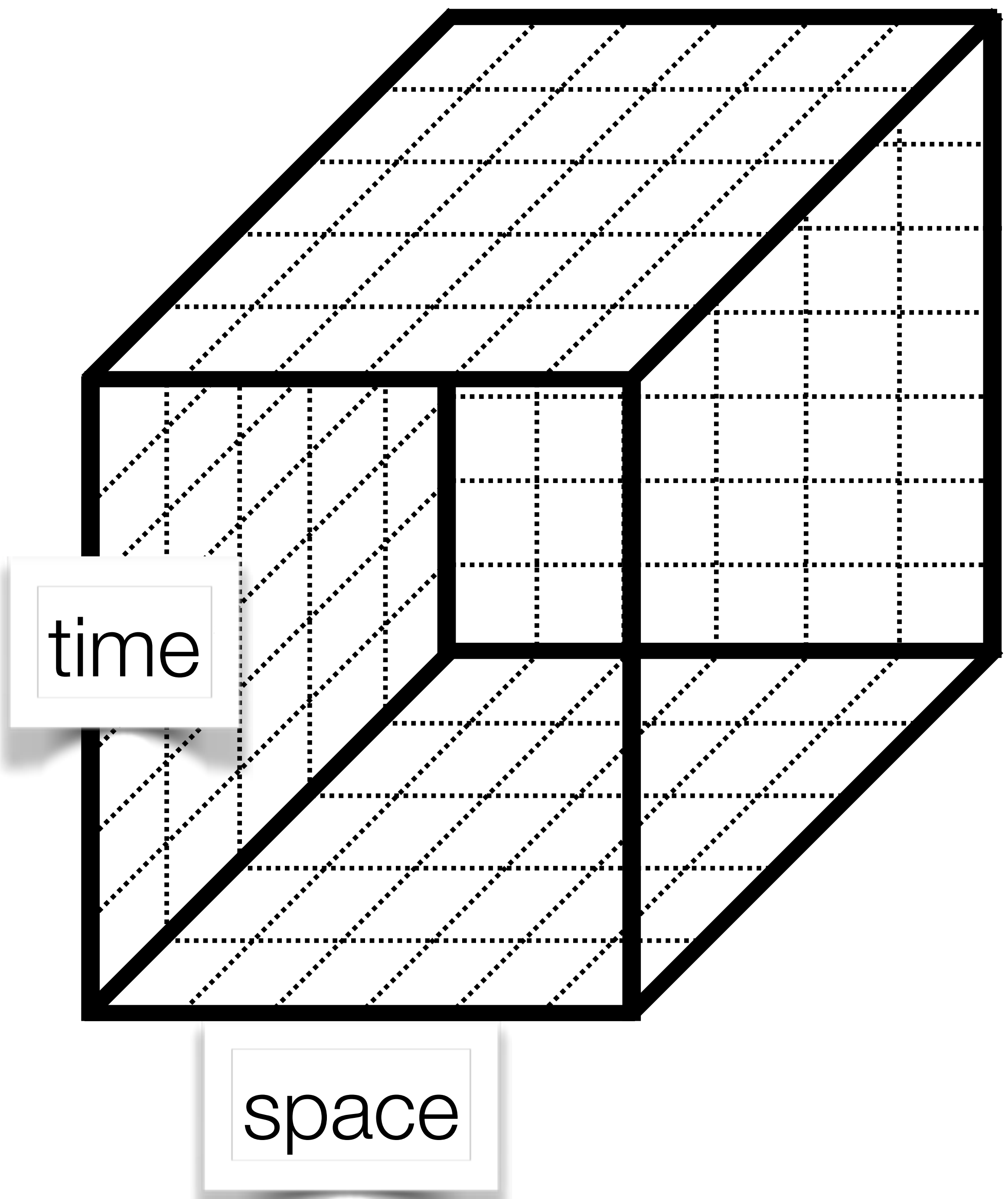
New Physics Searches

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}U\ \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det\left(\not{D} + M\right) e^{-S[U]}\ \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

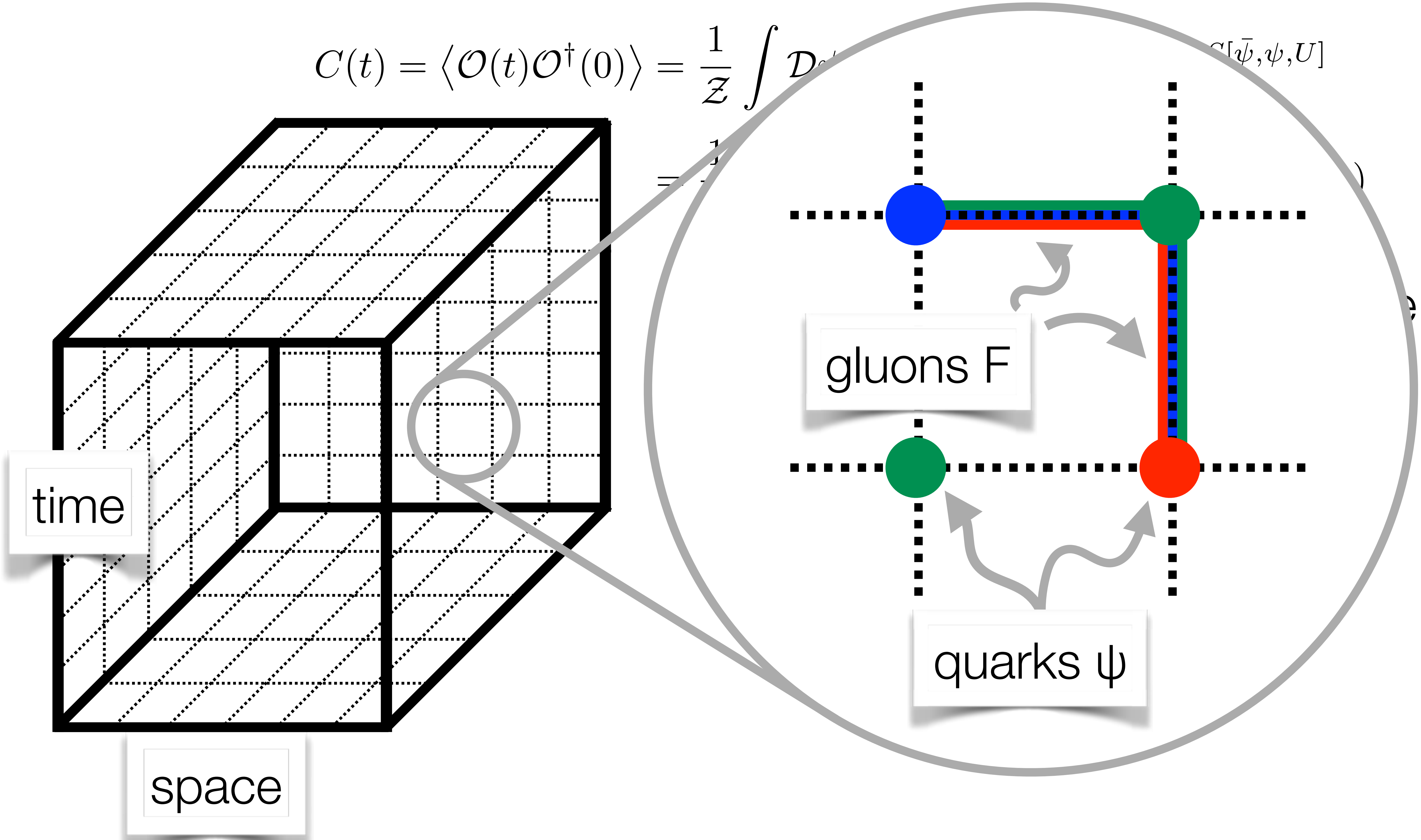


lattice
finite volume

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}$$

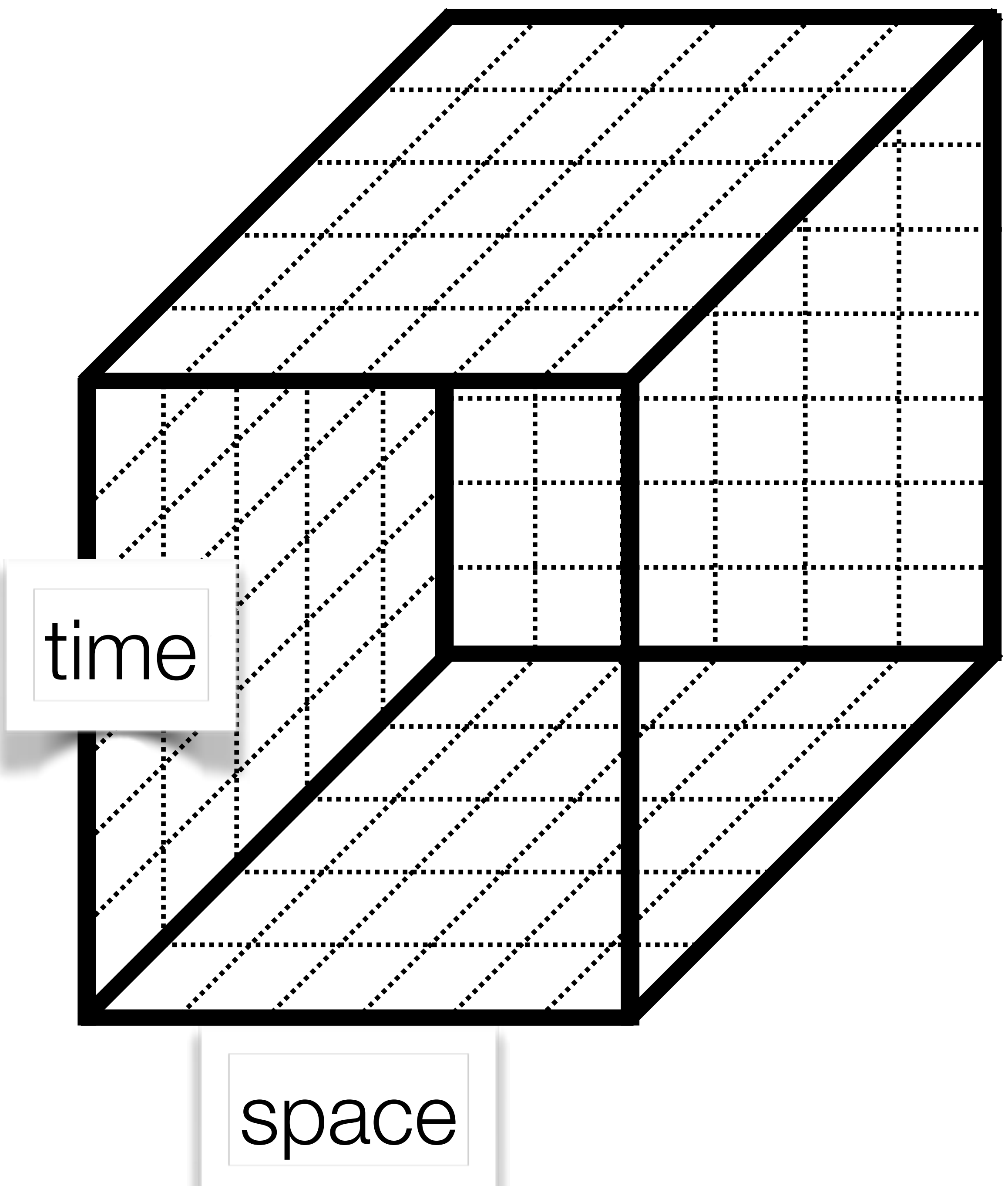


Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \mathcal{D}U \; \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, \underbrace{\det\left(\not{D} + M\right) e^{-S[U]}}_{\text{Probability}} \; \mathcal{O}(t)\mathcal{O}^\dagger(0)$$



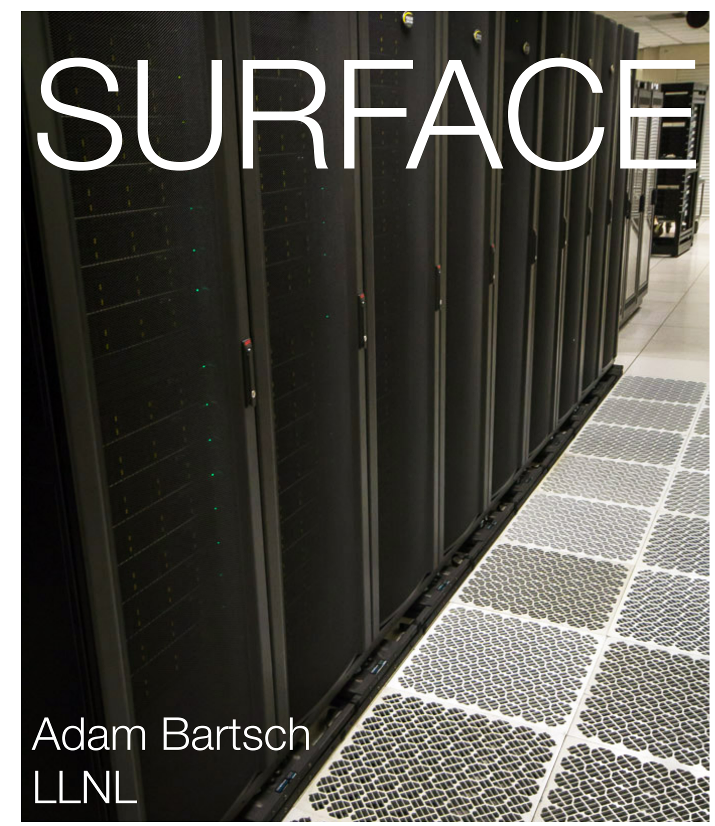
$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N}\sum_{i=1}^{N} \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i] \; + O\left(\frac{1}{\sqrt{N}}\right)$$

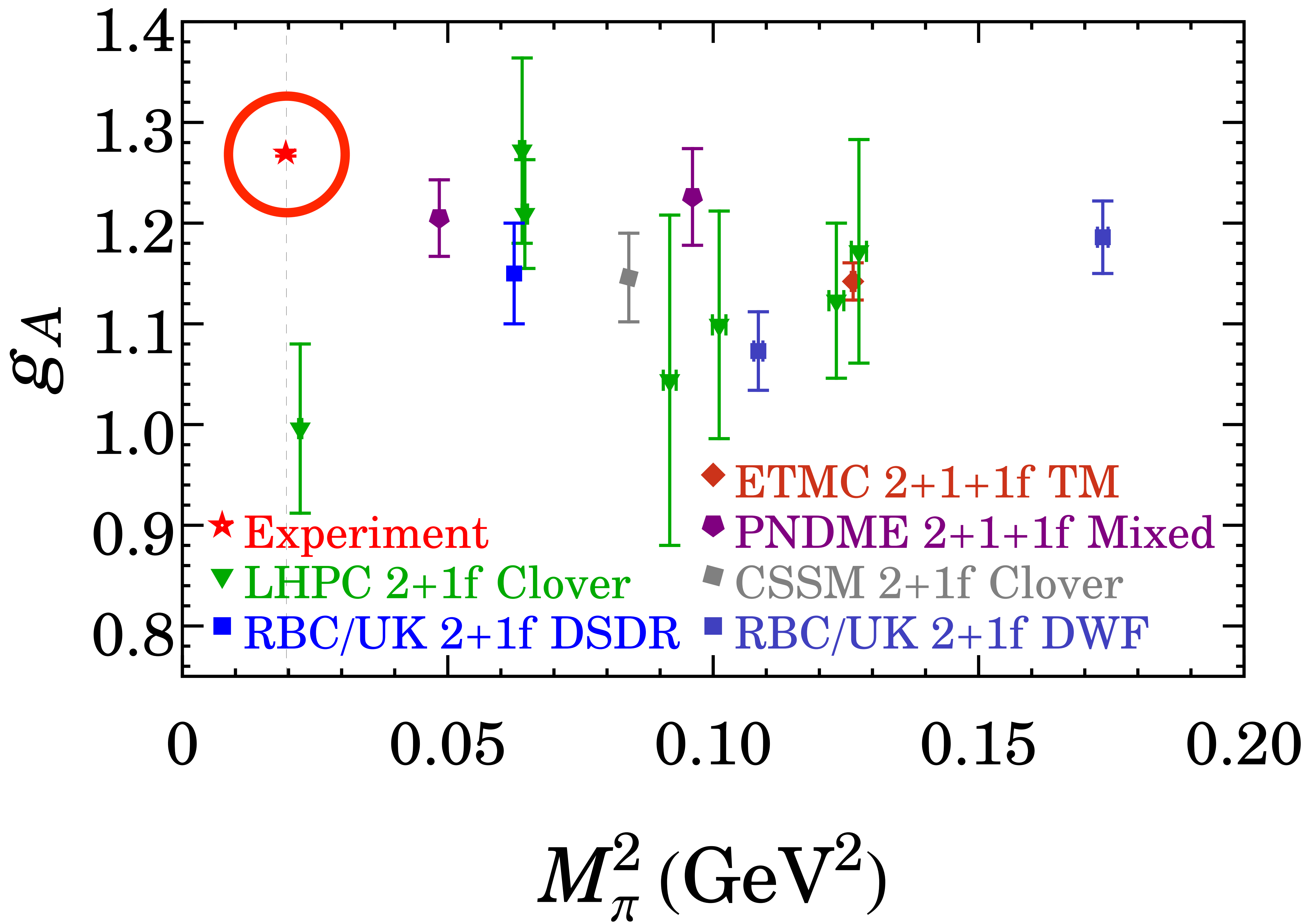

TITAN
OLCF


Edison
NERSC

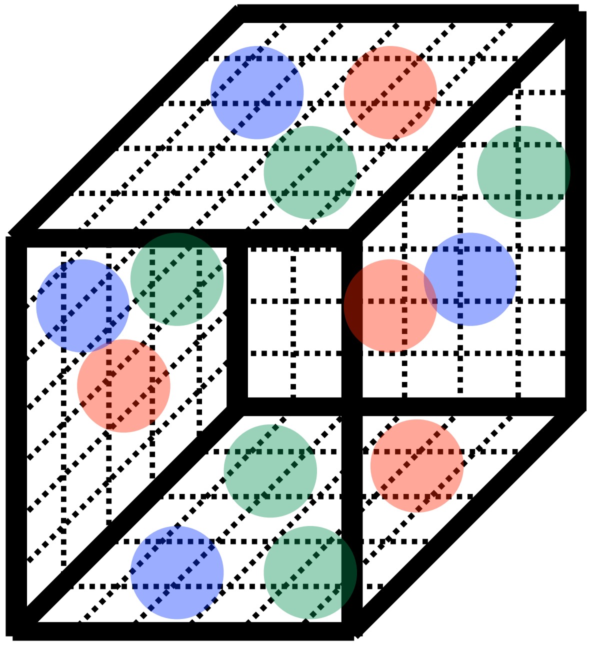

SURFACE
Adam Bartsch
LLNL

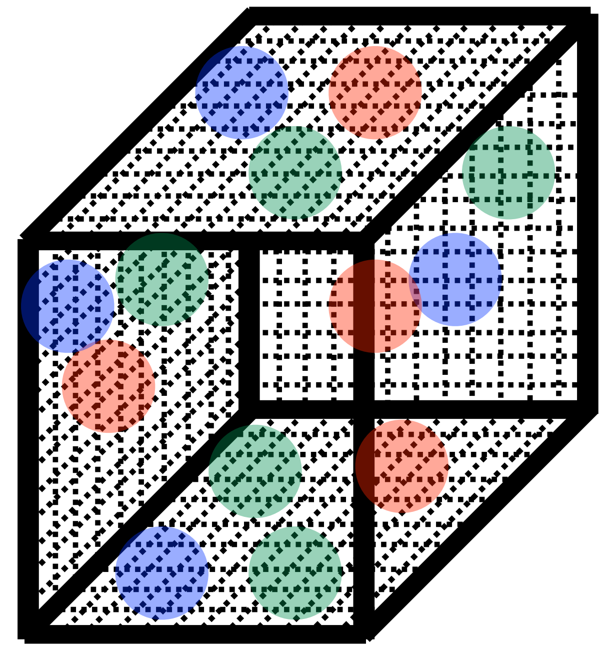
A long-outstanding problem for LQCD

Bhattacharya, Cohen, Gupta, Joseph, Lin, Yoon PRD 89 (2014) arXiv:1306.5435

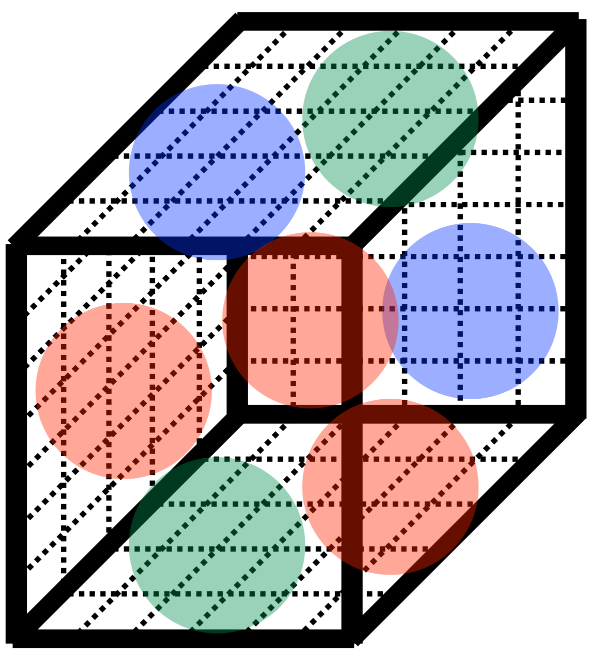


LQCD Systematics

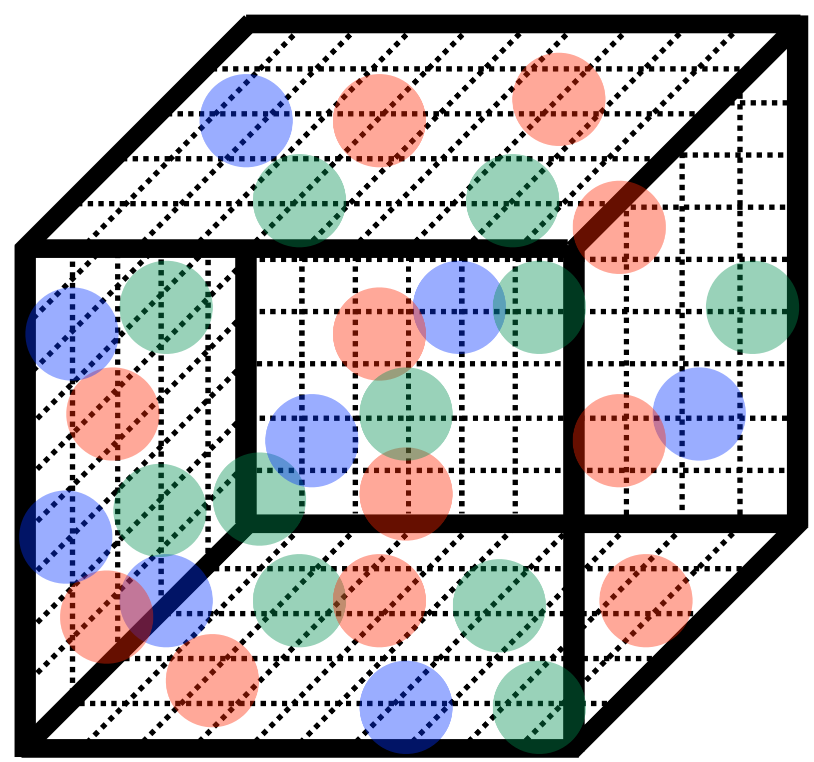




continuum limit



physical quark masses



infinite volume limit

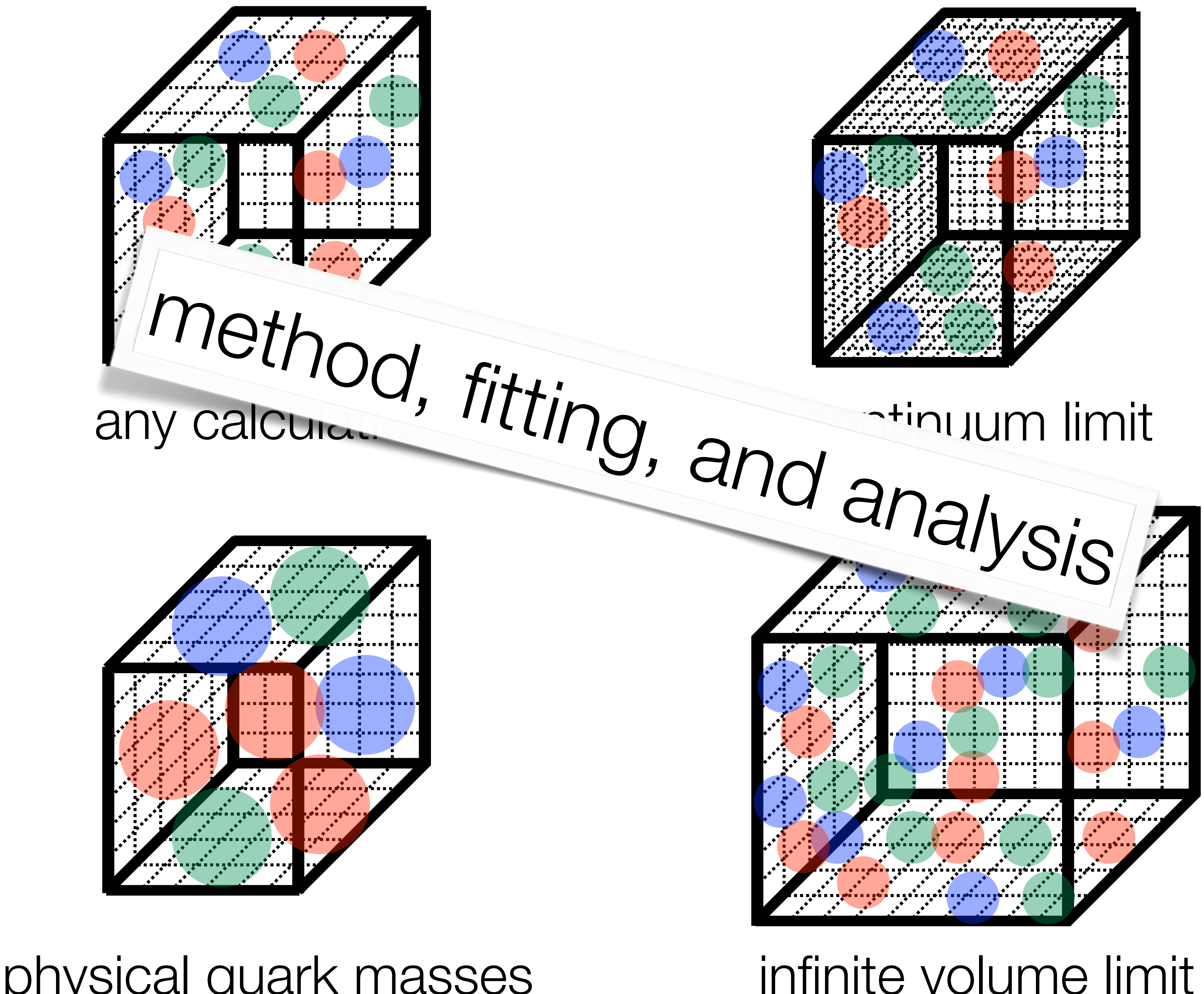
MILC Ensembles

MILC Collaboration Phys. Rev. D87 (2013) 054505

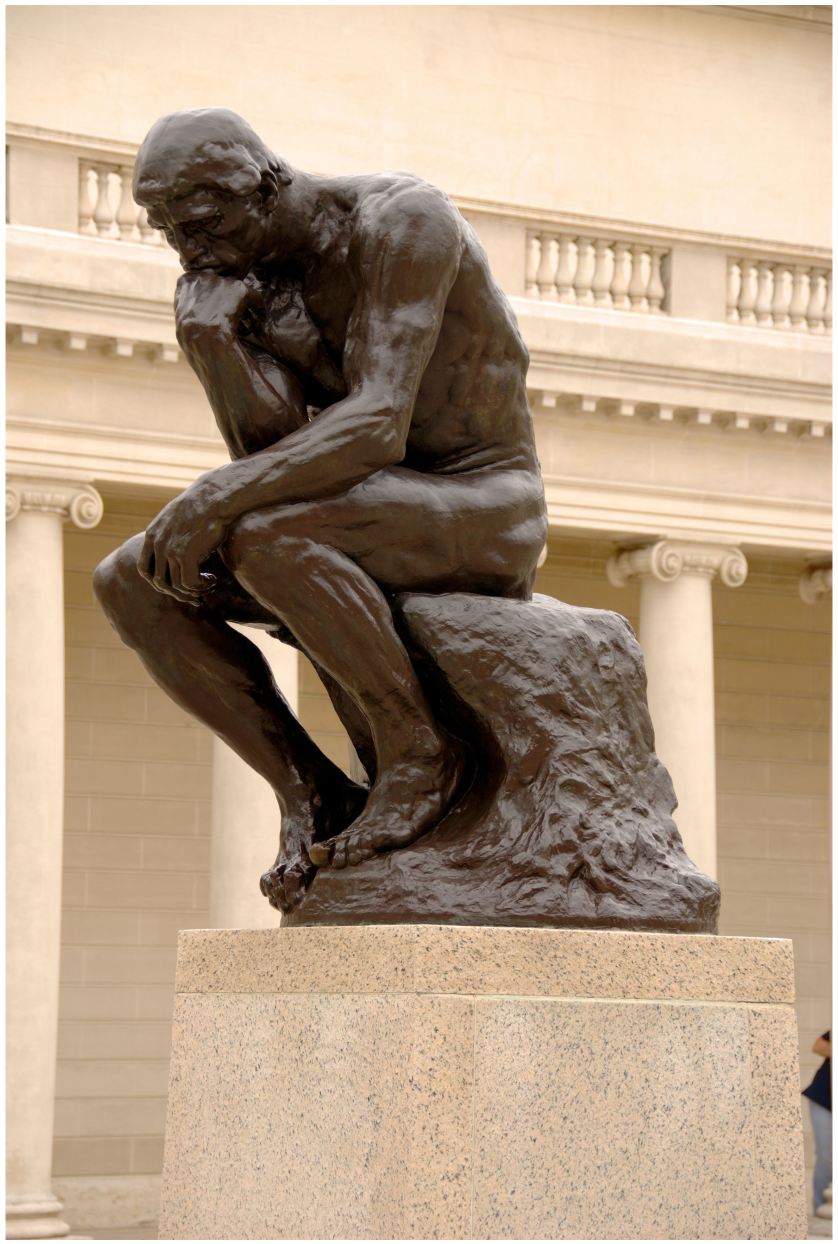
	abbr.	a [fm]	m_l/m_s	volume	m_π [MeV]	$m_\pi L$	N_{cfg}	M_5	α	L_5	N_{src}
coarser	a15m310	0.15	0.2	$16^3 \times 48$	310	3.8	1960	1.3	2.0	12	24
	a15m220	0.15	0.1	$24^3 \times 48$	220	4.0	1000	1.3	2.5	16	12
	a15m130	0.15	0.036	$32^3 \times 48$	135	3.2	1000	1.3	3.5	24	5
middle	a12m400	0.12	0.334	$24^3 \times 64$	400	5.8	1000	1.2	1.5	8	8
	a12m350	0.12	0.255	$24^3 \times 64$	350	5.1	1000	1.2	1.5	8	8
	a12m310	0.12	0.2	$24^3 \times 64$	310	4.5	1053	1.2	1.5	8	4
	a12m220L	0.12	0.1	$40^3 \times 64$	220	5.4	1000	1.2	2.0	12	4
	a12m220	0.12	0.1	$32^3 \times 64$	220	4.3	1000	1.2	2.0	12	4
	a12m220S	0.12	0.1	$24^3 \times 64$	220	3.2	1000	1.2	2.0	12	4
	a12m130	0.12	0.036	$48^3 \times 64$	135	3.9	1000	1.2	3.0	20	3
finer	a09m310	0.09	0.2	$32^3 \times 96$	310	4.5	784	1.1	1.5	6	8
	a09m220	0.09	0.1	$48^3 \times 96$	220	4.7	1001	1.1	1.5	8	6

- Anyone is free to use them
- Large statistics available
- Capable of controlling all systematic uncertainties
- We use domain wall valence on the HISQ sea, $\mathcal{O}(a^2)$ errors [1701.07559].

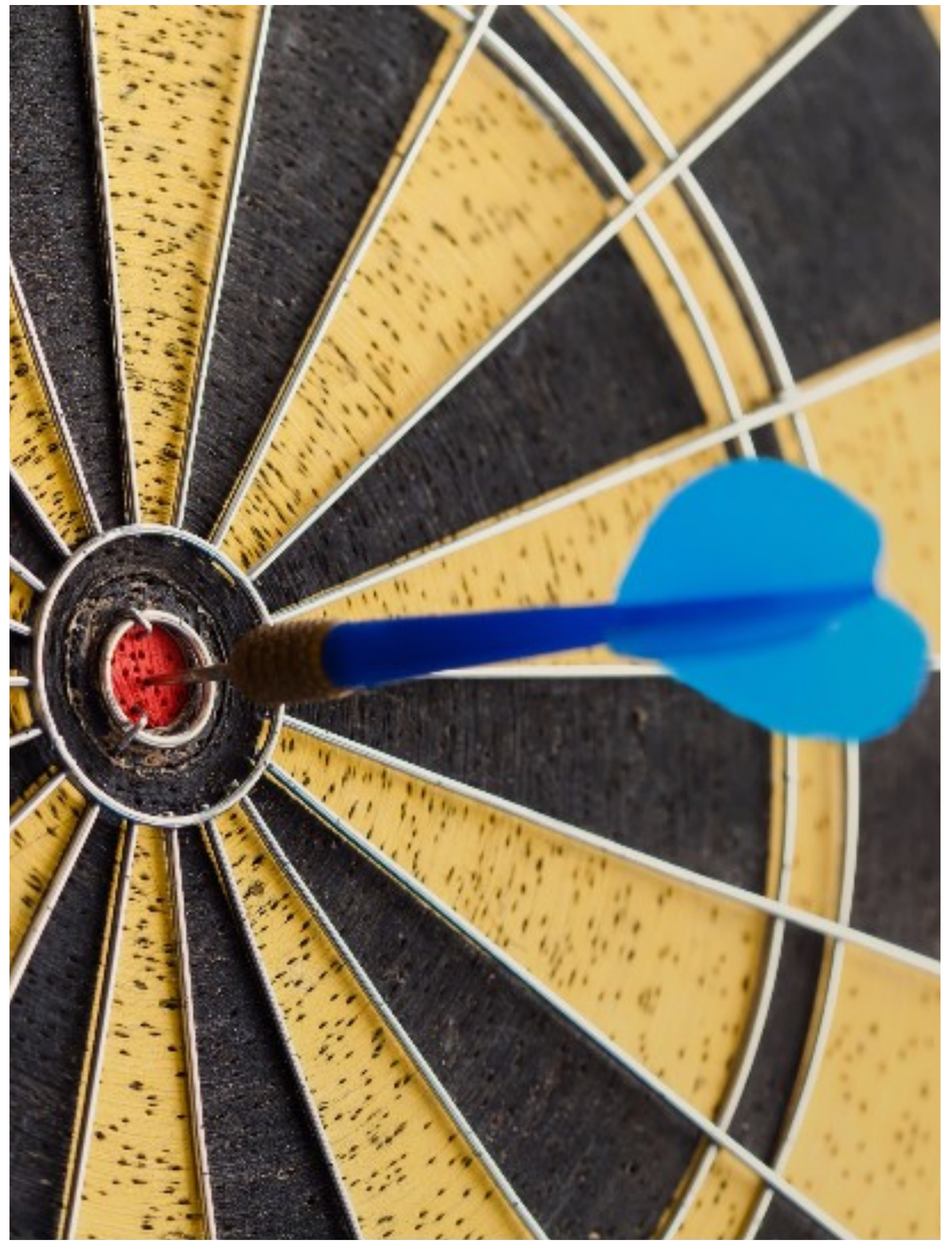
LQCD Systematics



New Methods



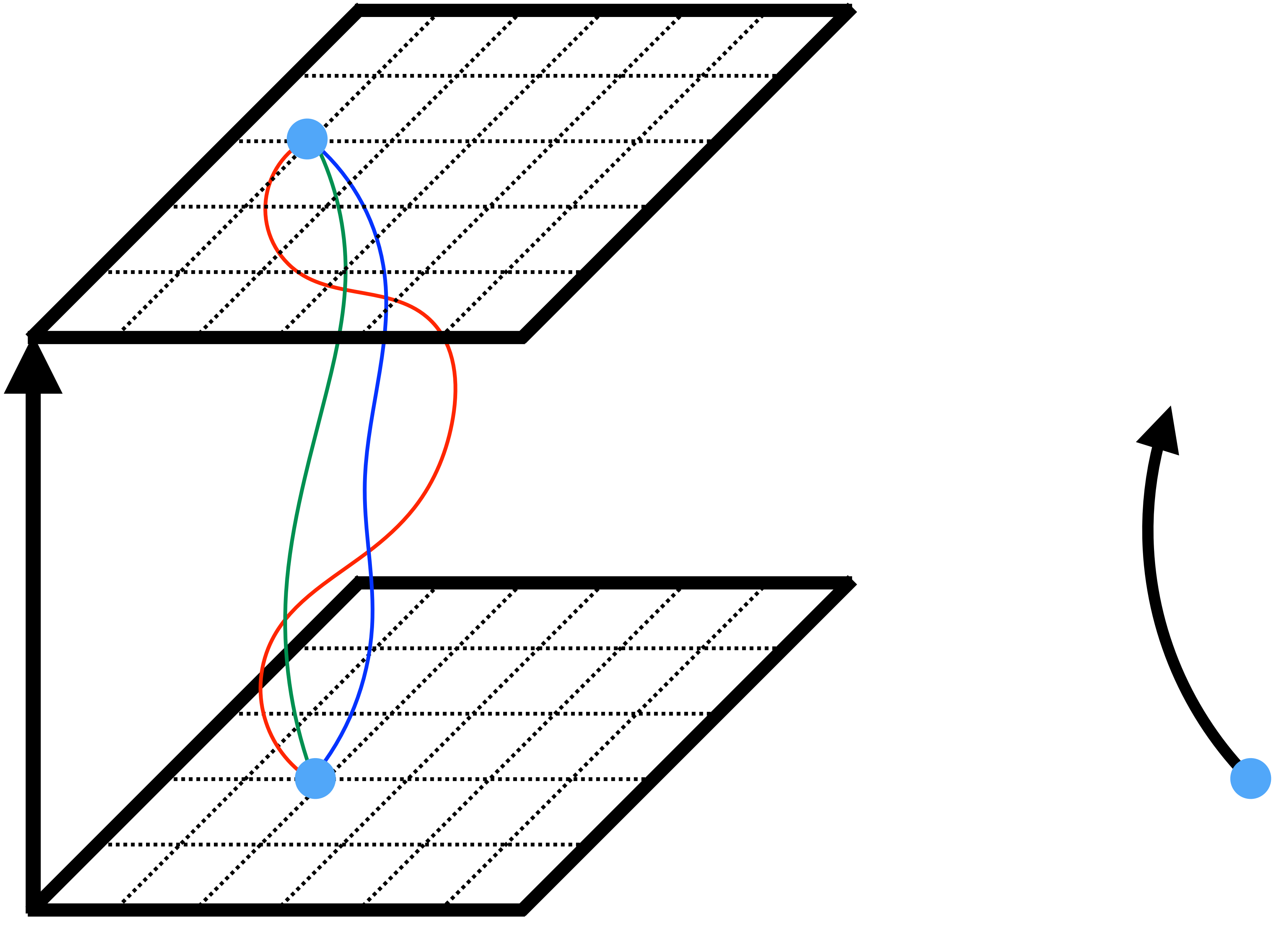
New Analytic Tools



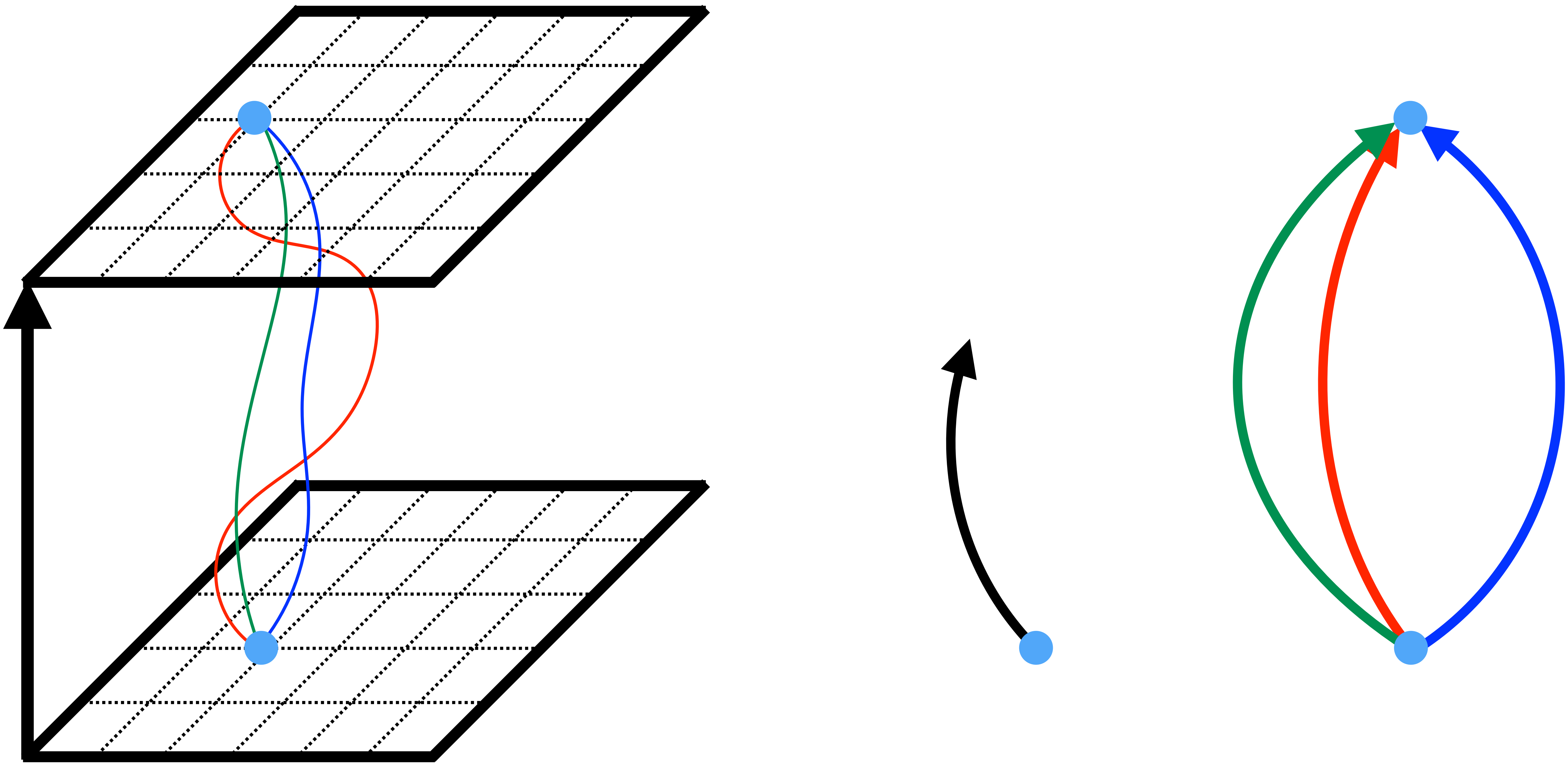
Improved Systematics

Computationally Affordable

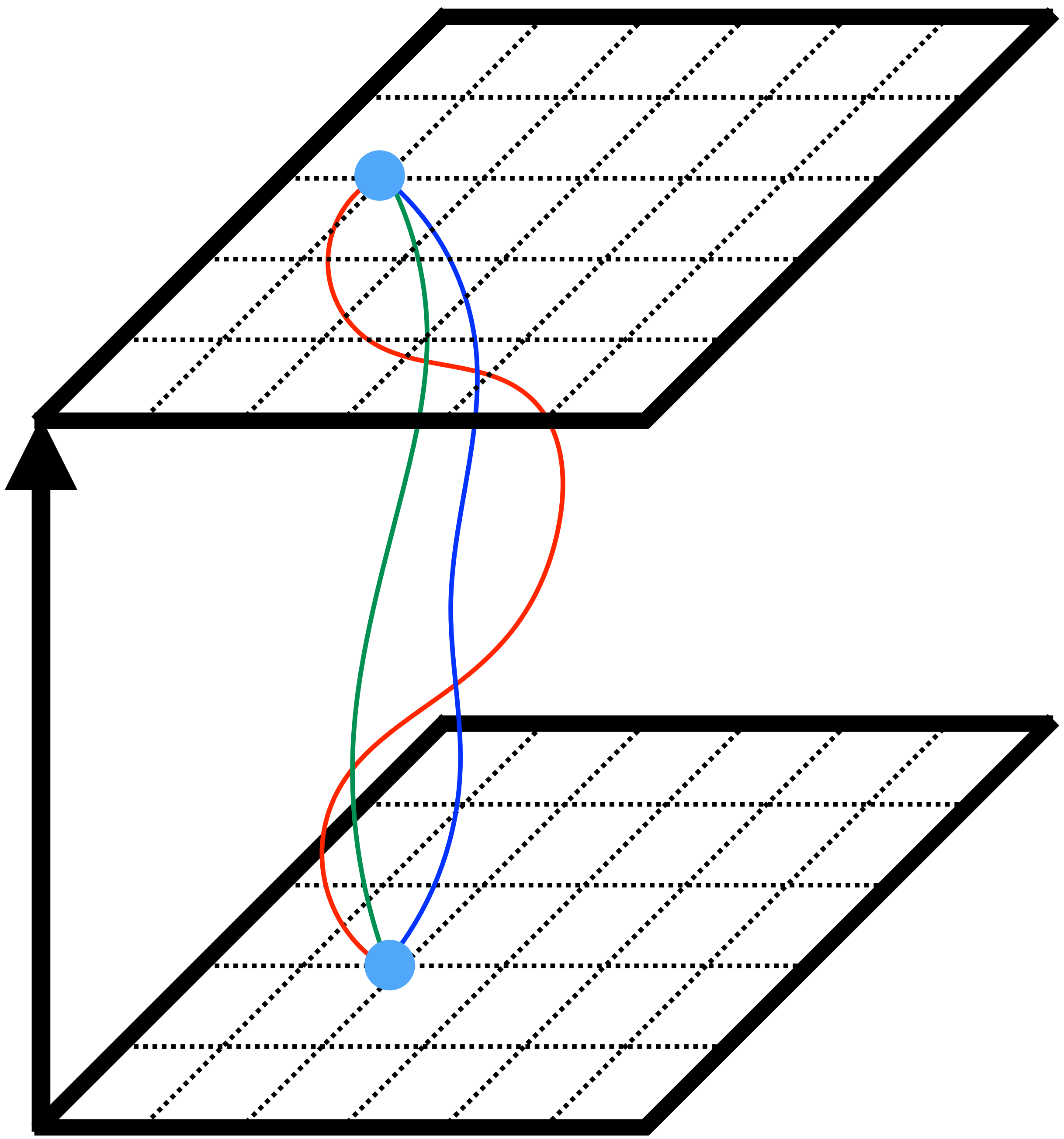
2-Point Standard Method



2-Point Standard Method



Effective Mass



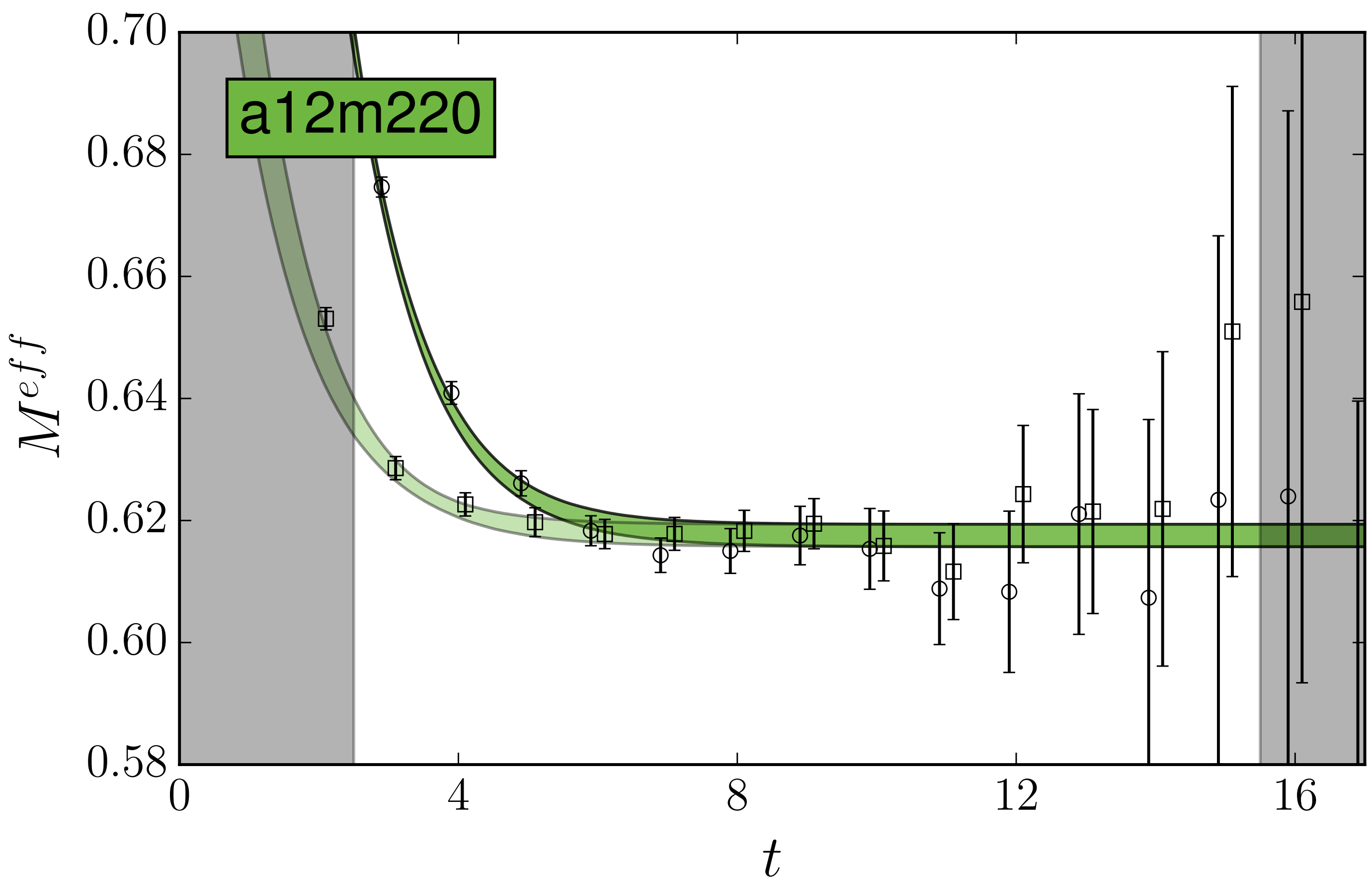
$$C(t) = \langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle$$

$$= \sum_n \langle\Omega|e^{\hat{H}t}\mathcal{O}(0)e^{-\hat{H}t}\frac{|n\rangle\langle n|}{2E_n}\mathcal{O}^\dagger(0)|\Omega\rangle$$

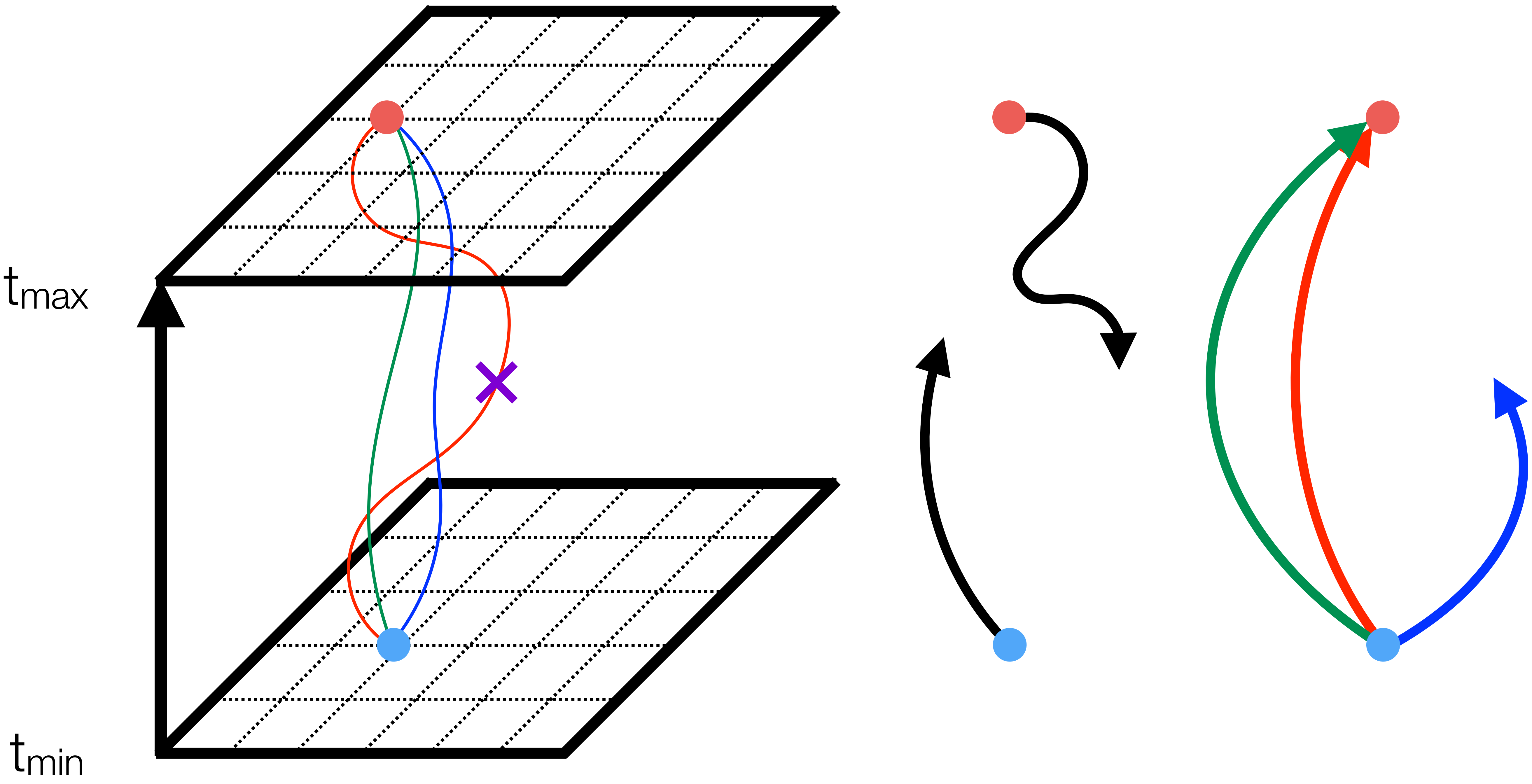
$$= \sum_n Z_n Z_n^\dagger \frac{e^{-E_n t}}{2E_n}$$

$$M^{eff}(t) = -\partial_t \ln\left(C(t)\right)$$

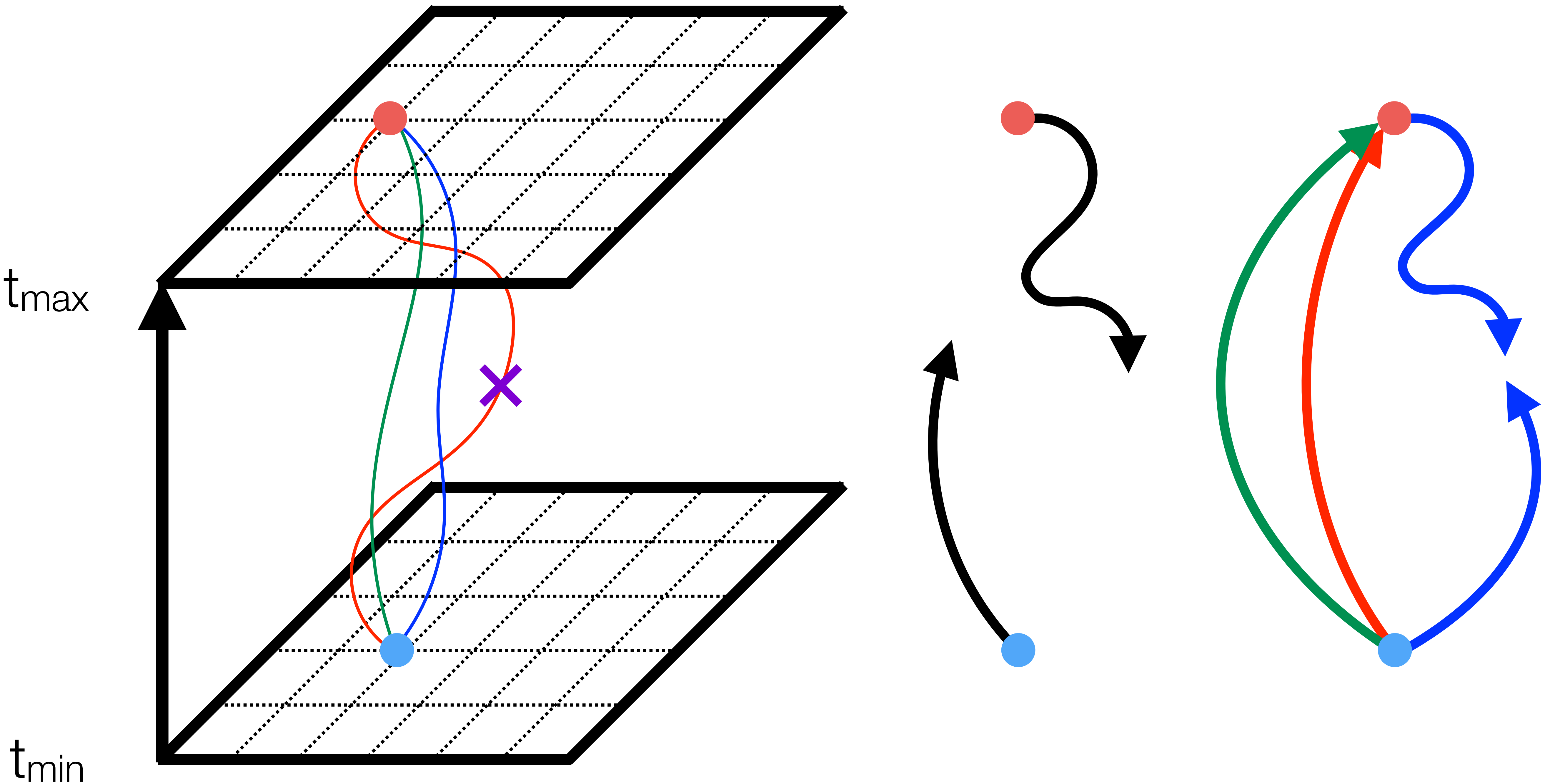
$$\lim_{t\to\infty} M^{eff}(t) = E_0$$



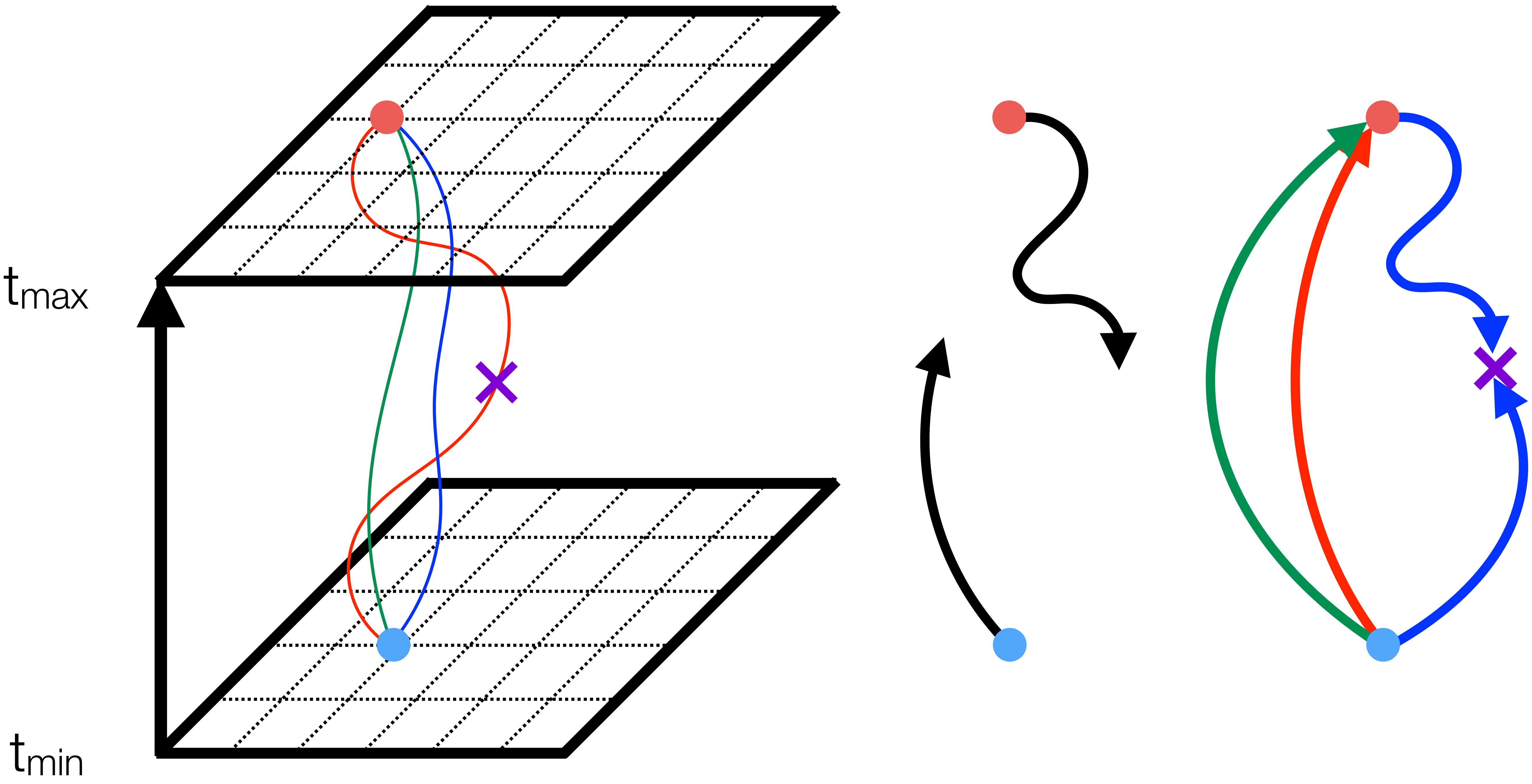
3-Point Standard Method



3-Point Standard Method

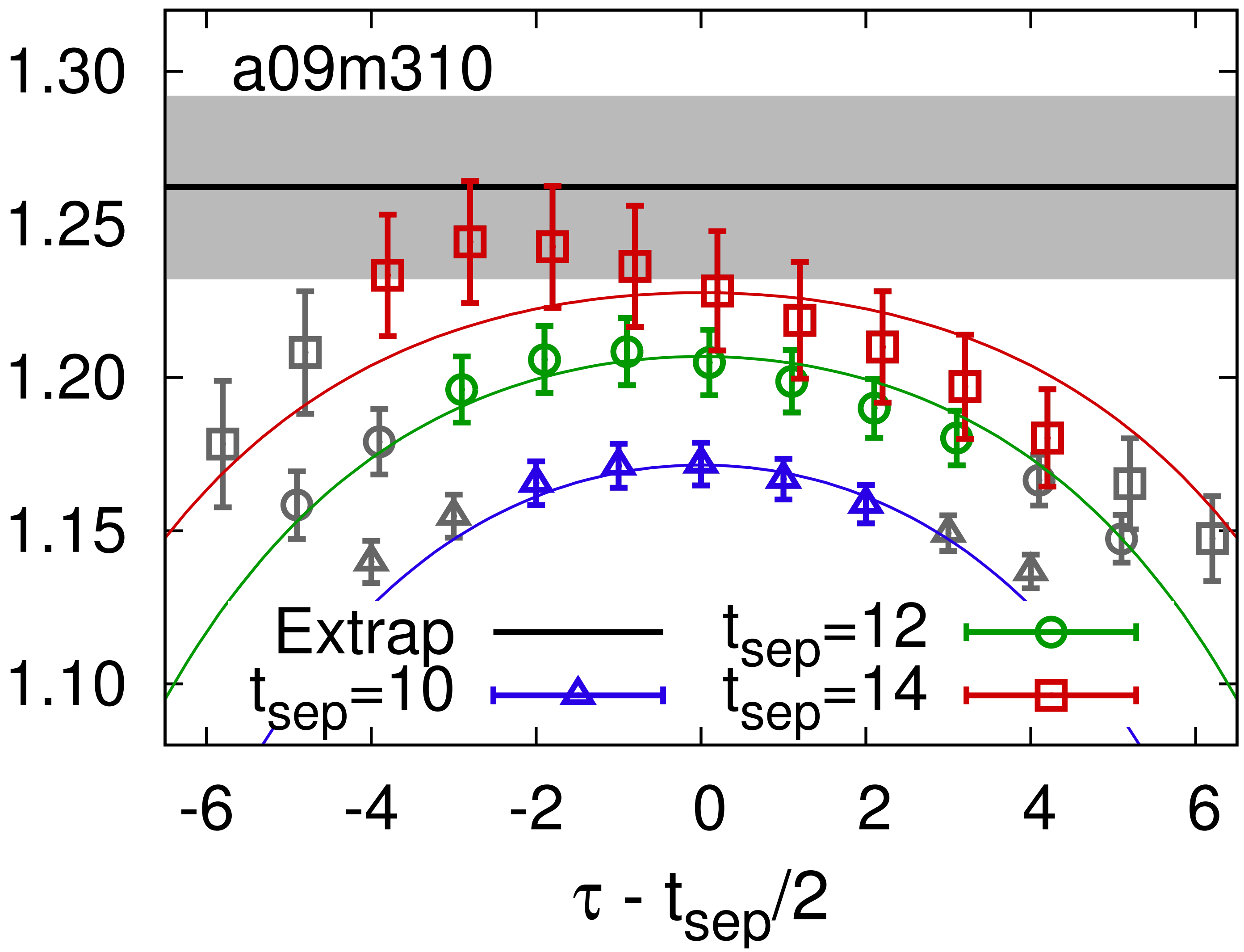


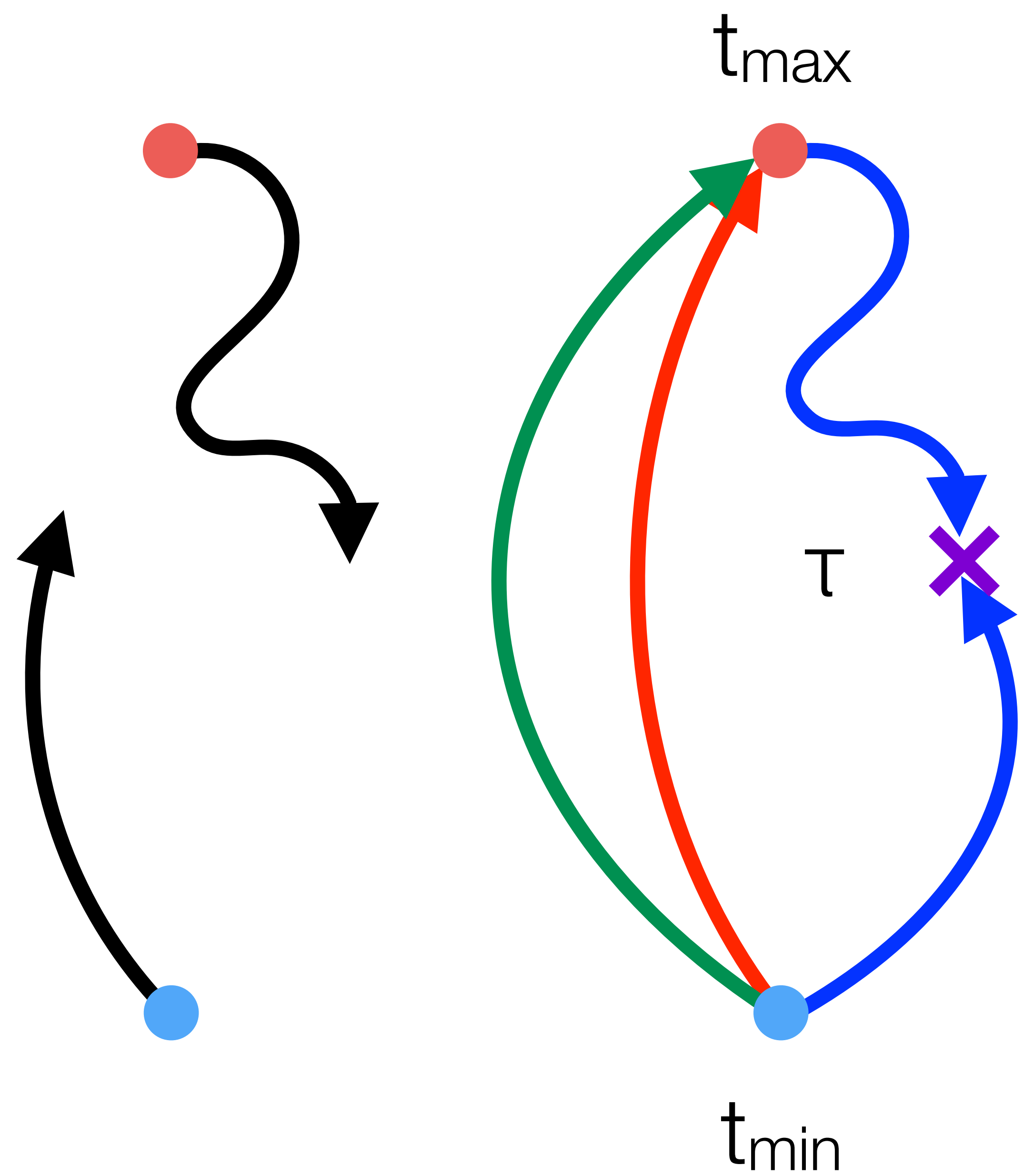
3-Point Standard Method



Standard Method

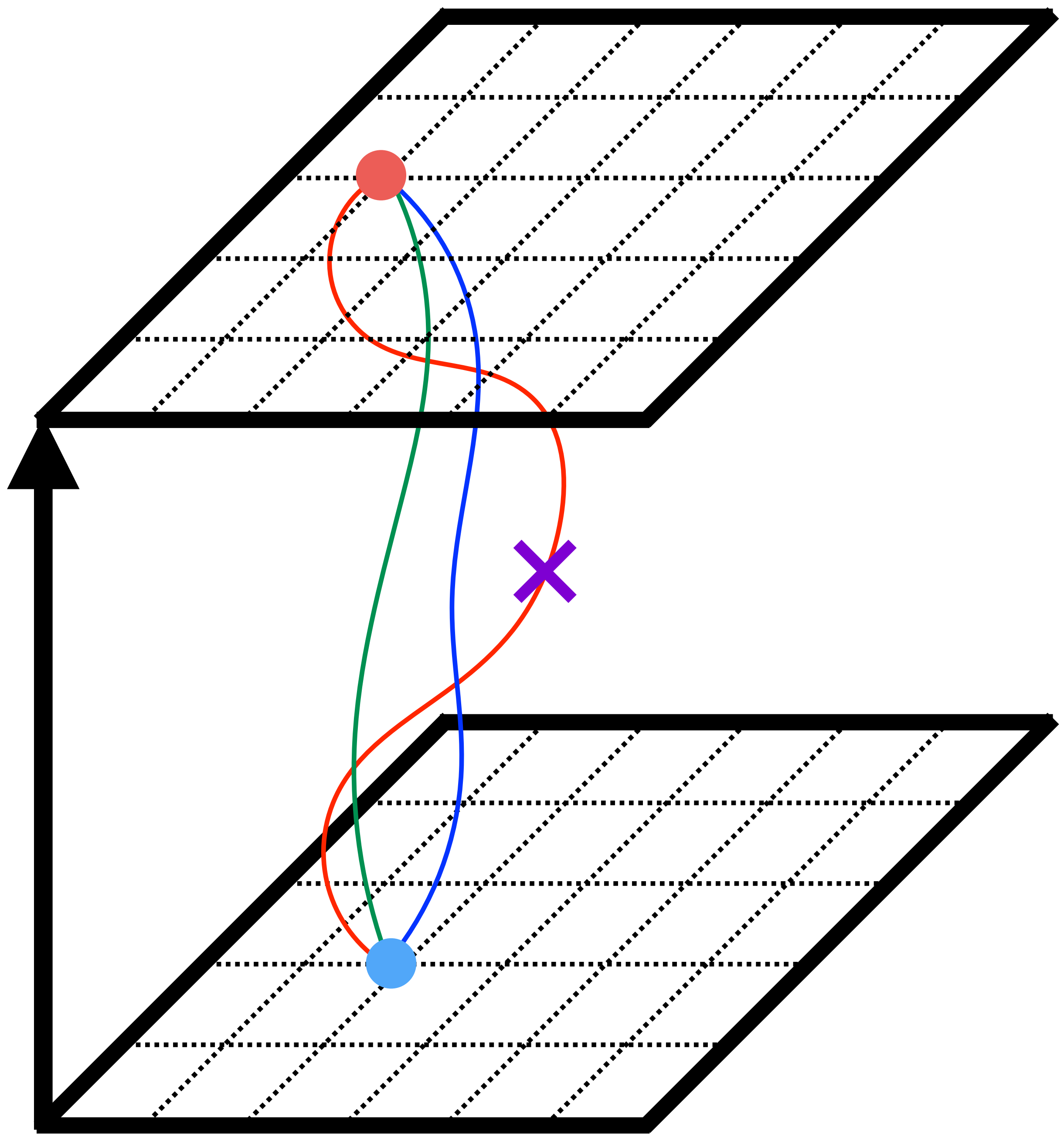
PNDME Phys. Rev. D94 (2016) arXiv:1606.07049



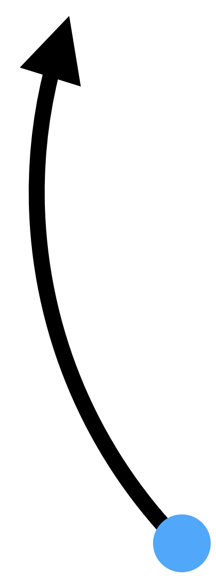


Feynman-Hellman Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963



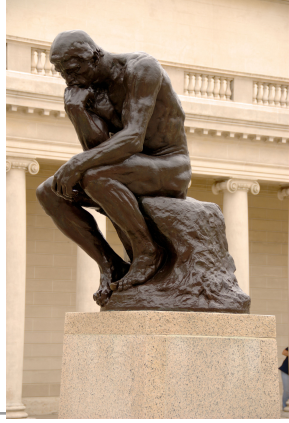
t_{min}

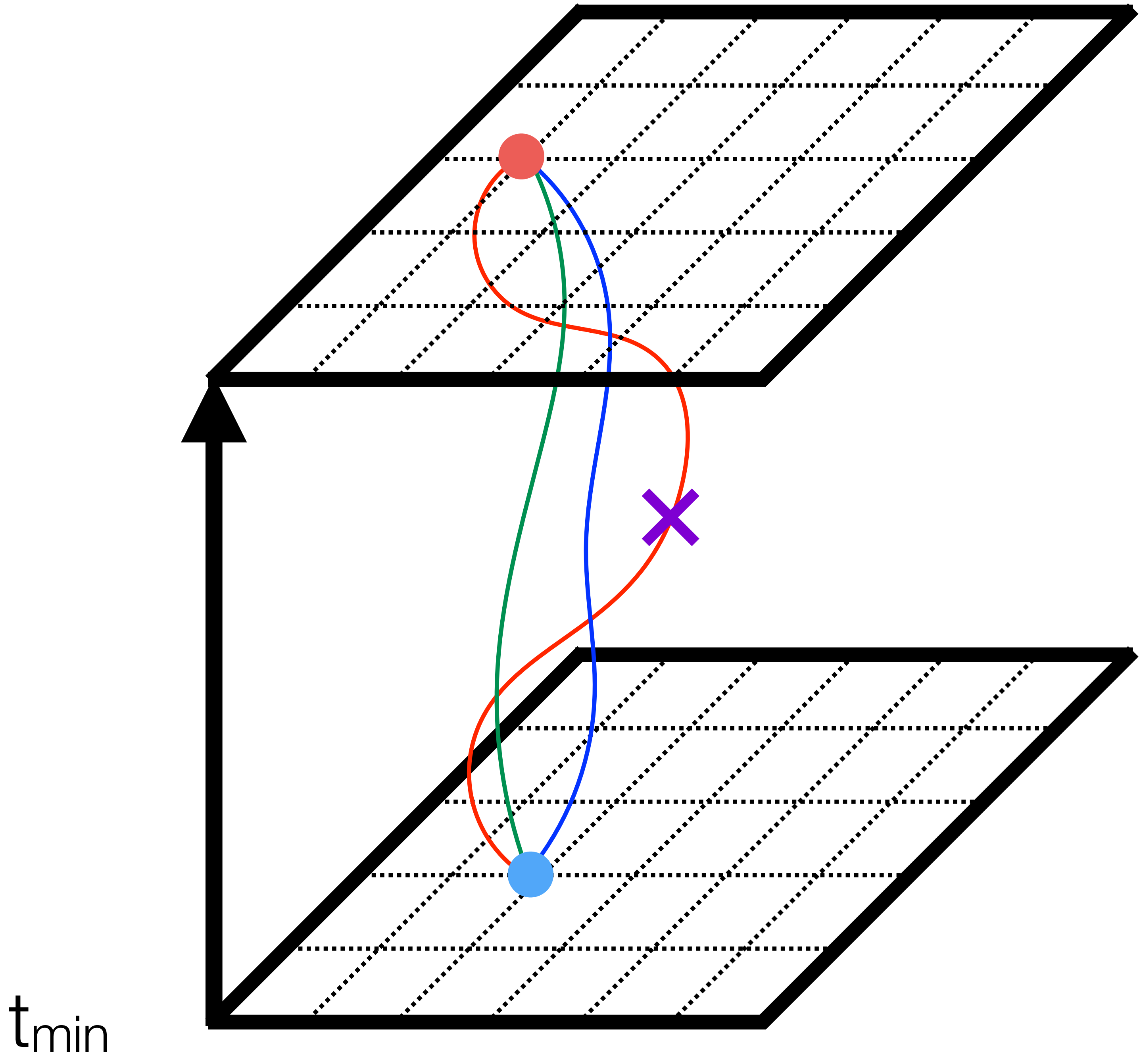


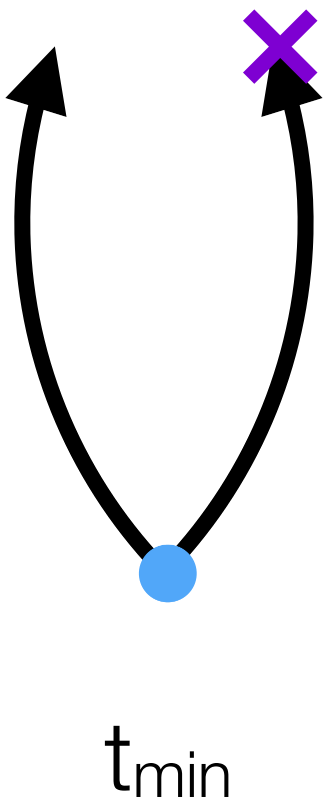
t_{min}

Feynman-Hellman Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963







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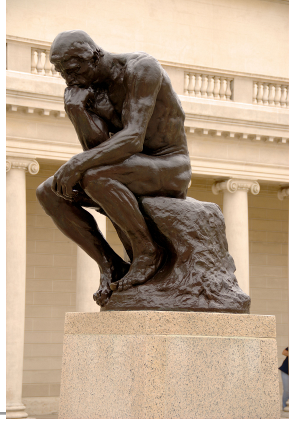
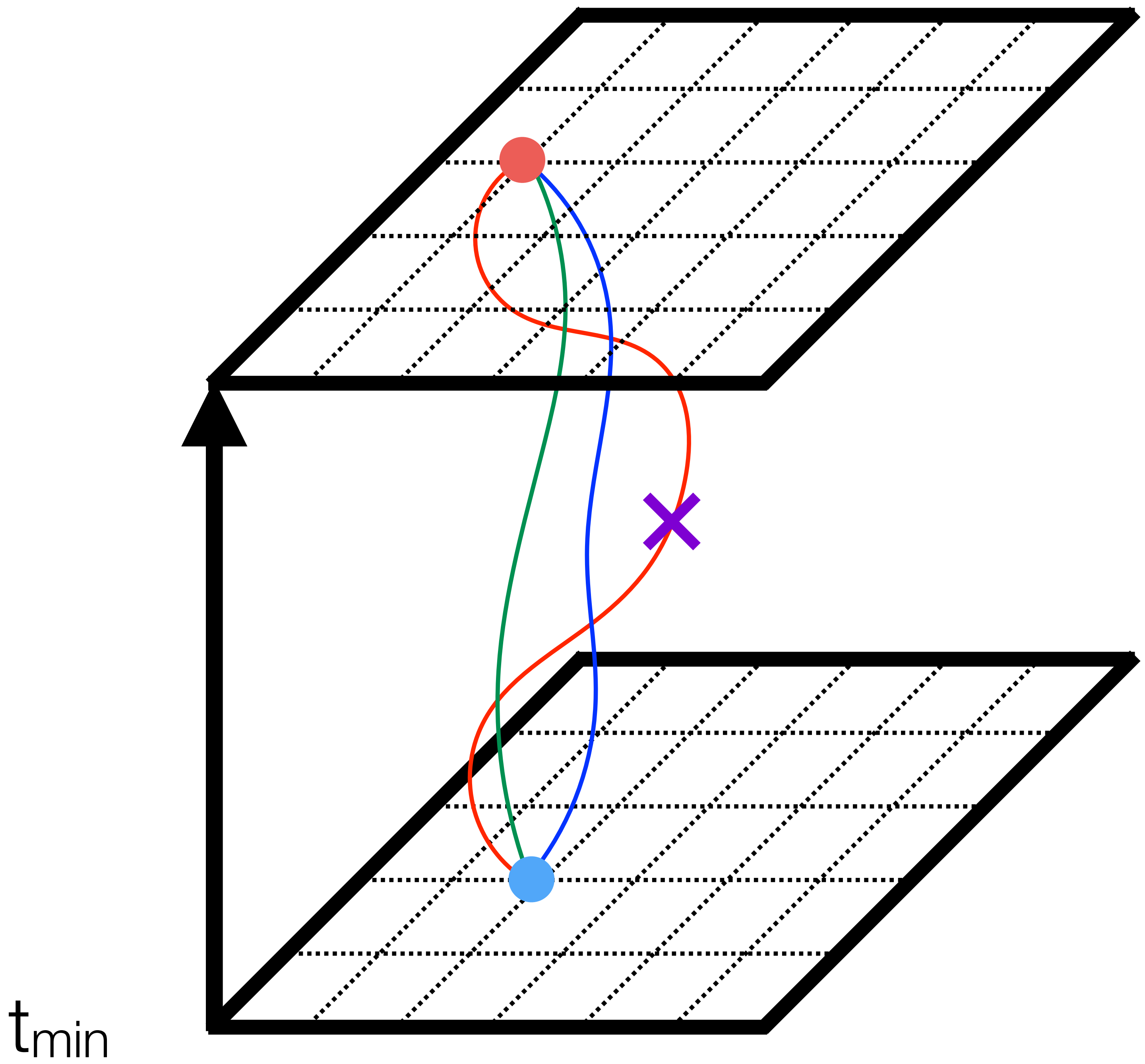
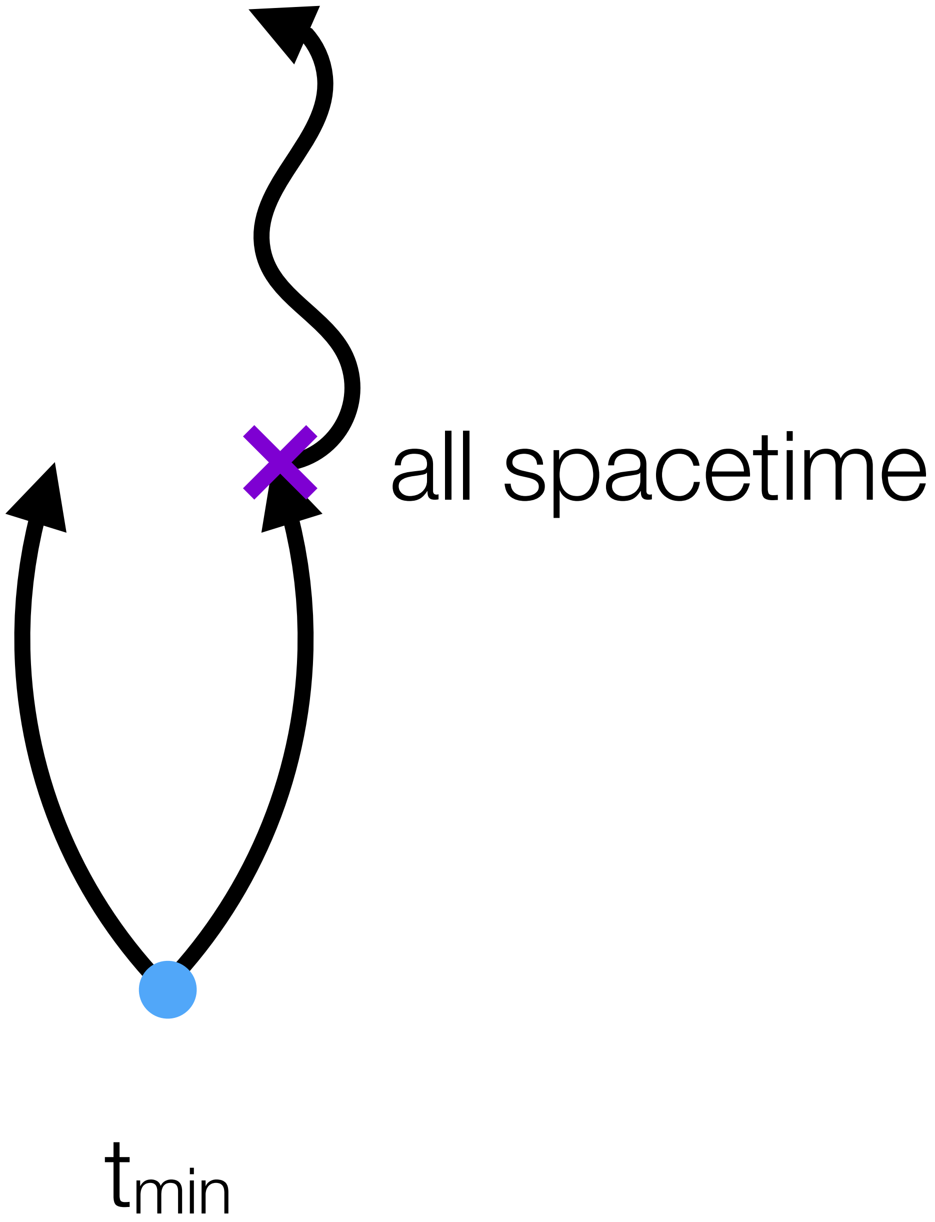
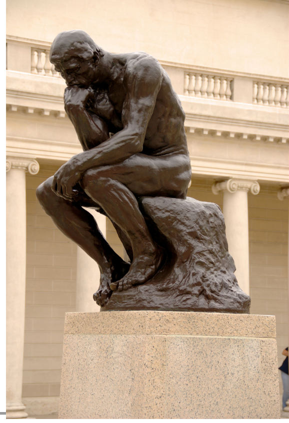


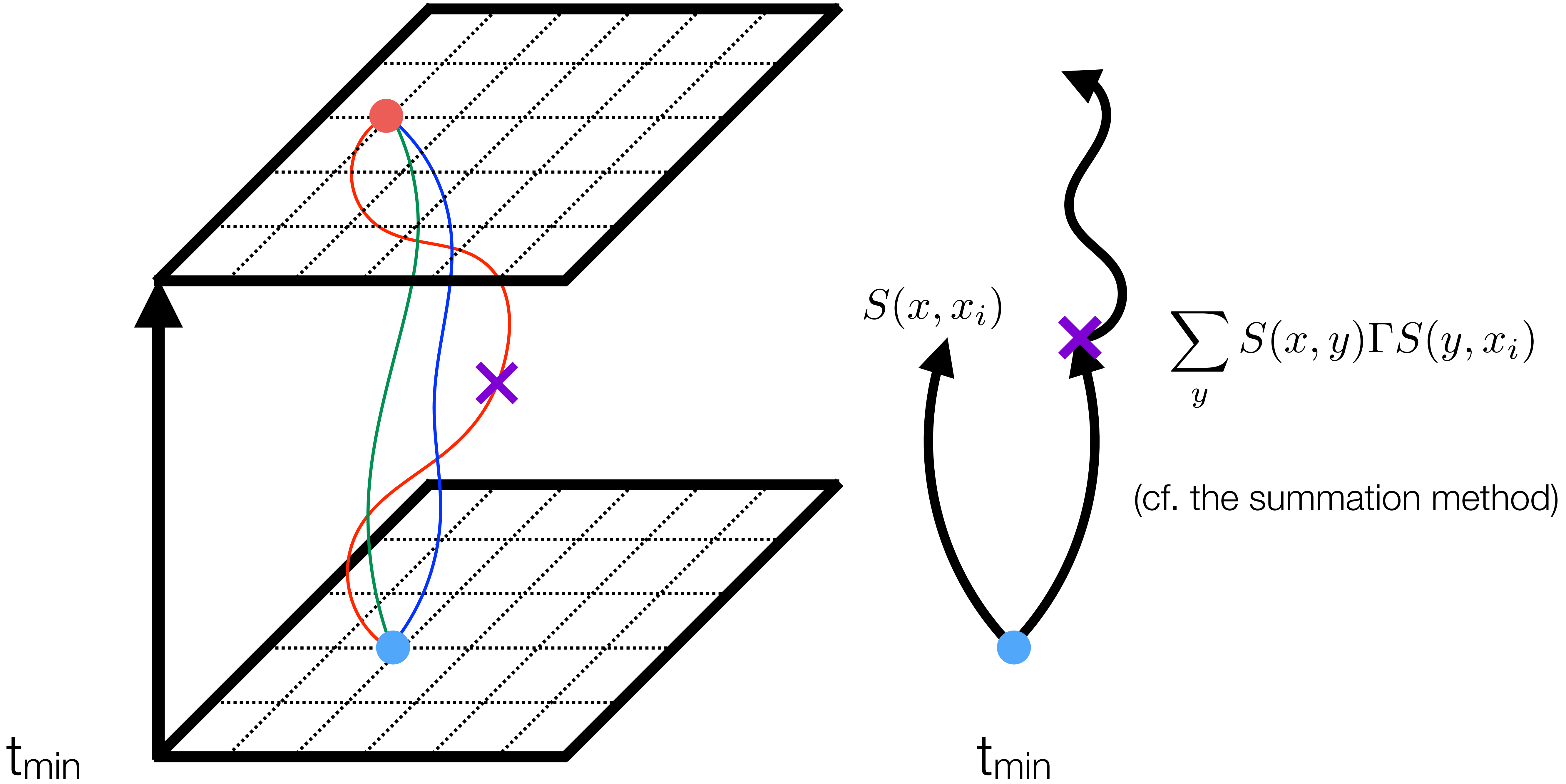




Feynman-Hellman Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963





Feynman-Hellman Method

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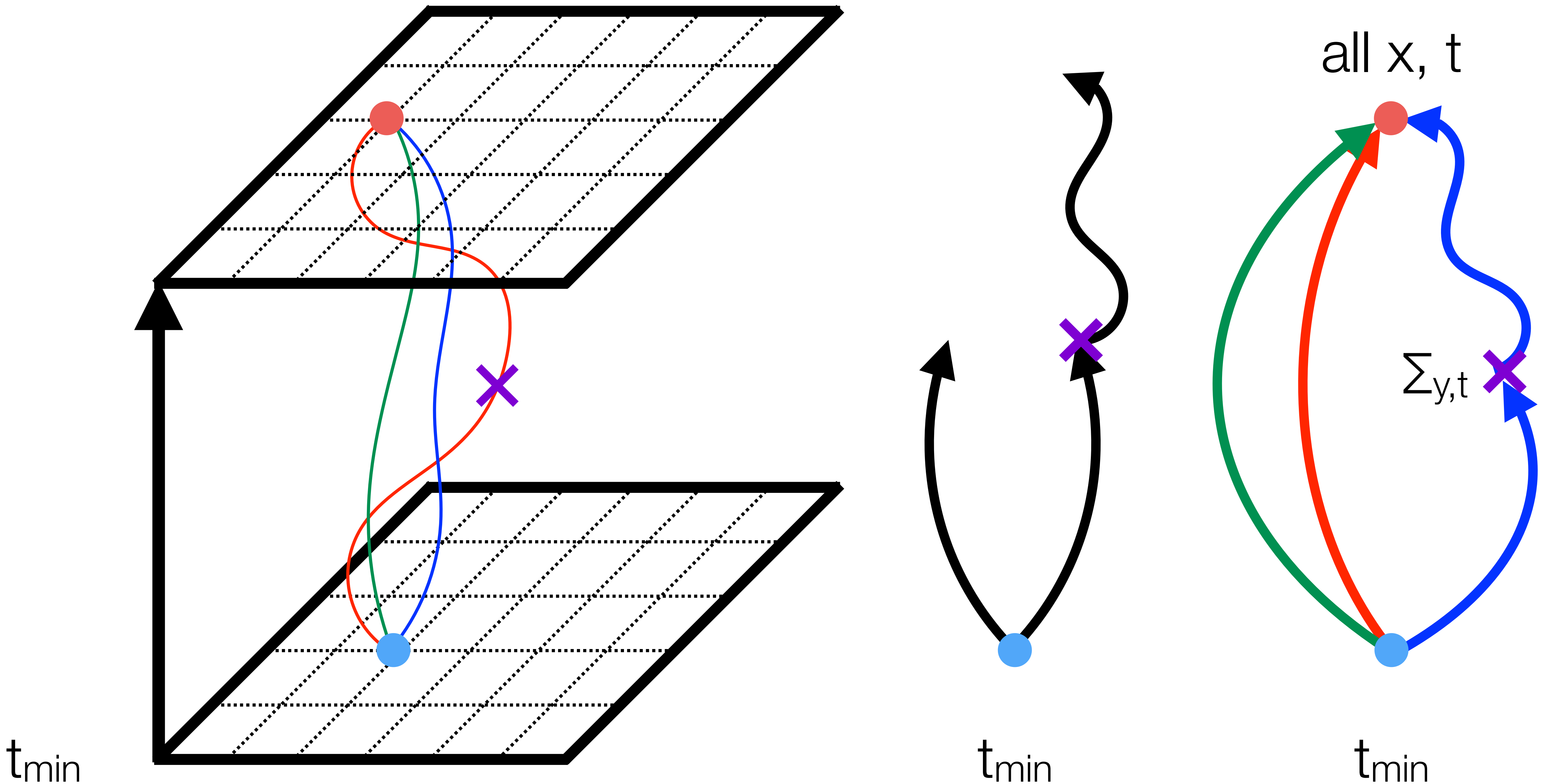
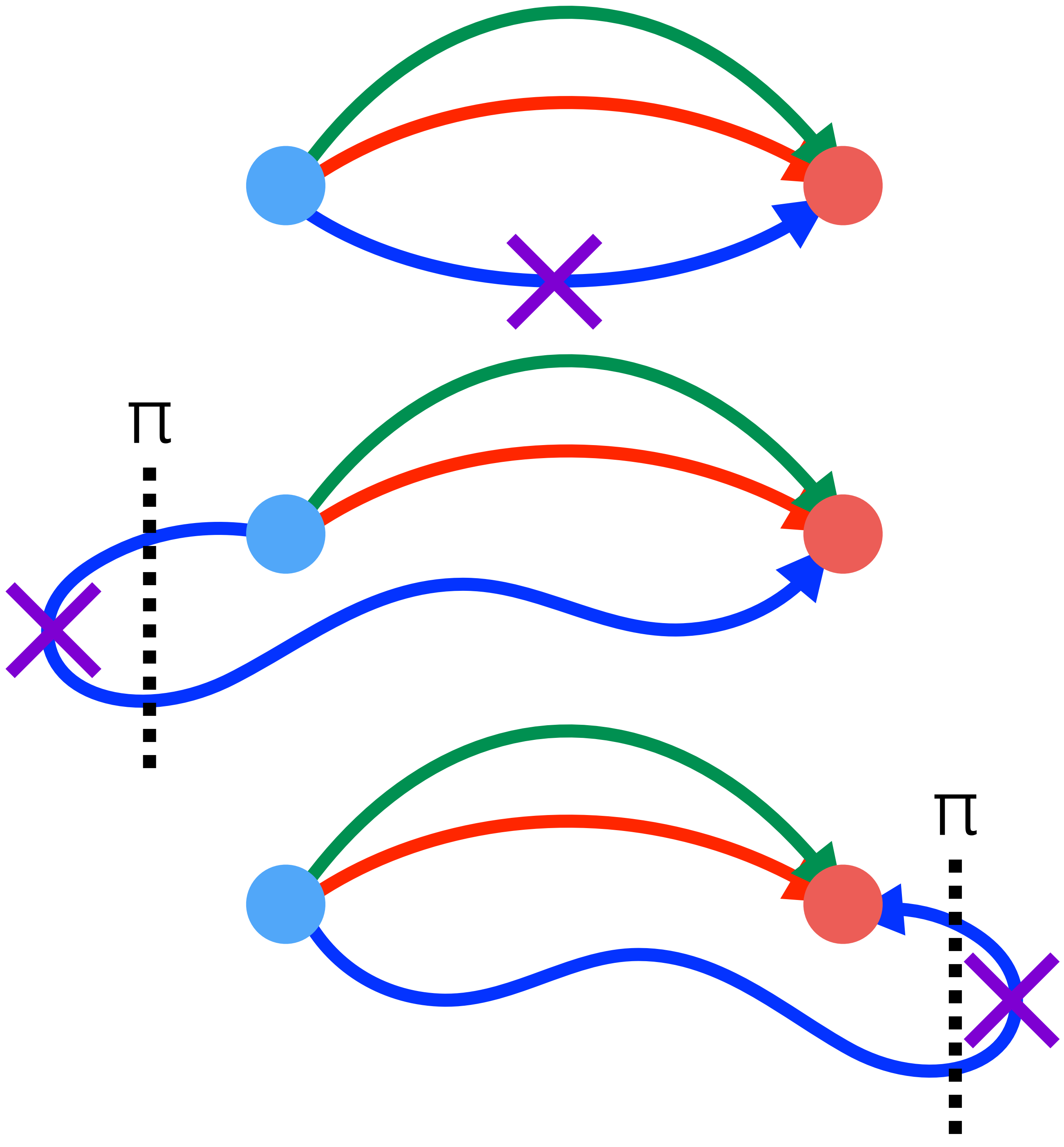
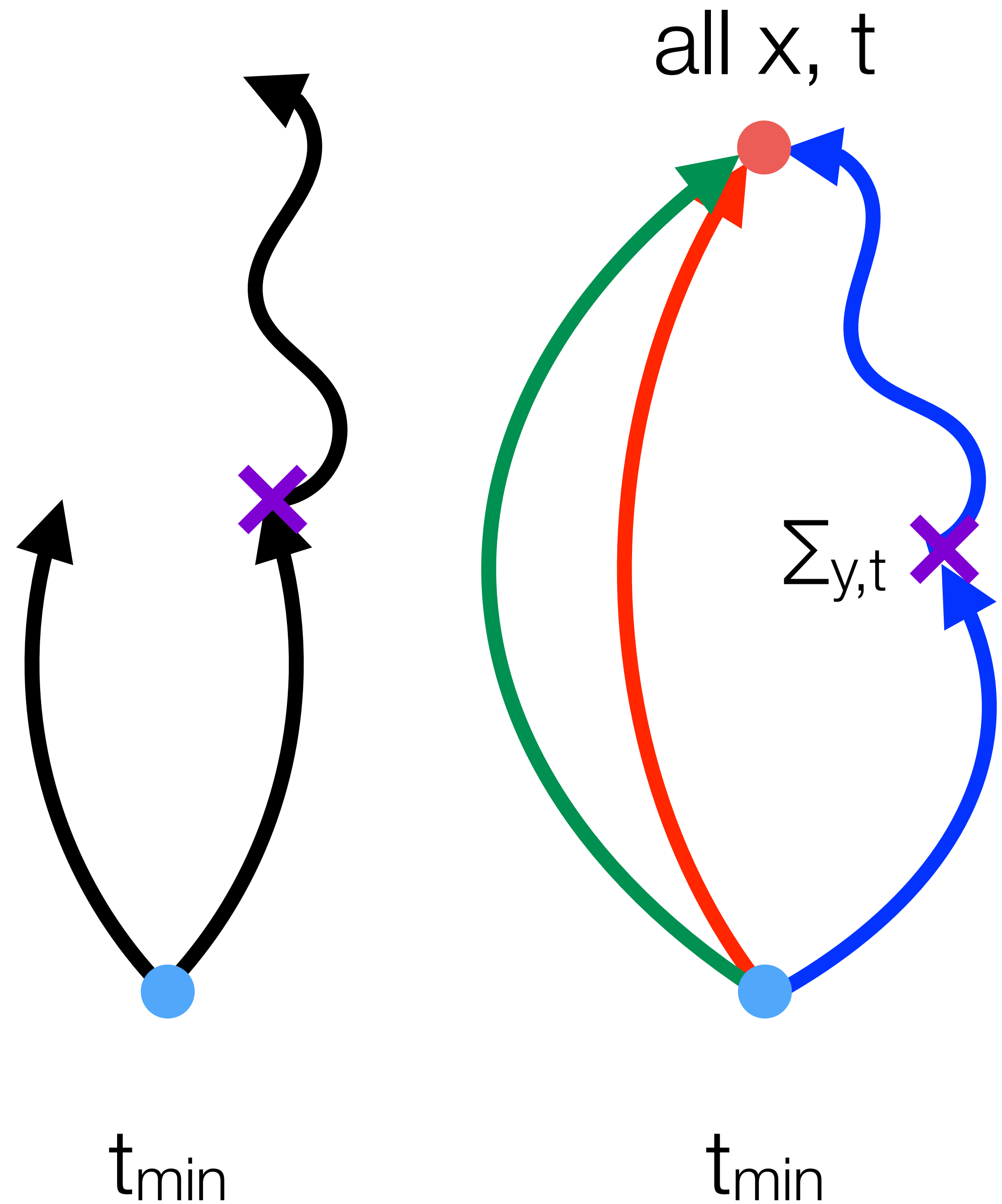


Improved systematics

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963





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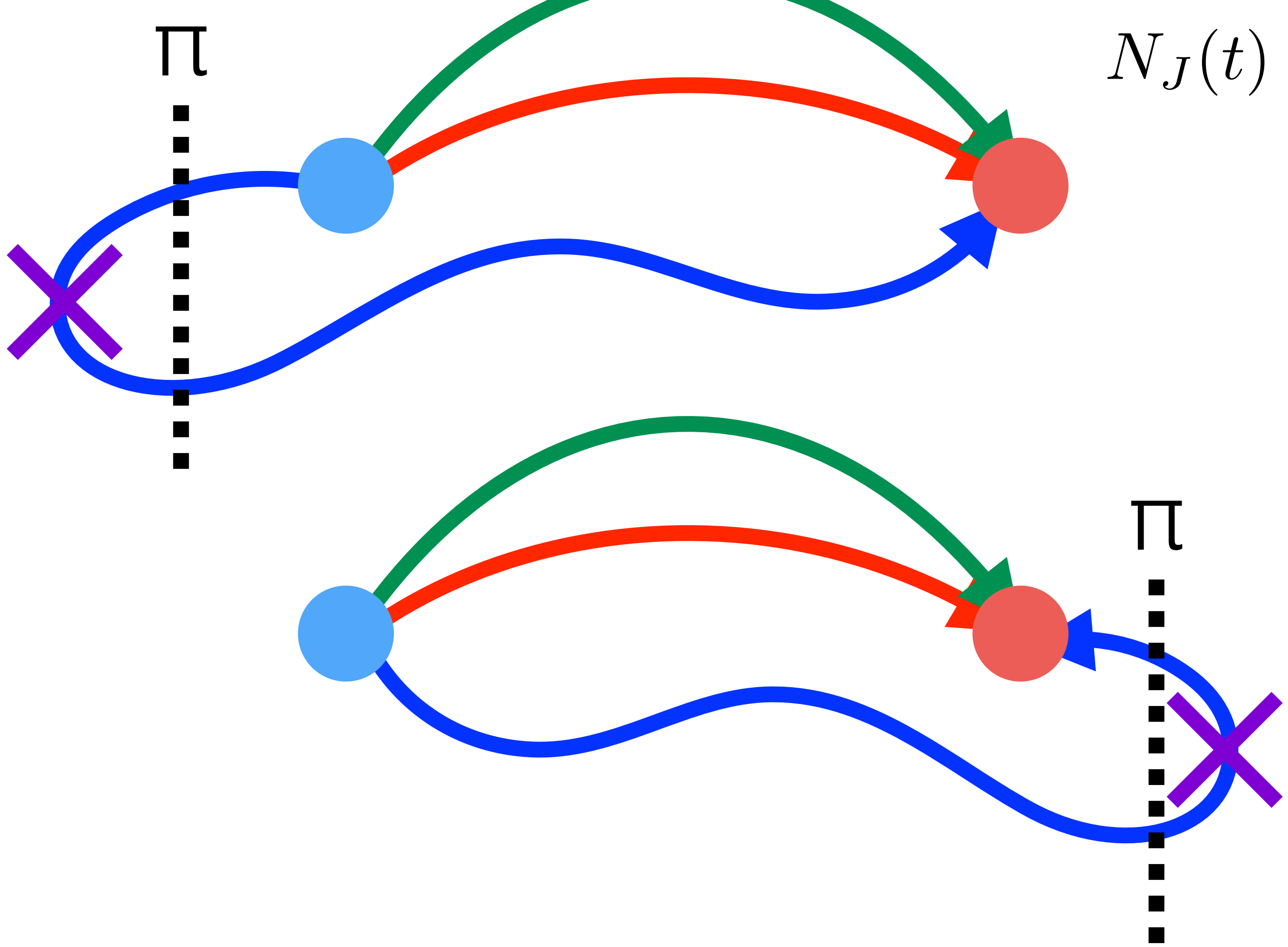


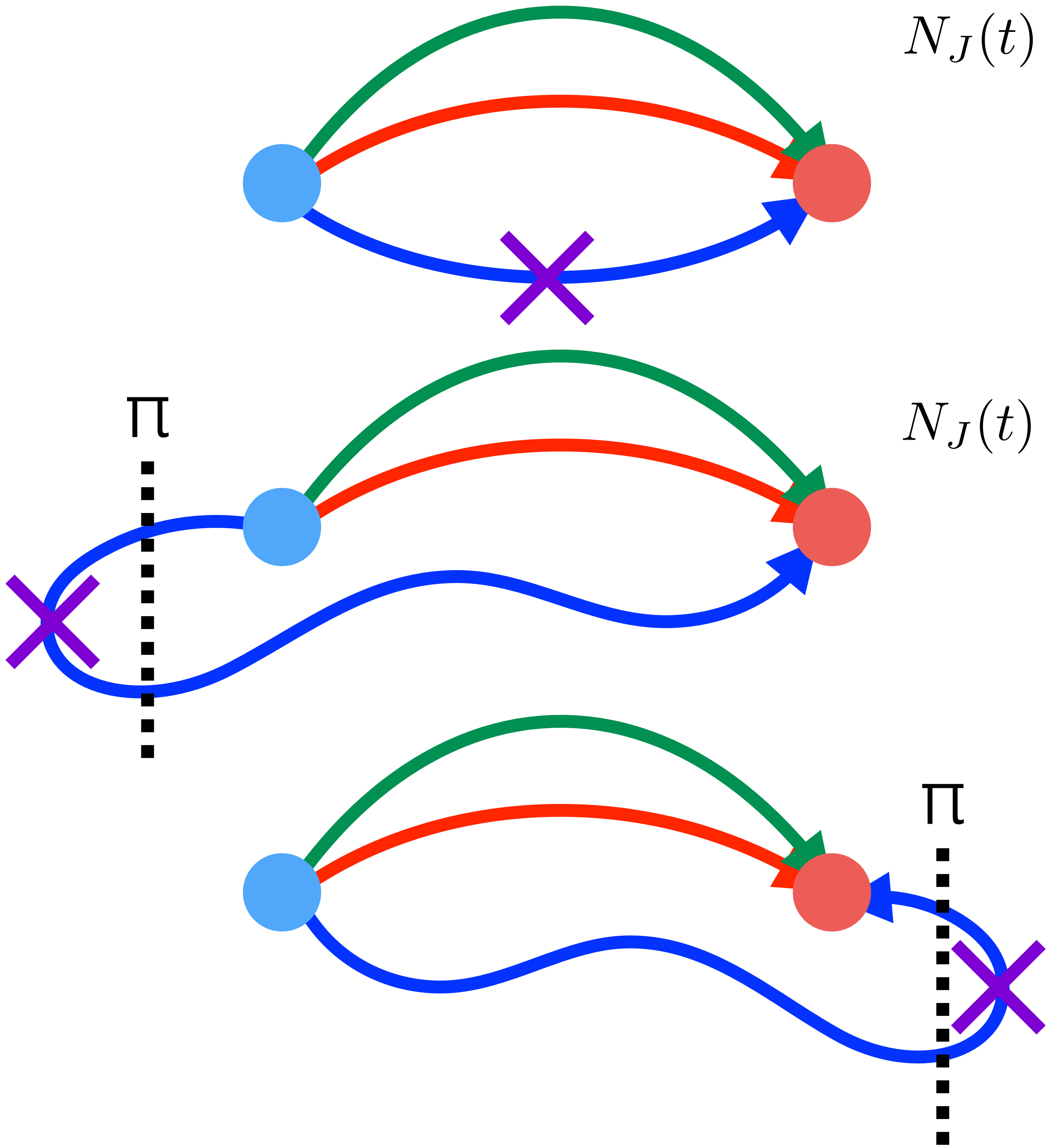
$$N_J(t) = \sum_{t'} \langle \Omega | T\{O(t) J(t') O^\dagger(0)\} | \Omega \rangle$$

$$N_J(t) = \sum_n \left[(t-1) z_n g^J_{nn} z^\dagger_n + d^J_n \right] e^{-E_n t}$$

$$+ \sum_{\substack{n \\ m \neq n}} z_n g^J_{nm} z^\dagger_m \frac{e^{-E_n t + \frac{\Delta_{nm}}{2}} - e^{-E_m t + \frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{nm}}{2}} - e^{\frac{\Delta_{mn}}{2}}}$$

Improved systematics

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963



$$N_J(t) = \sum_{t'} \langle \Omega | T\{O(t) J(t') O^\dagger(0)\} | \Omega \rangle$$

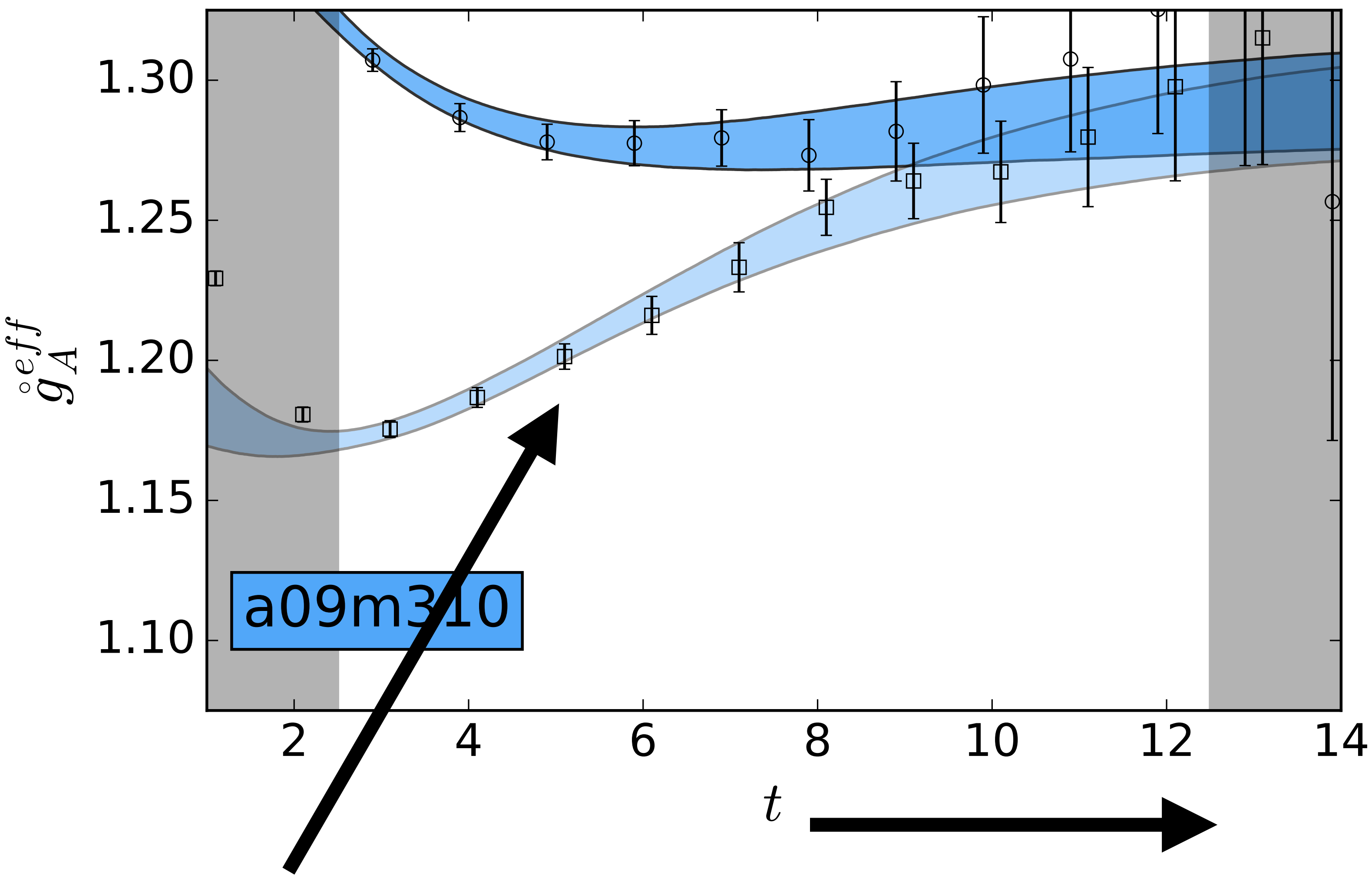
time dependence of
what you want

$$N_J(t) = \sum_n \left[(t-1) z_n g^J_{nn} z^\dagger_n + d^J_n \right] e^{-E_n t} + \sum_{\substack{n \\ m \neq n}} z_n g^J_{nm} z^\dagger_m \frac{e^{-E_n t + \frac{\Delta_{nm}}{2}} - e^{-E_m t + \frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{nm}}{2}} - e^{\frac{\Delta_{mn}}{2}}}$$

differs from the time dependence of
pieces you don't care about

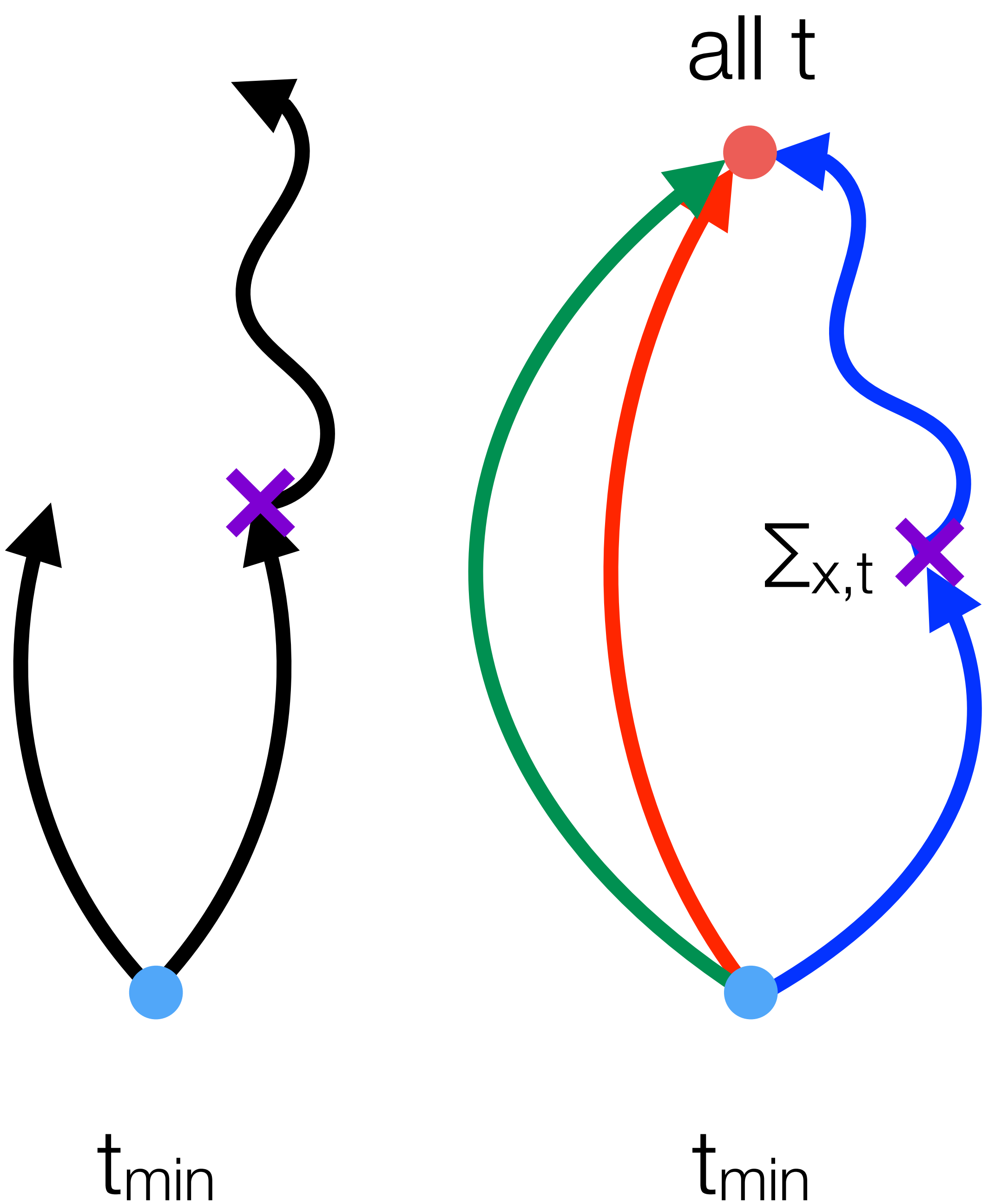
Example Effective Matrix Element

arXiv:1704.01114



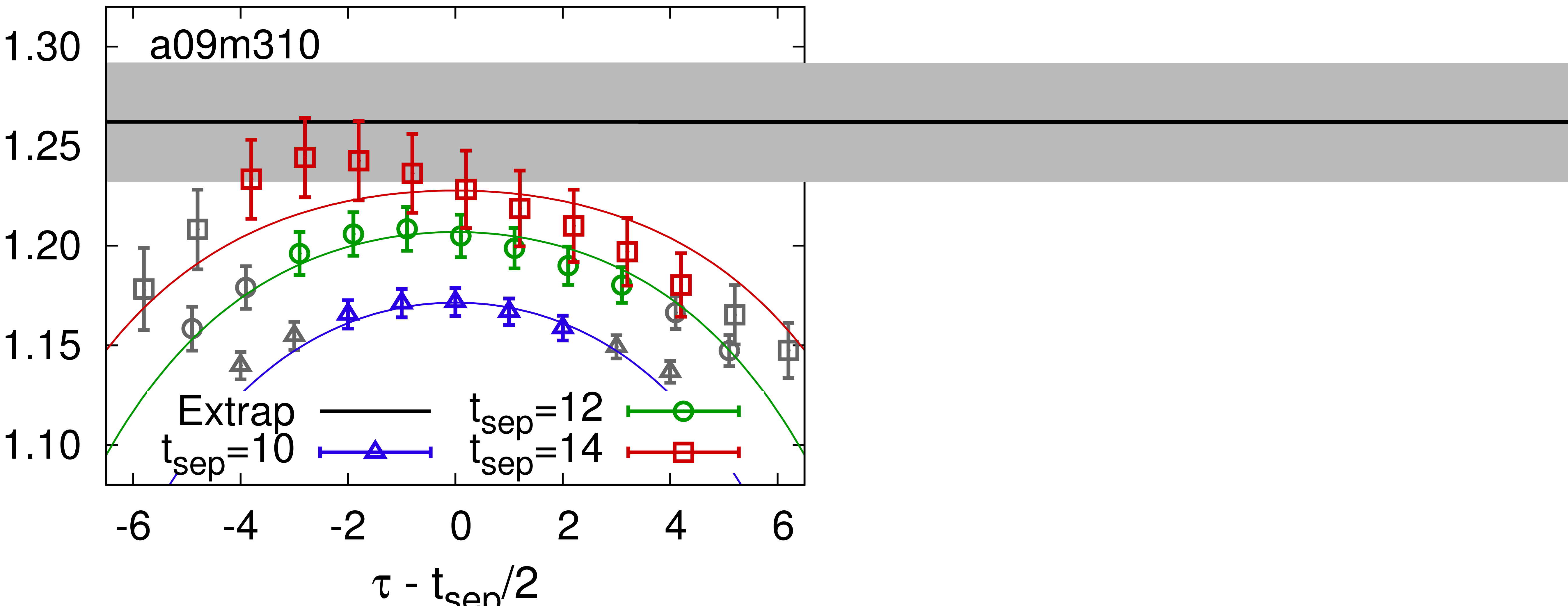
known functional form

asymptotes in just one time variable



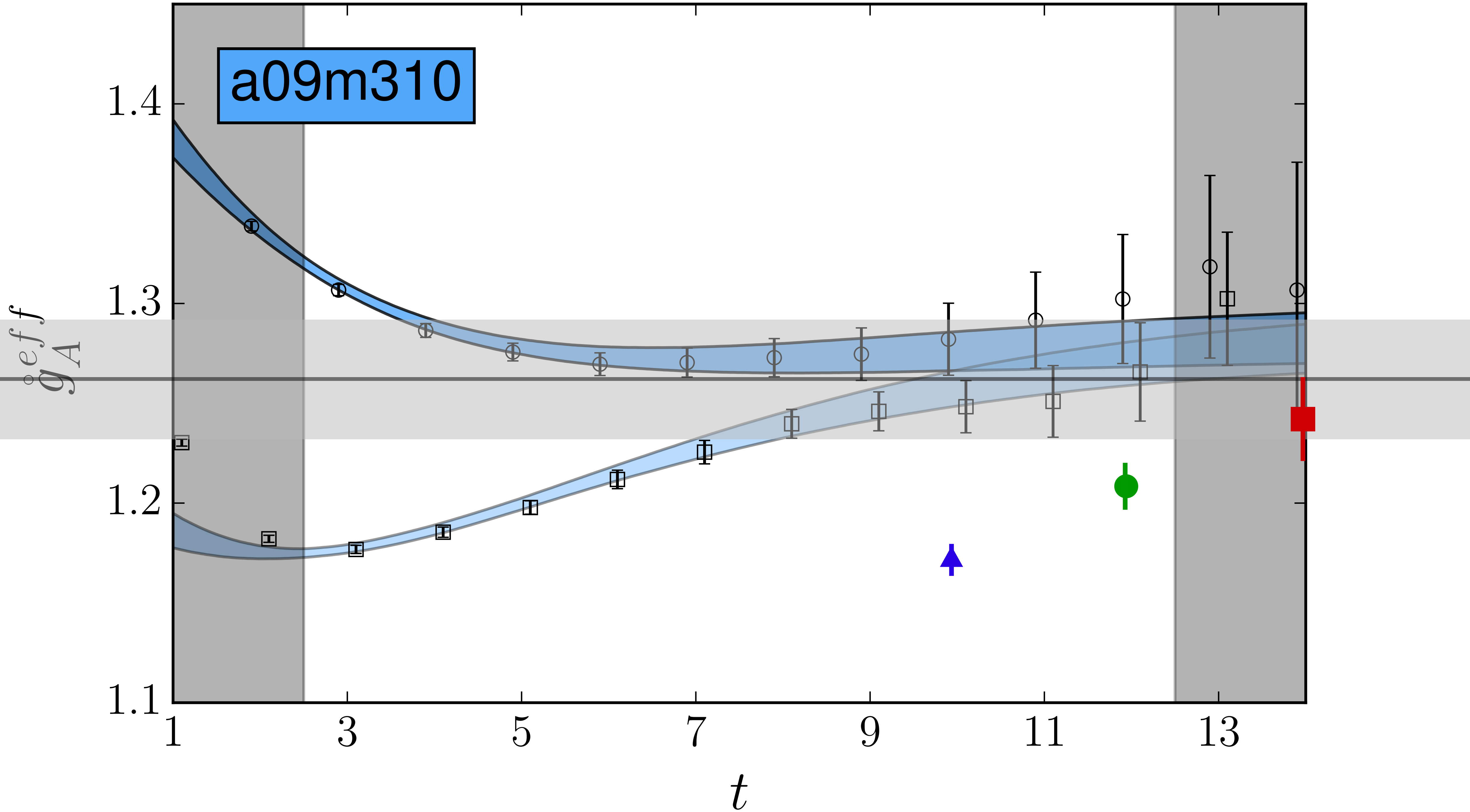
Improved Systematics

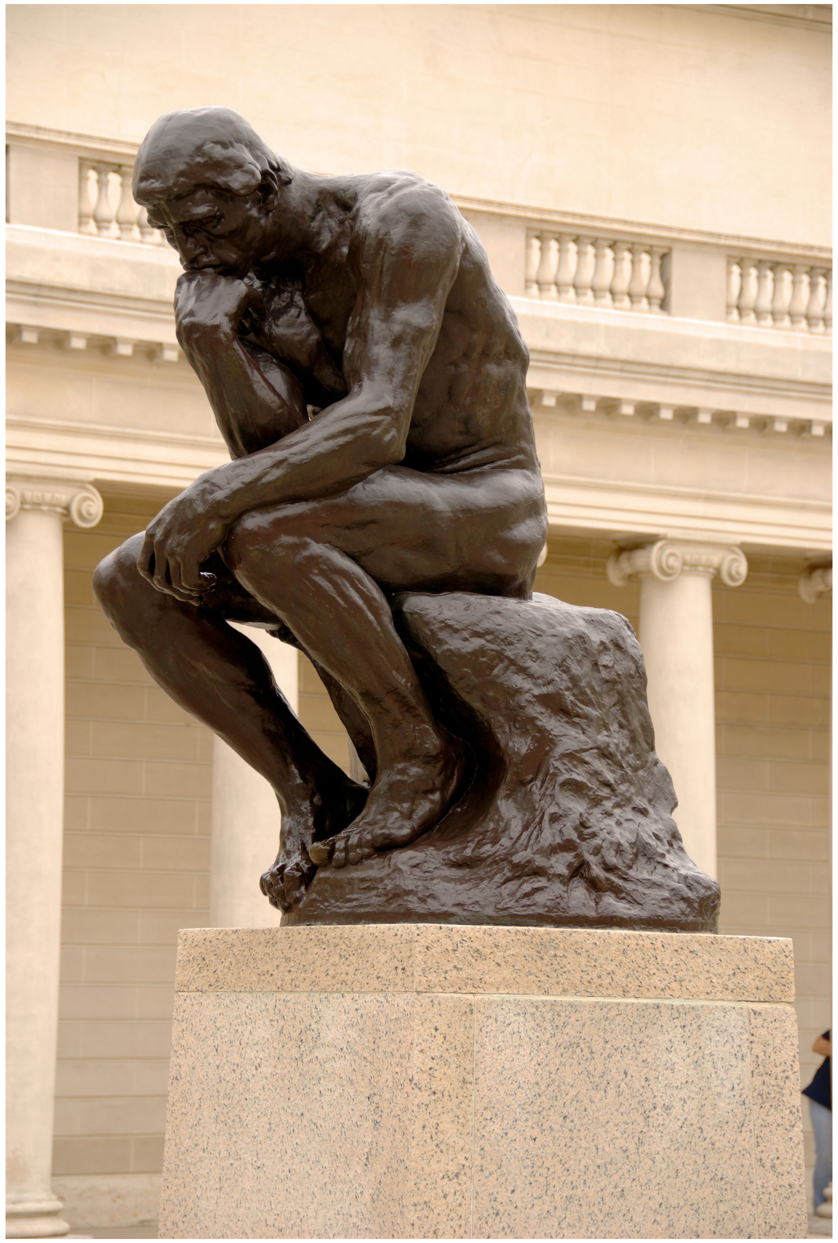
PNDME Phys. Rev. D94 (2016) arXiv:1606.07049

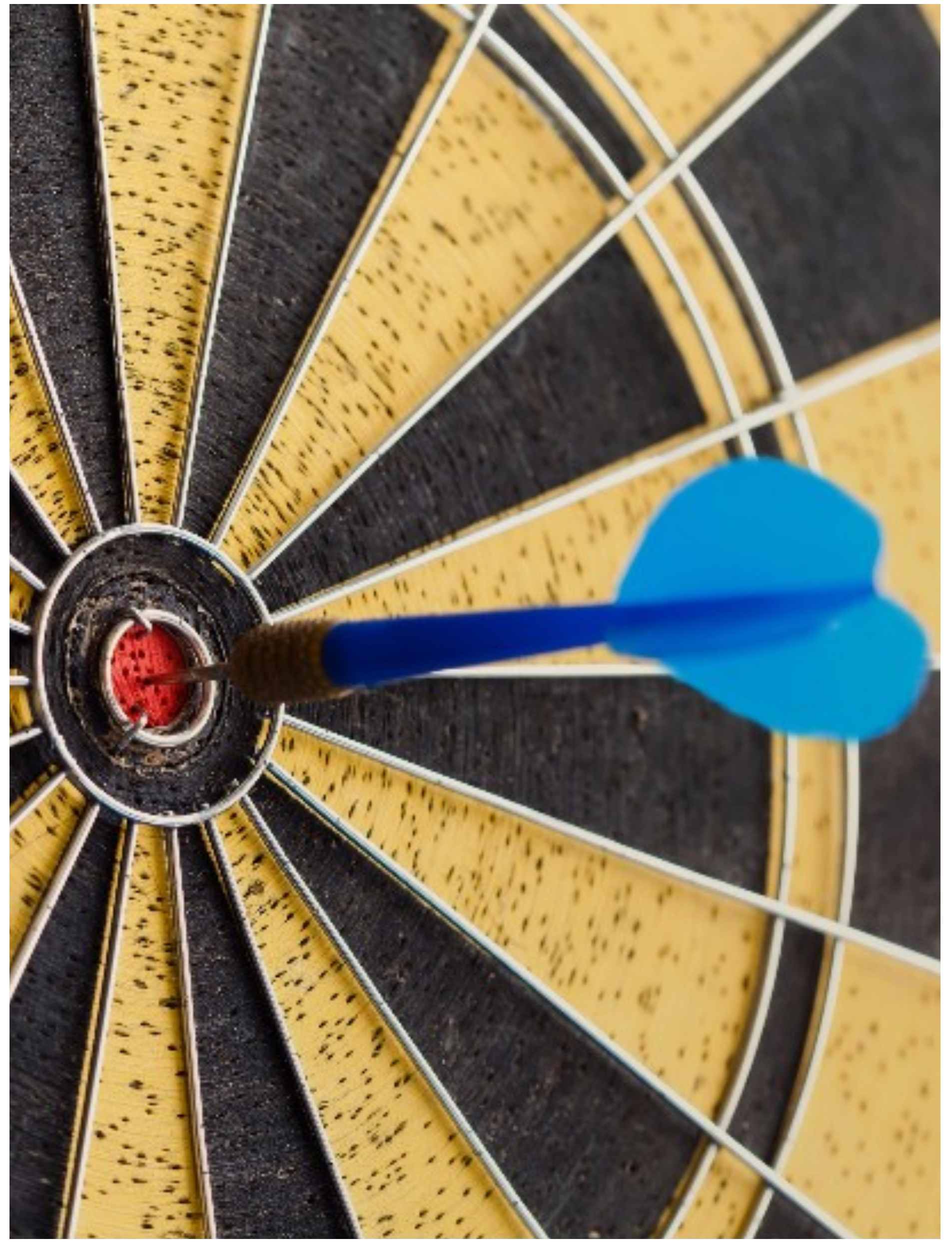


Improved Systematics

arXiv:1704.01114







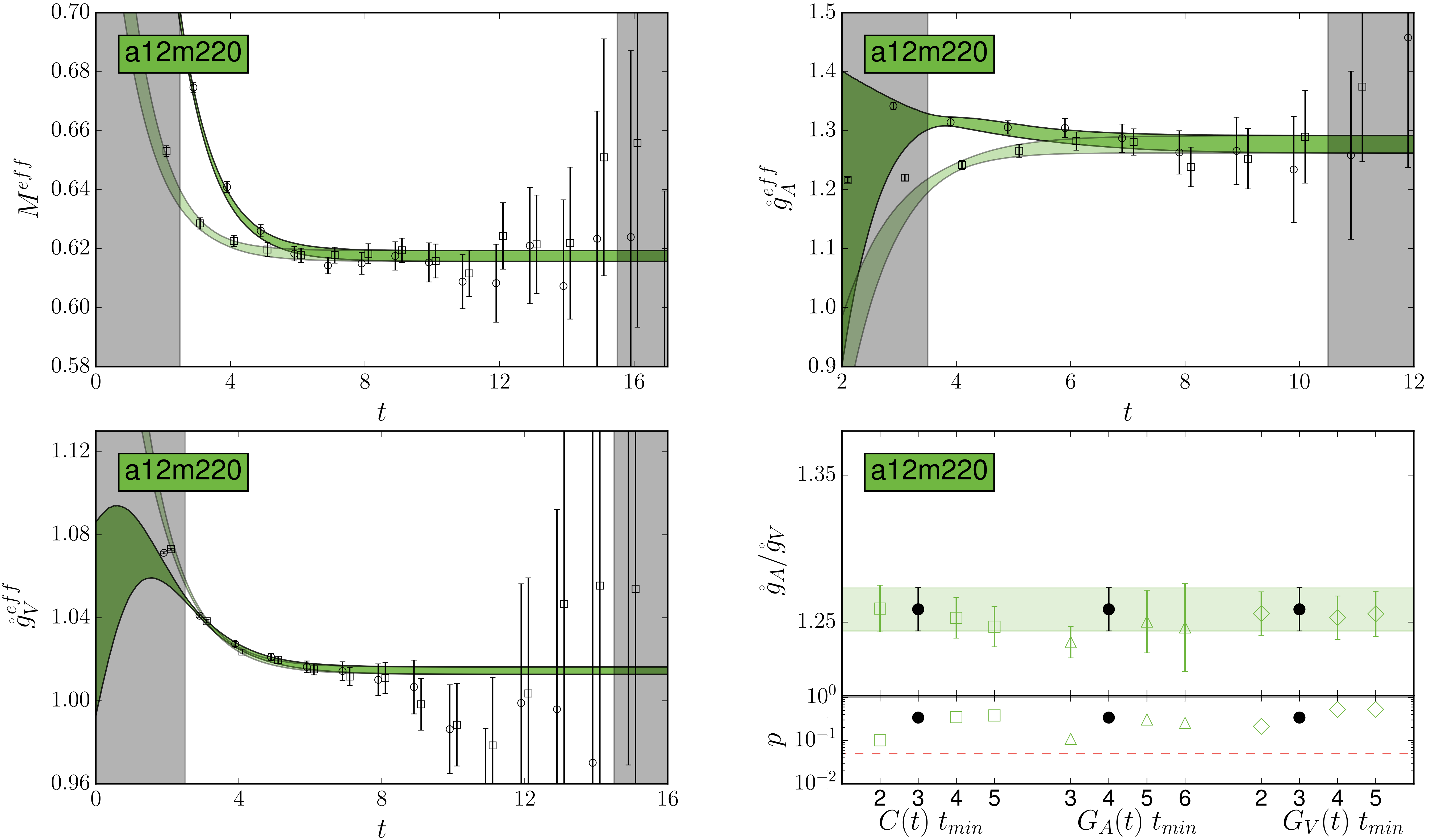
- Not QCD Specific
- Any fermion bilinear matrix element

- 3-point → 2-point function: easier fits
- Known spectral decomposition

- Stochastic enhancement
- 3/2 the cost of one temporal separation

Systematics for an example point

arXiv:1704.01114



Systematics for an example point

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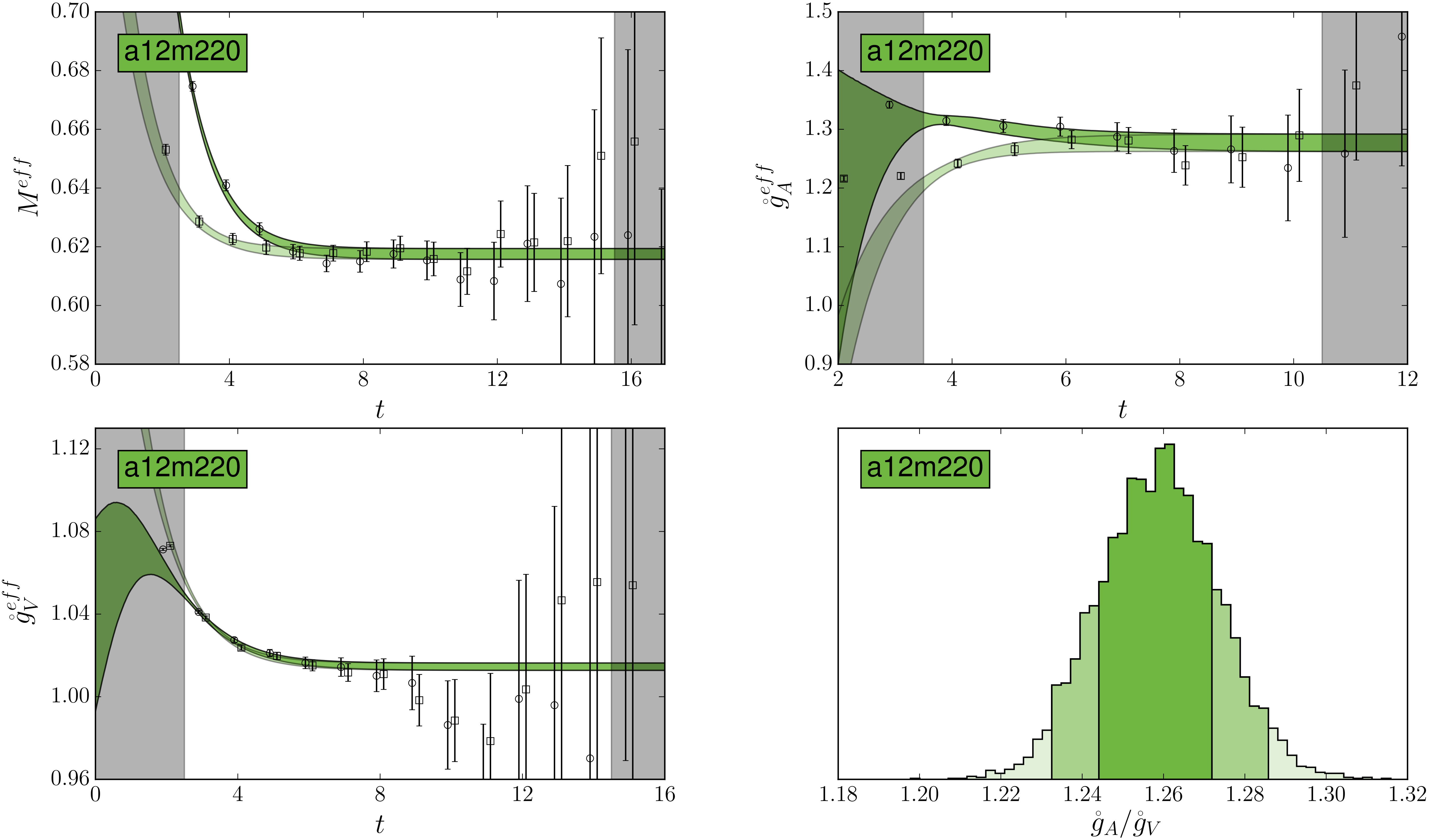
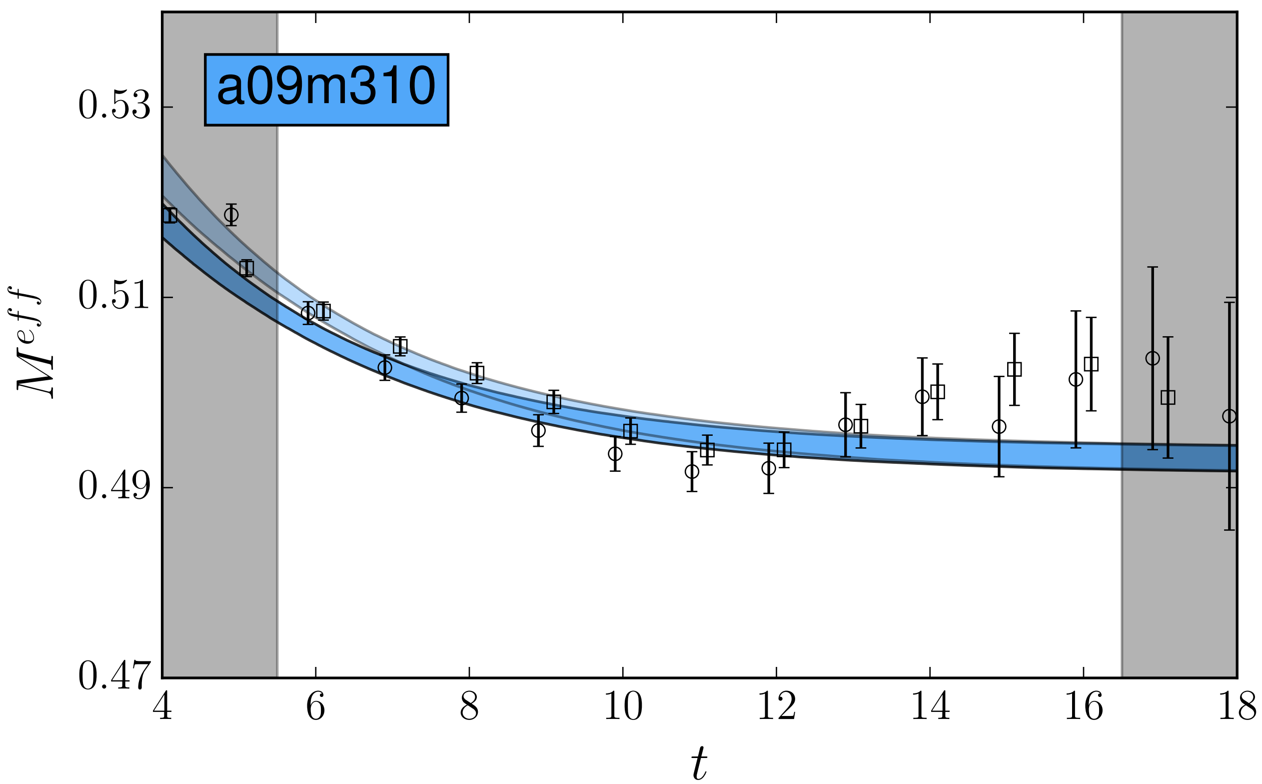
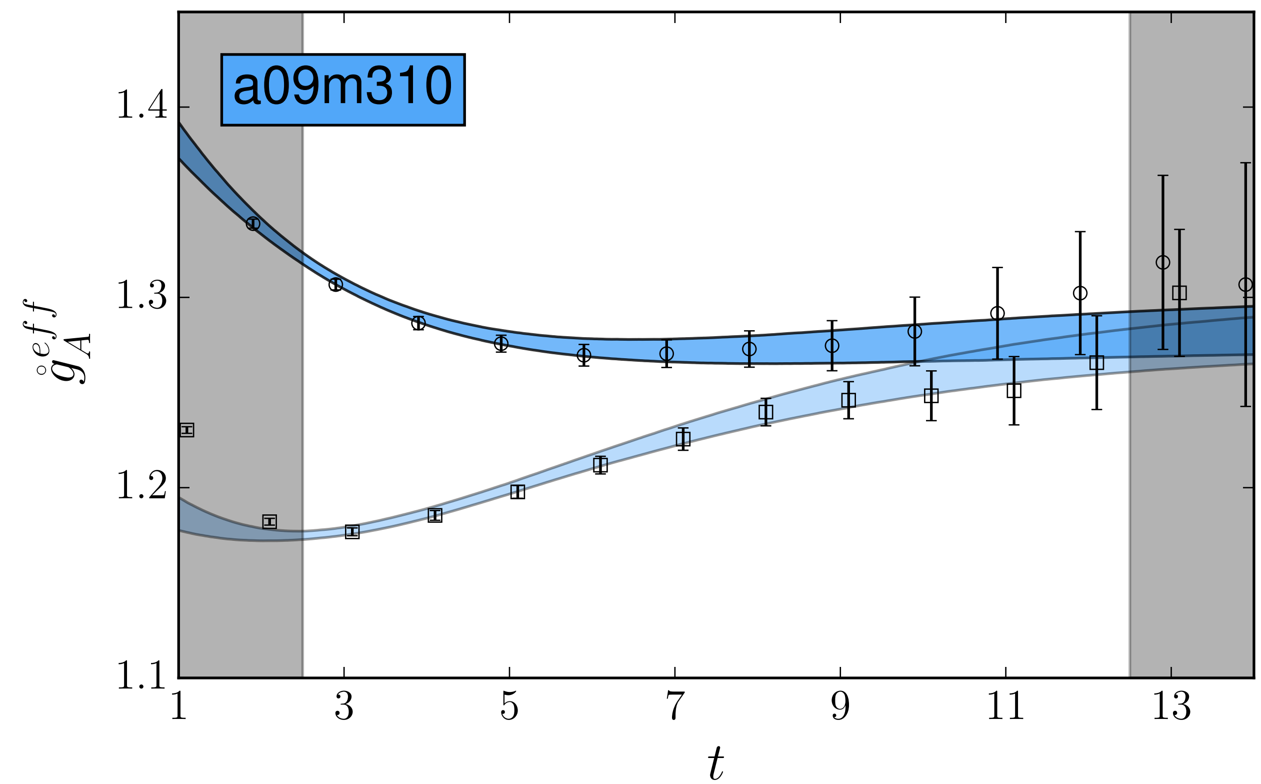
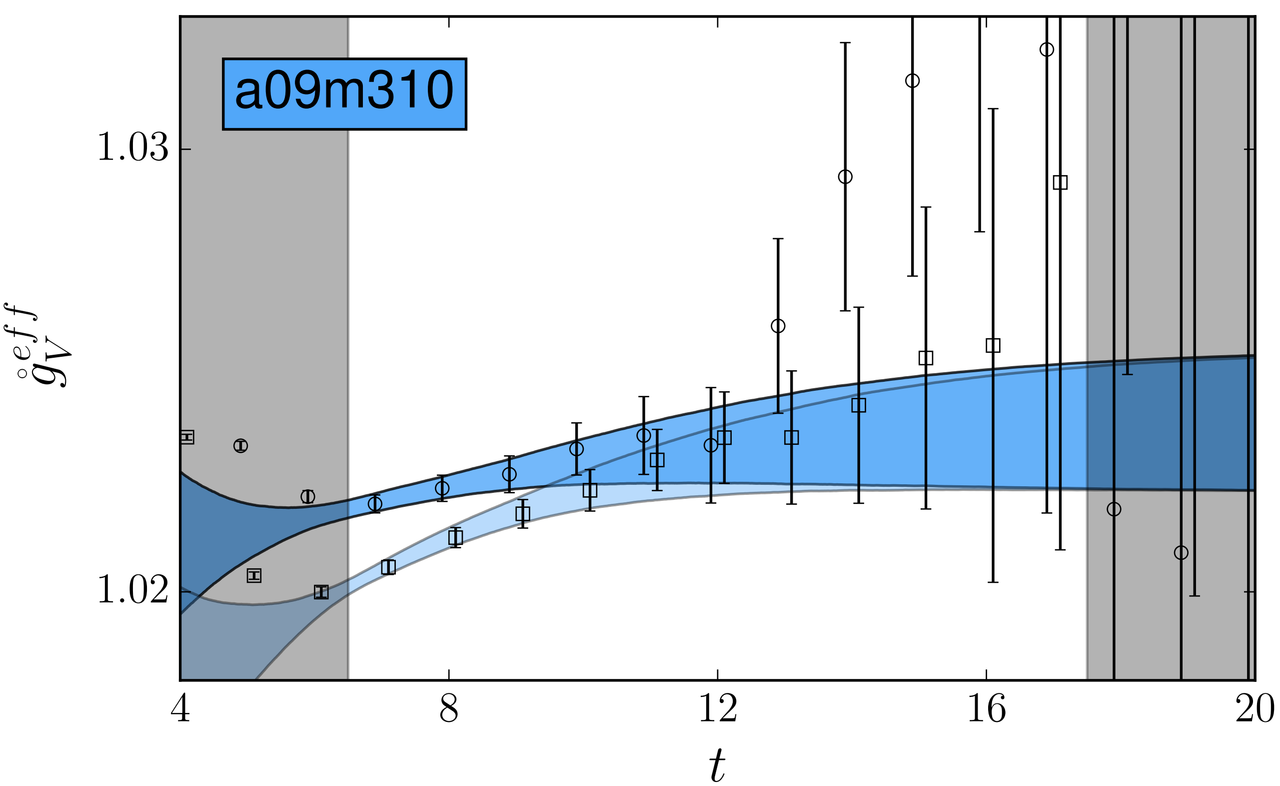


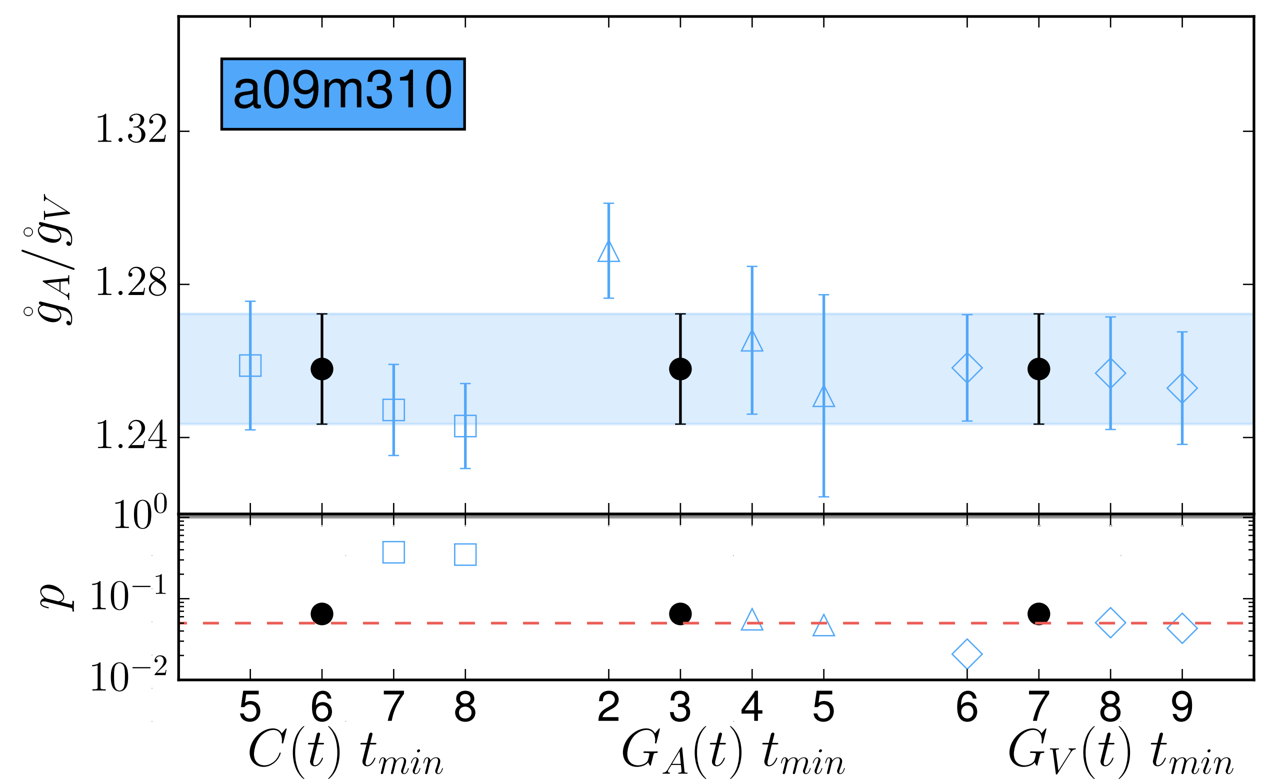
Another example point

arXiv:1704.01114









Fit to χPT

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \qquad \epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{\omega_0^2}$$

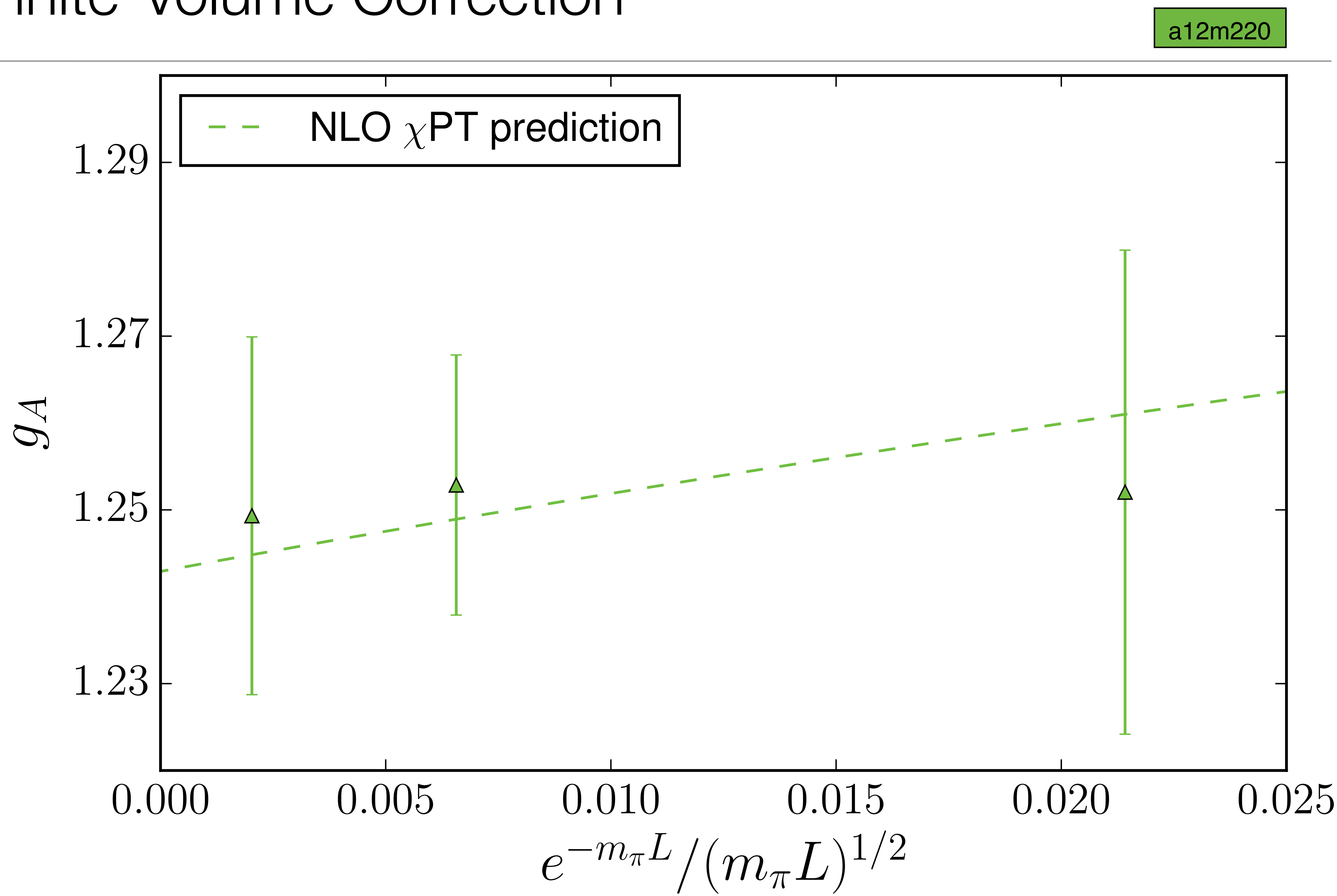
analytic pieces $g_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4$

non-analytic $-\epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$

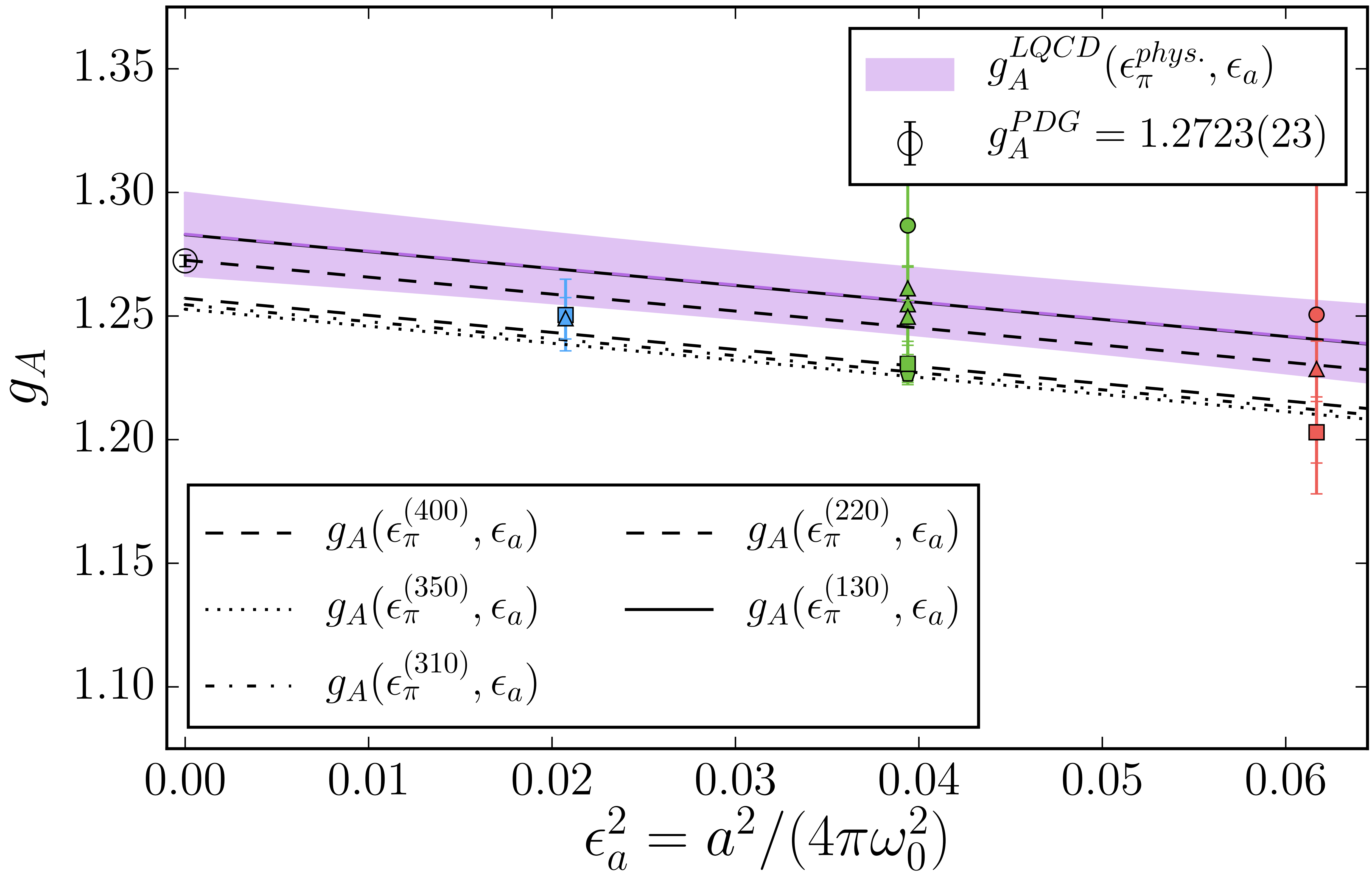
analytic in a^2 $a_2\epsilon_a^2 + b_4\epsilon_\pi^2\epsilon_a^2 + a_4\epsilon_a^4$

NLO FV $\frac{8}{3}\epsilon_\pi^2\left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)\right]$

Finite-Volume Correction

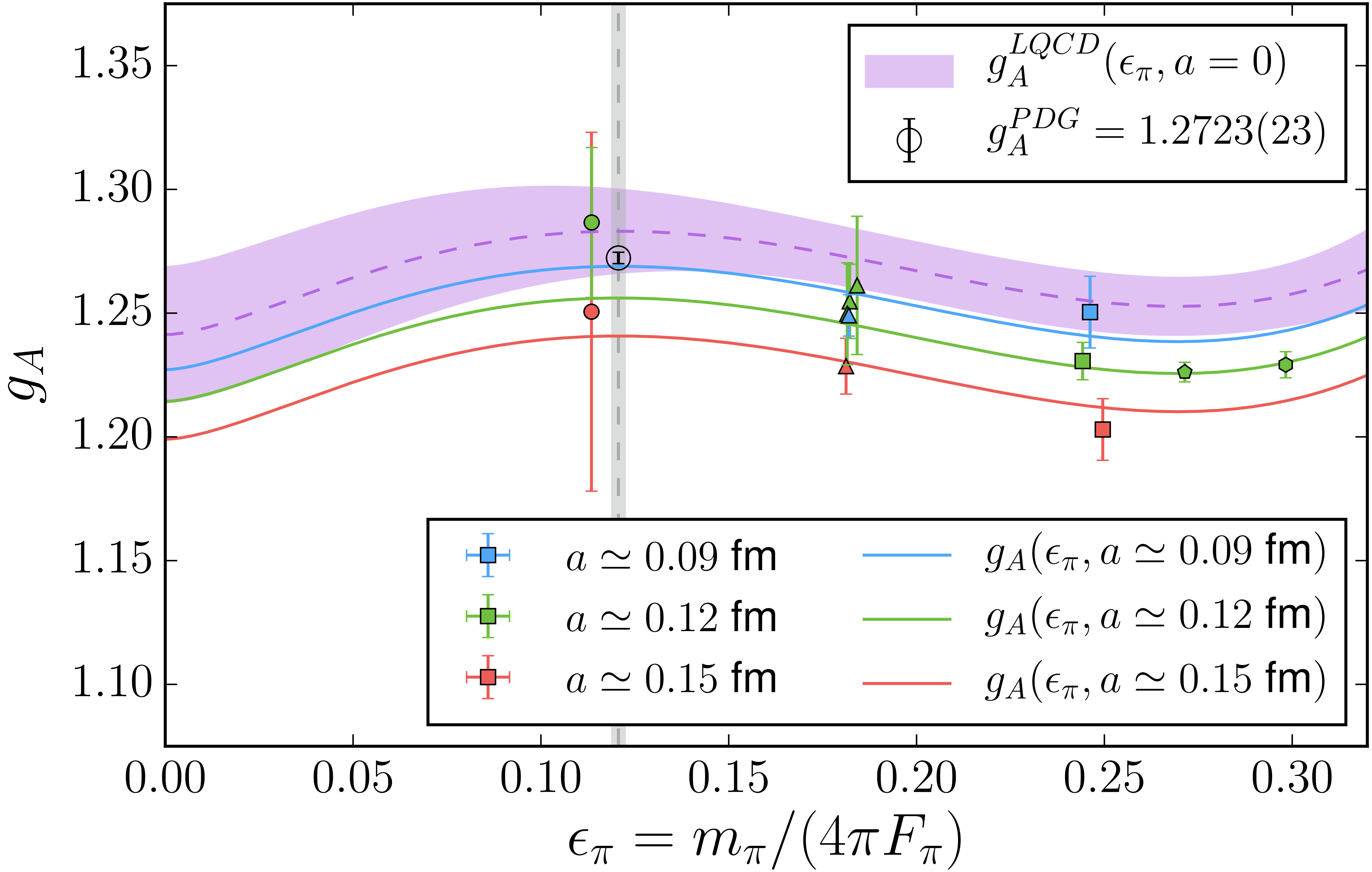


Continuum Extrapolation

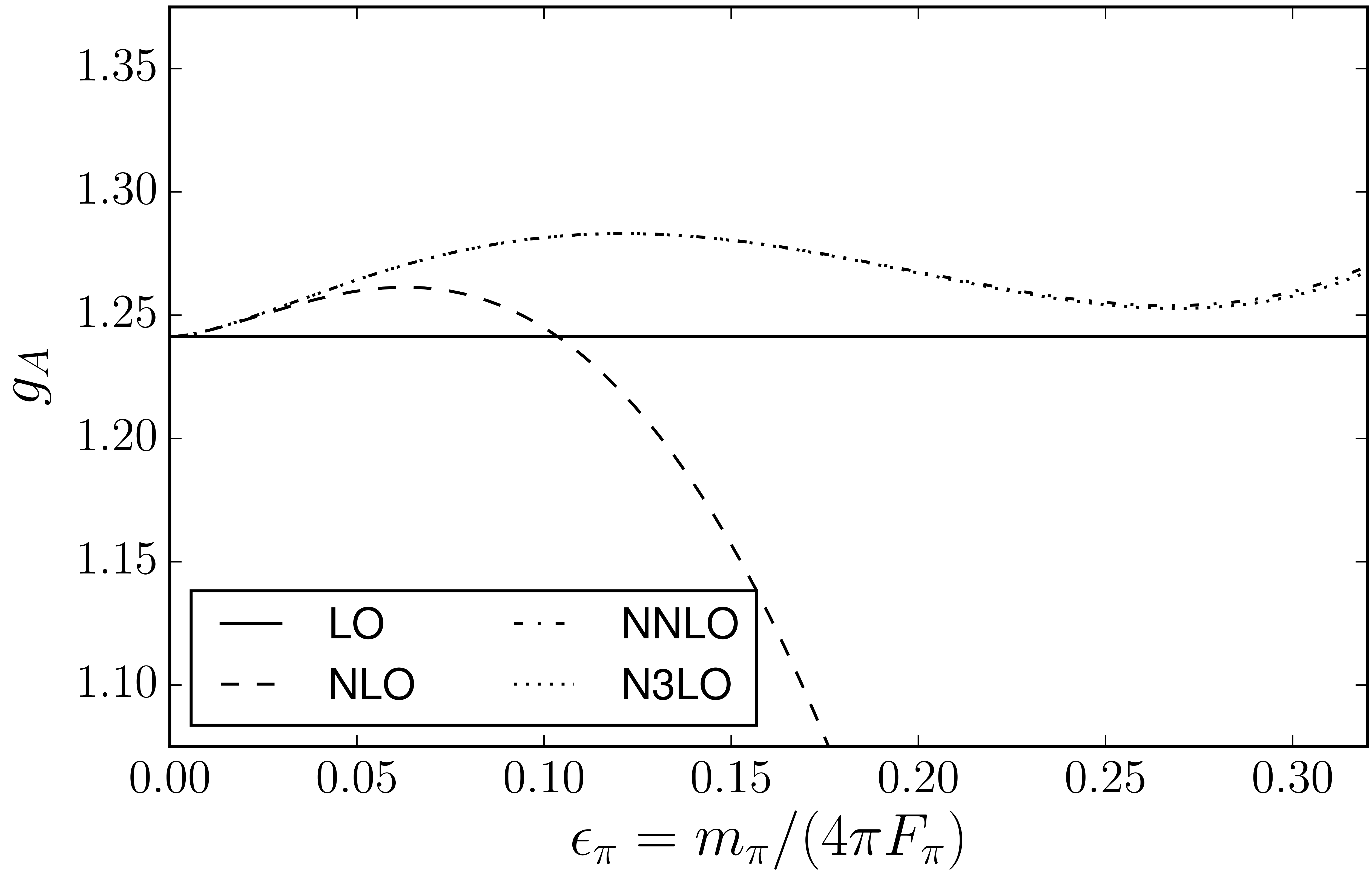


Chiral Extrapolation

arXiv:1704.01114



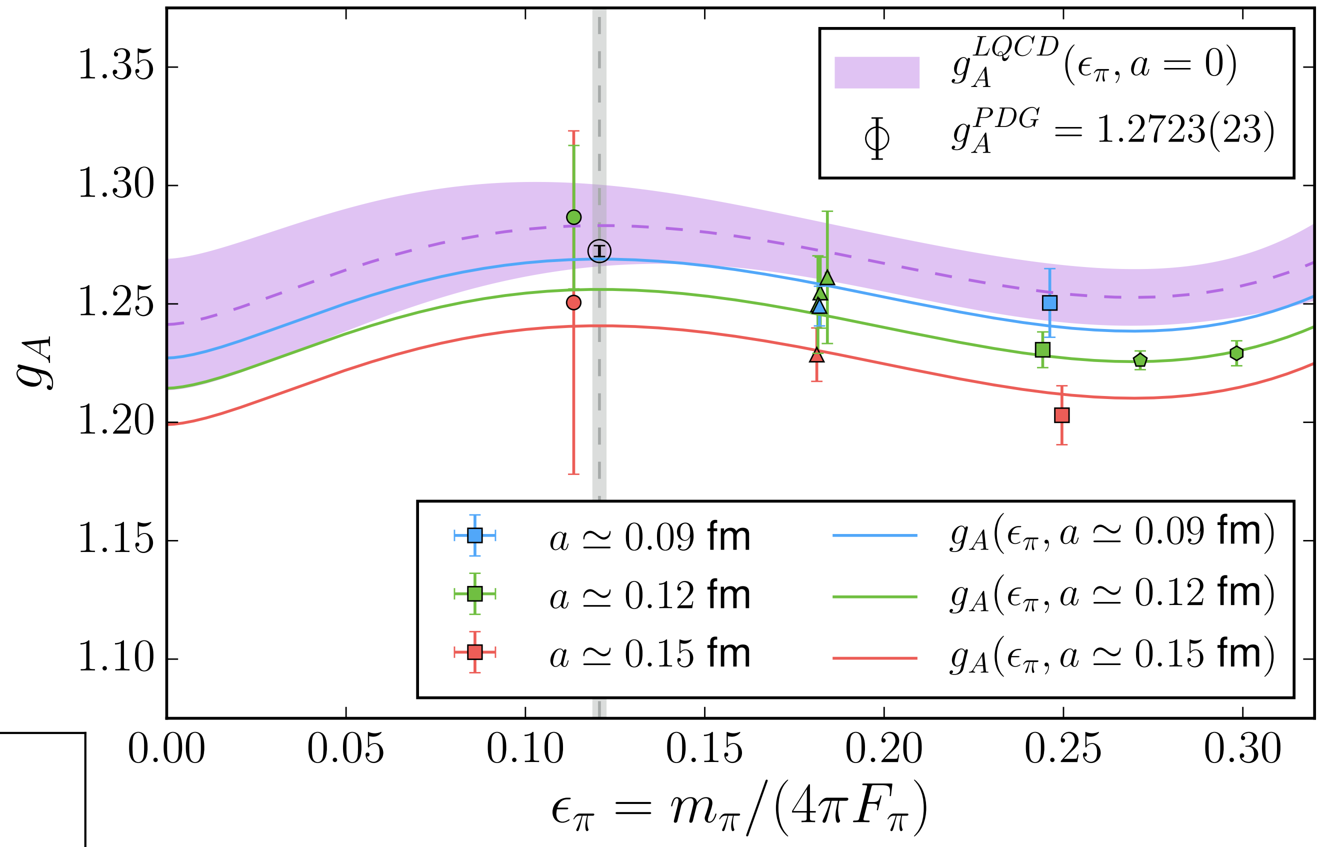
χPT Convergence

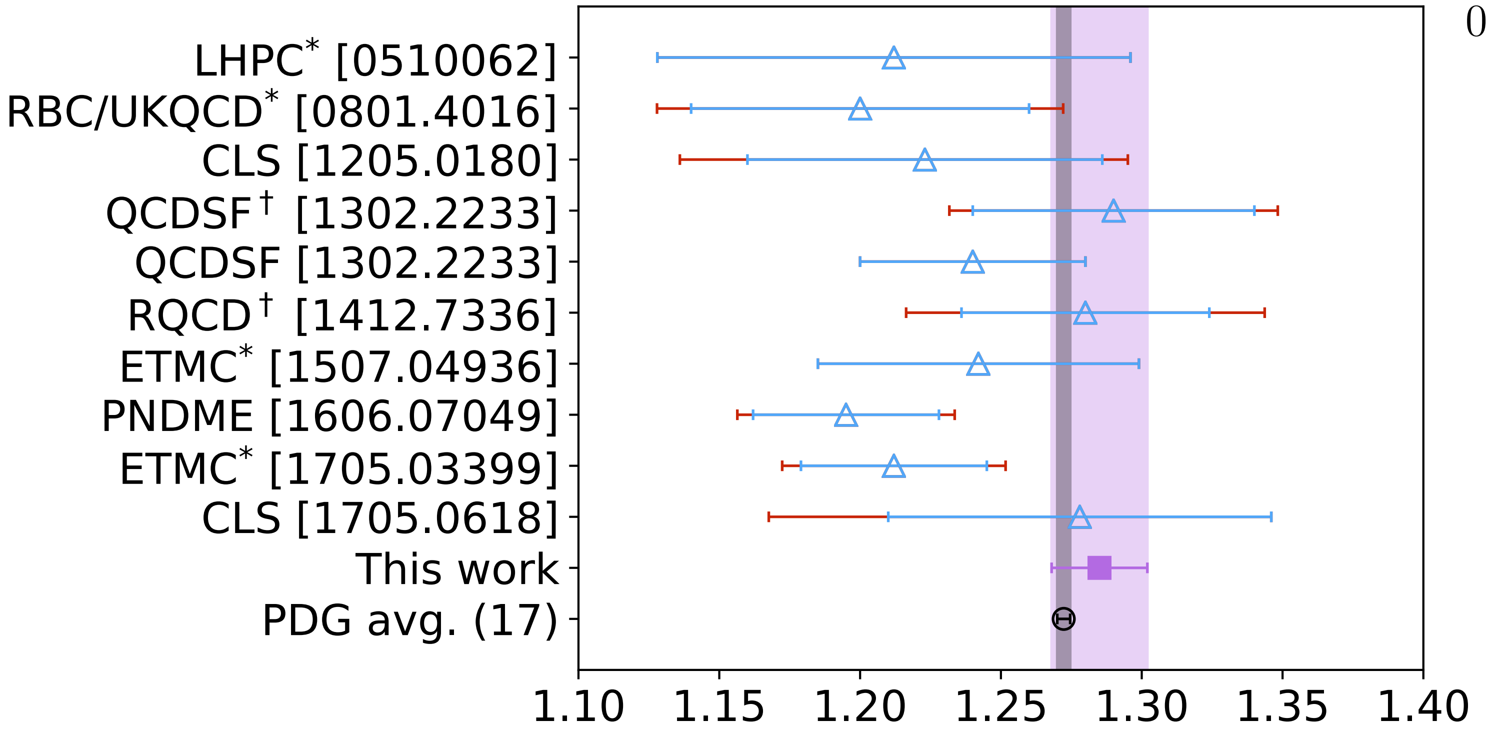


Error Budget

$$g_A = 1.283(17)\ [1.3\%]$$

statistical	1.29%
chiral extrapolation	0.21%
continuum extrapolation	0.10%
infinite volume	0.23%
isospin breaking	0.04%
total	1.33%

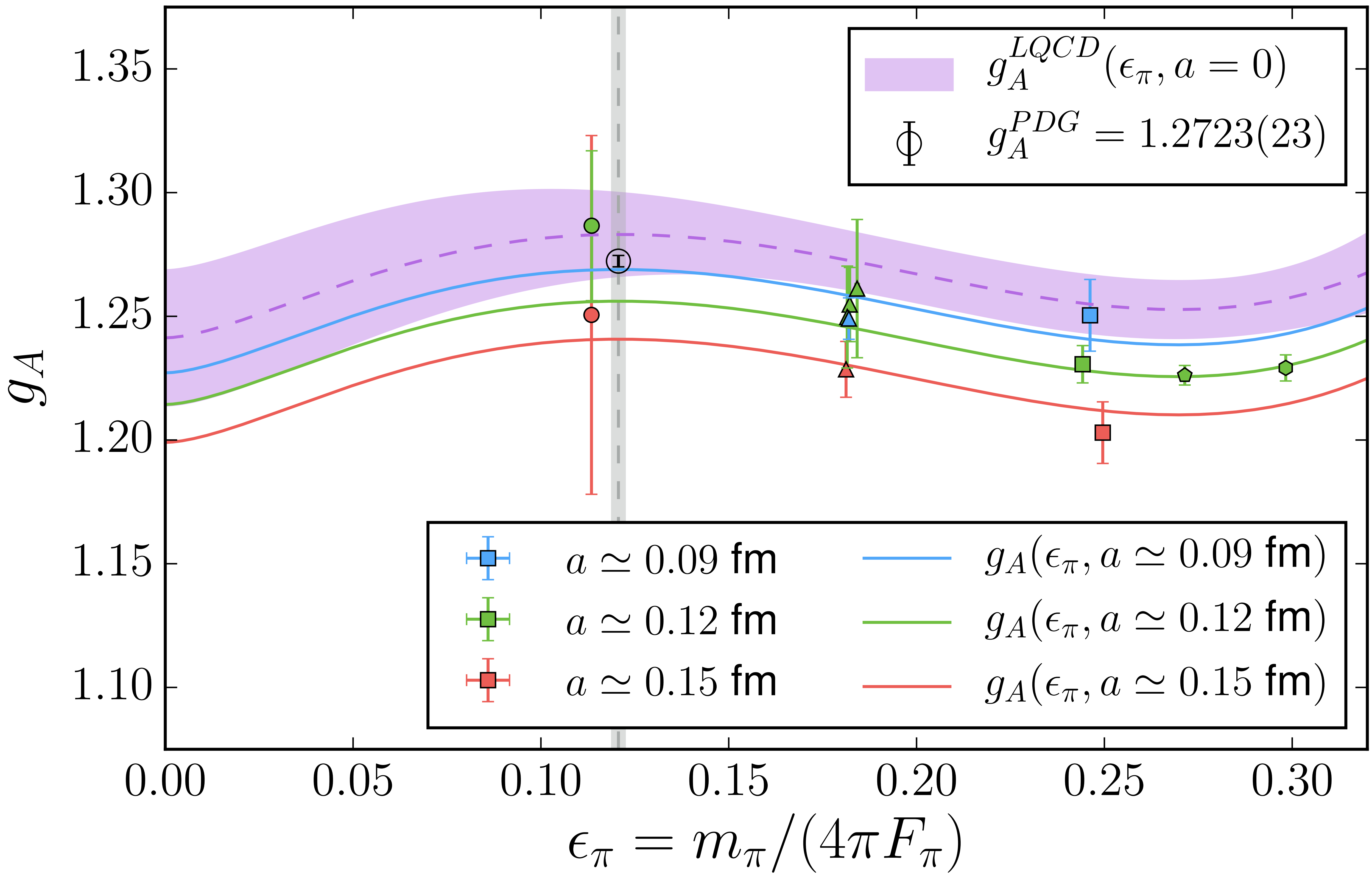




Chiral Extrapolation

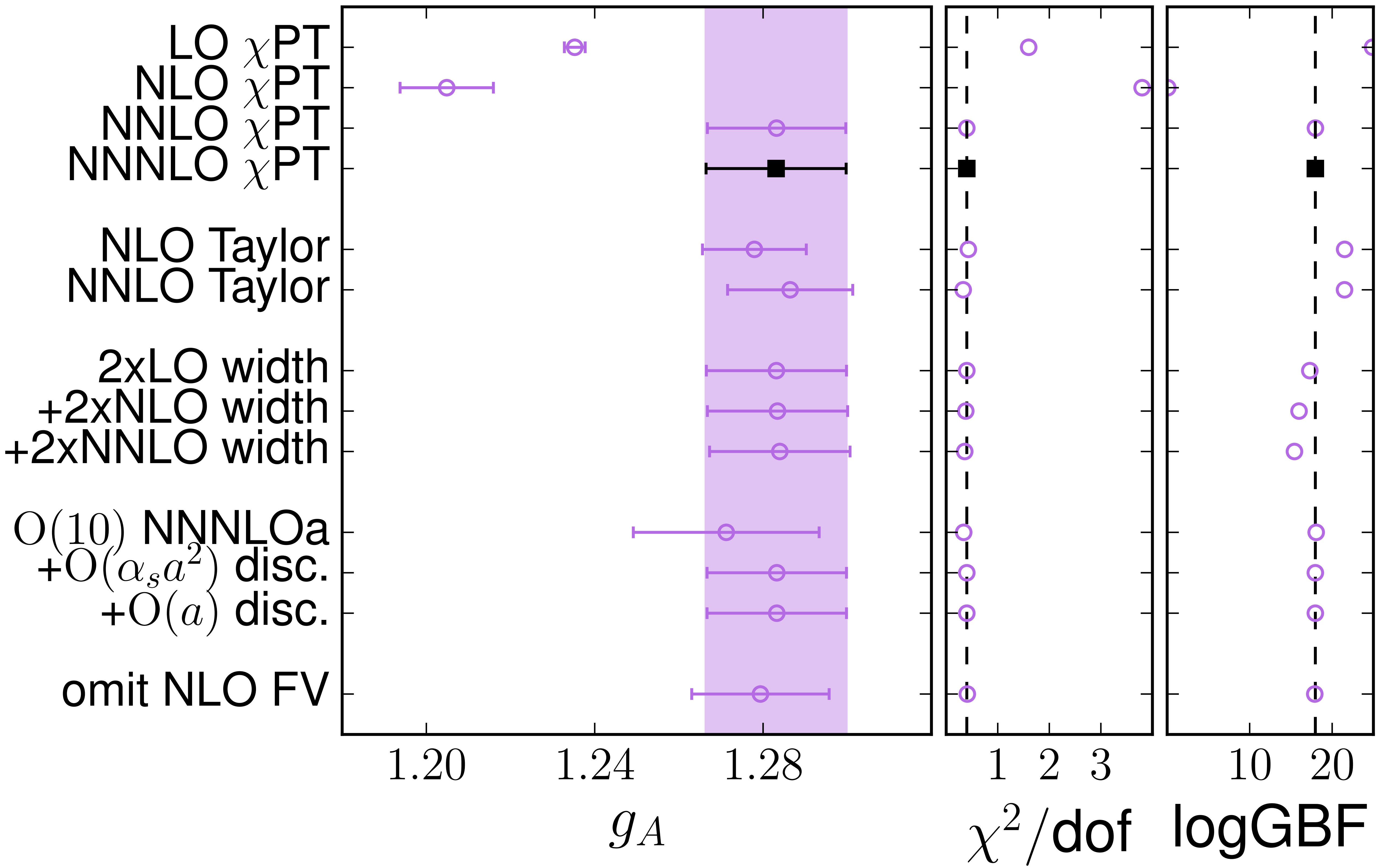
$g_A = 1.283(17)\ [1.3\%]$

arXiv:1704.01114

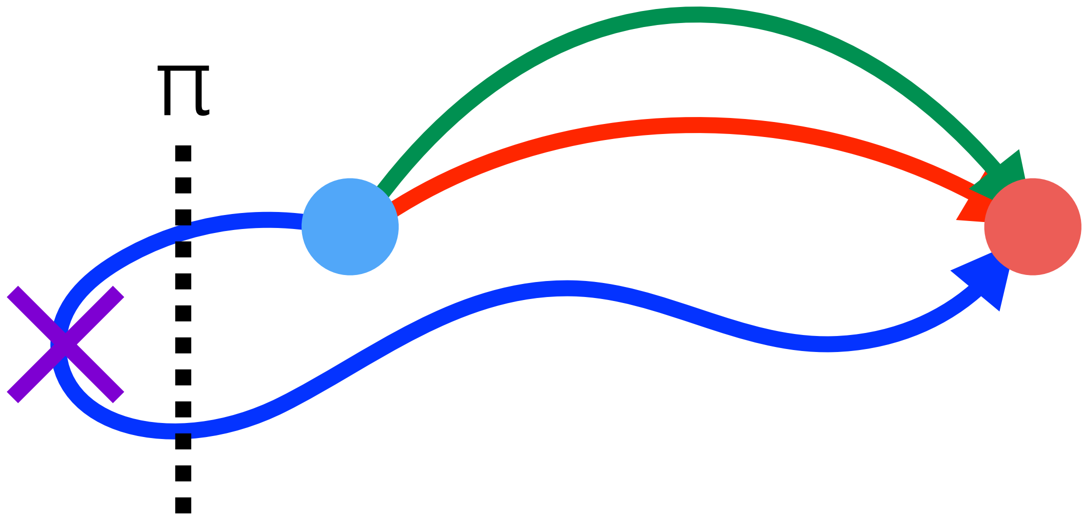


Backup Slides

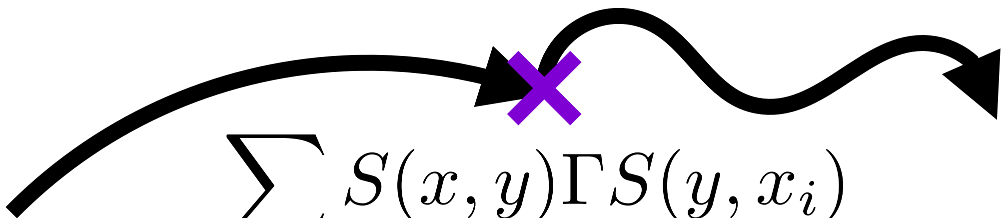
Fit Stability



Comparison with the summation method

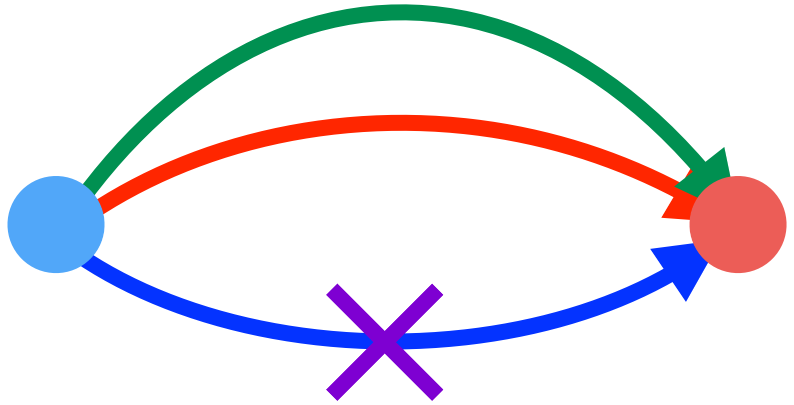


Summation method doesn't have this contamination

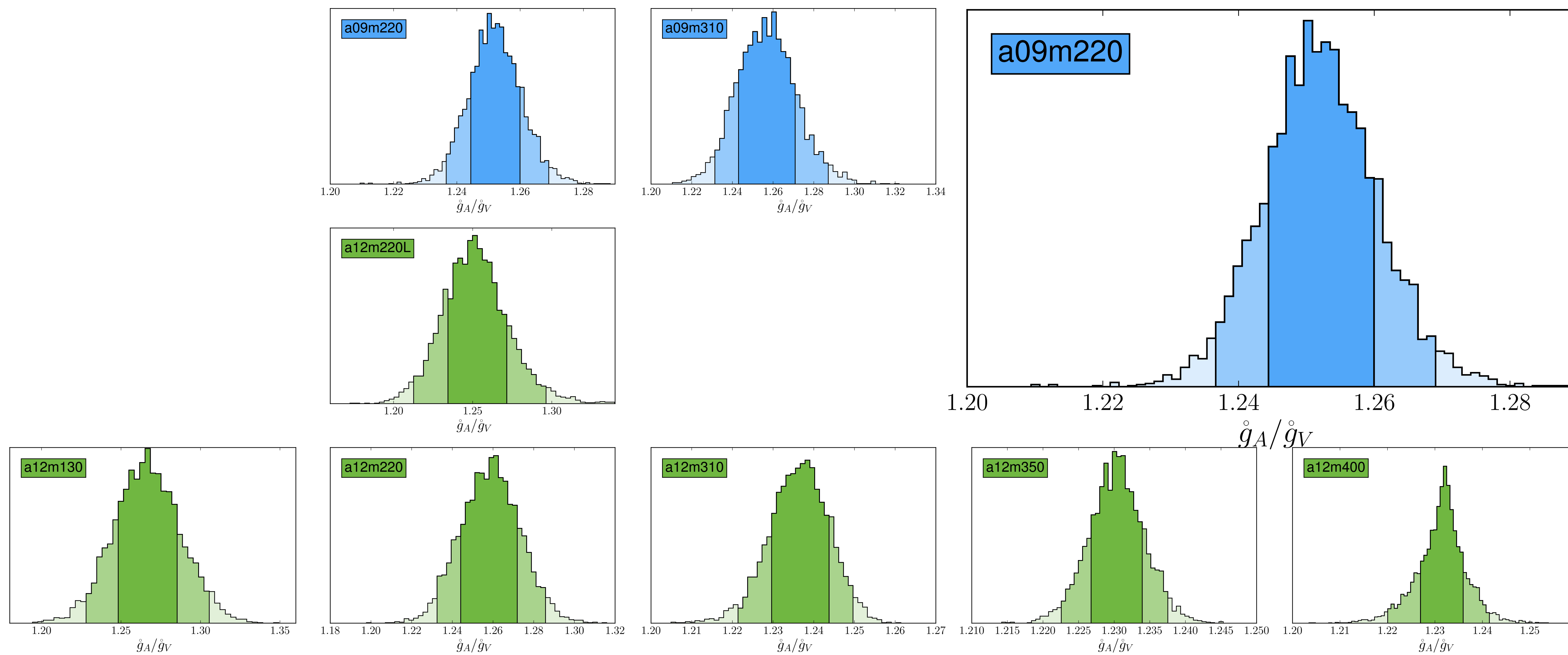


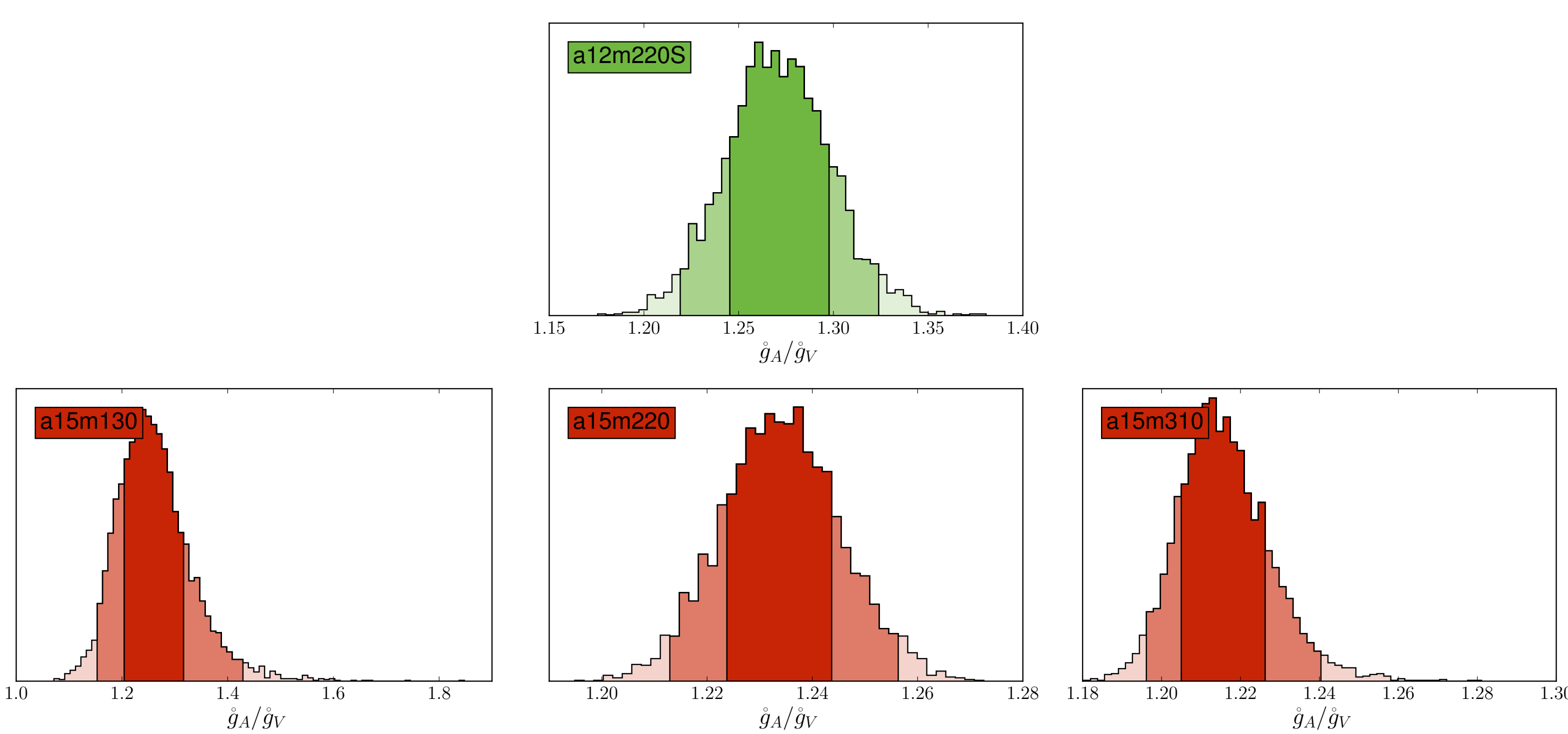
$$\sum_y S(x,y)\Gamma S(y,x_i)$$

FH method requires new solves to study different insertions



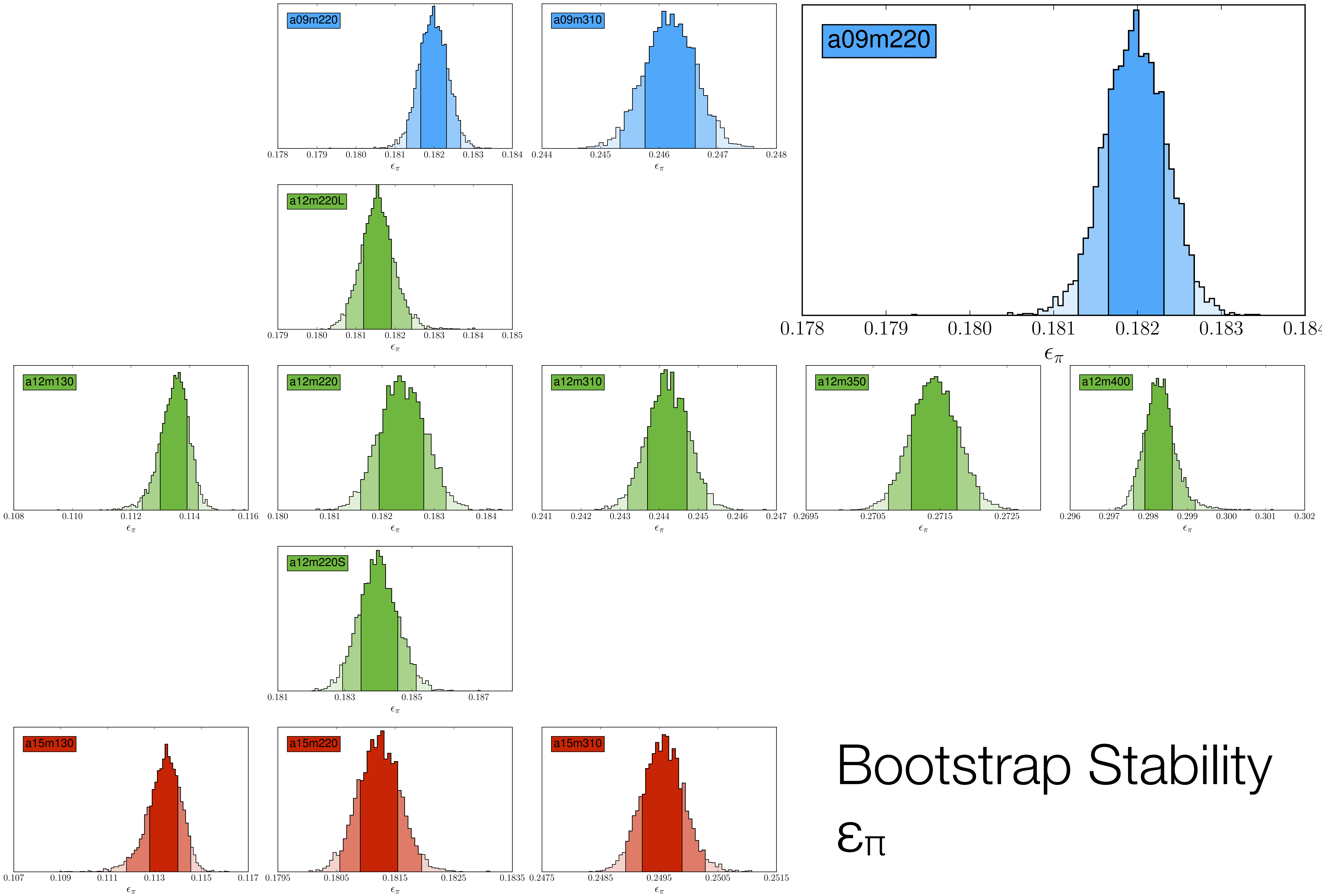
Summation method needs new solves for different source-sink separations





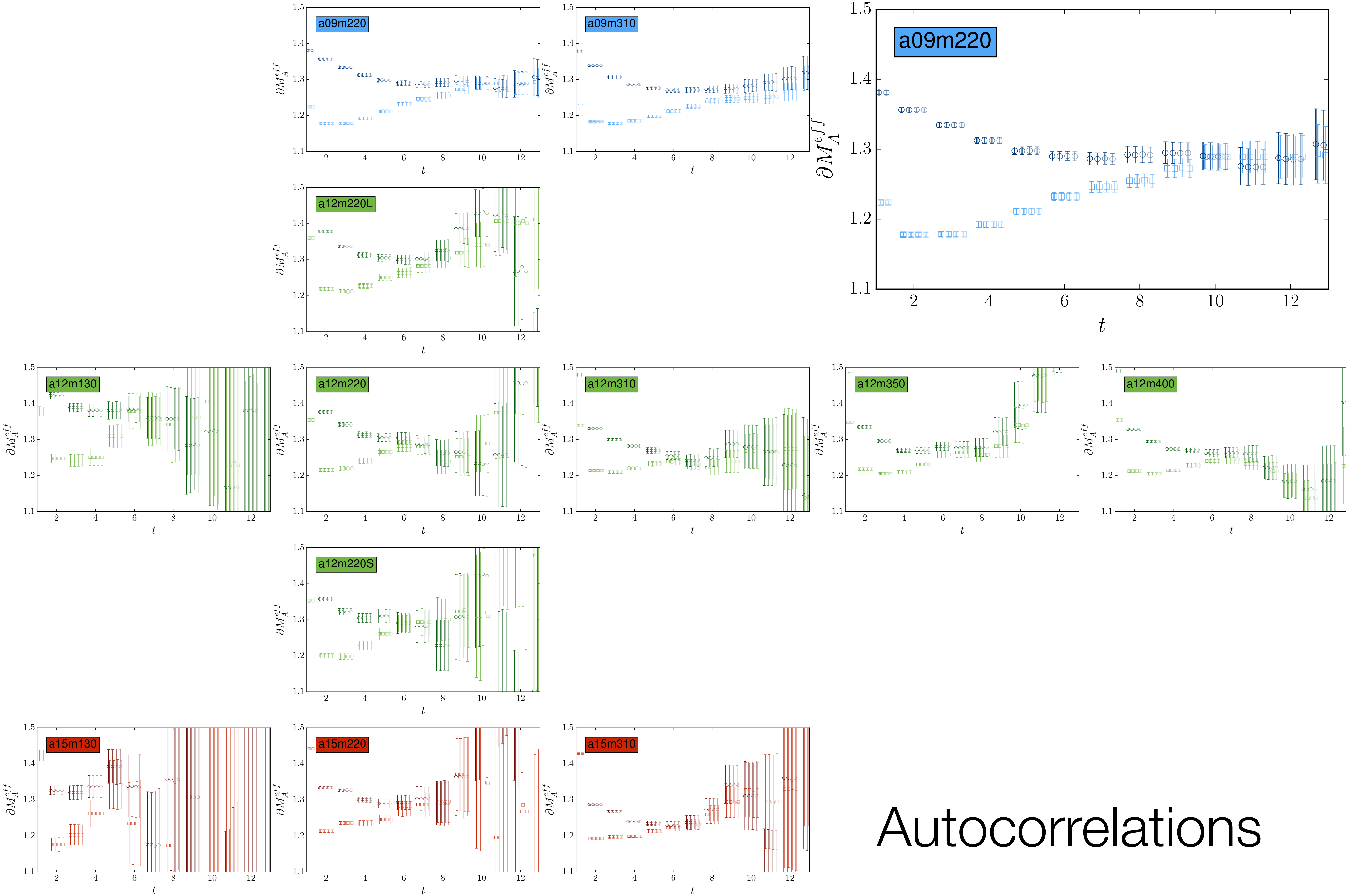
Bootstrap Stability

g_A

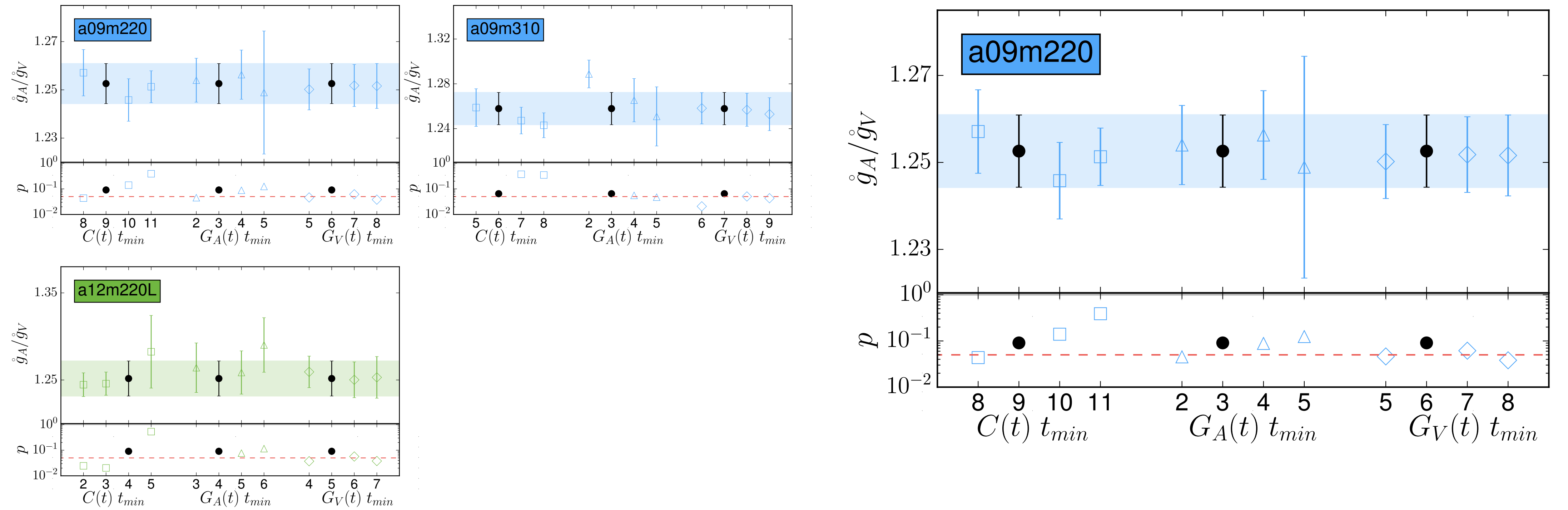


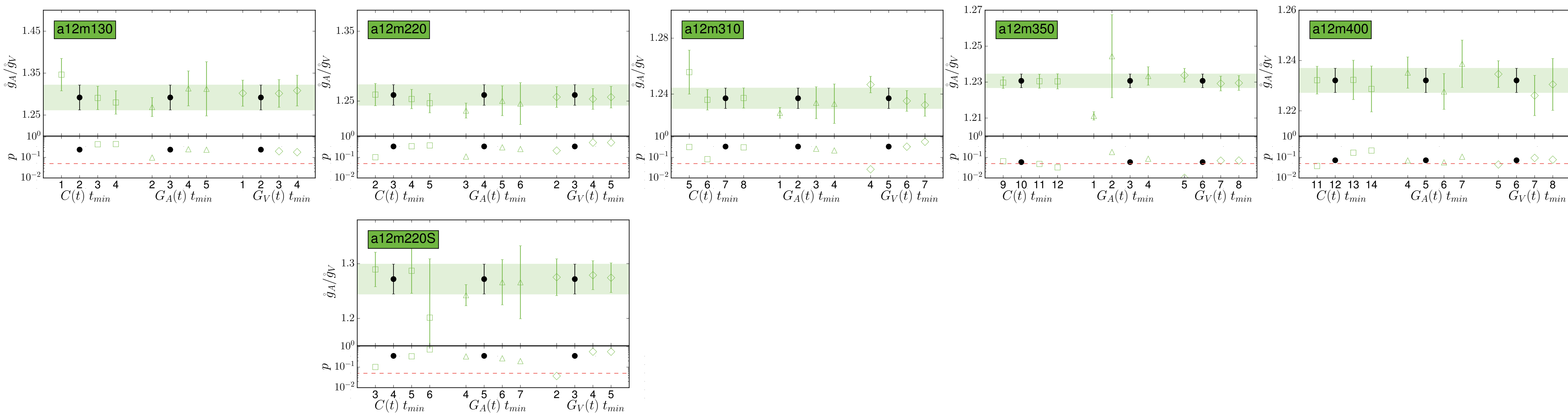
Bootstrap Stability

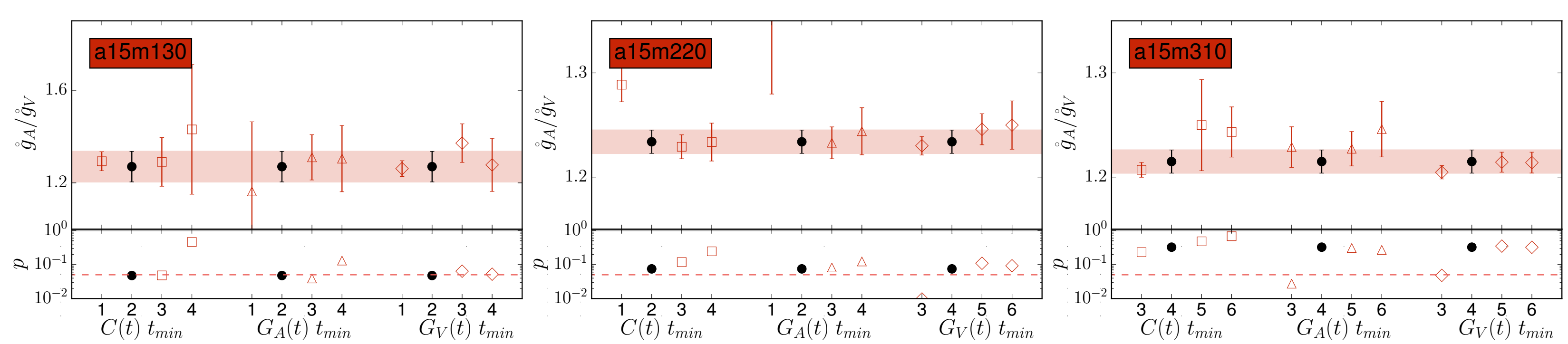
ϵ_π



Autocorrelations

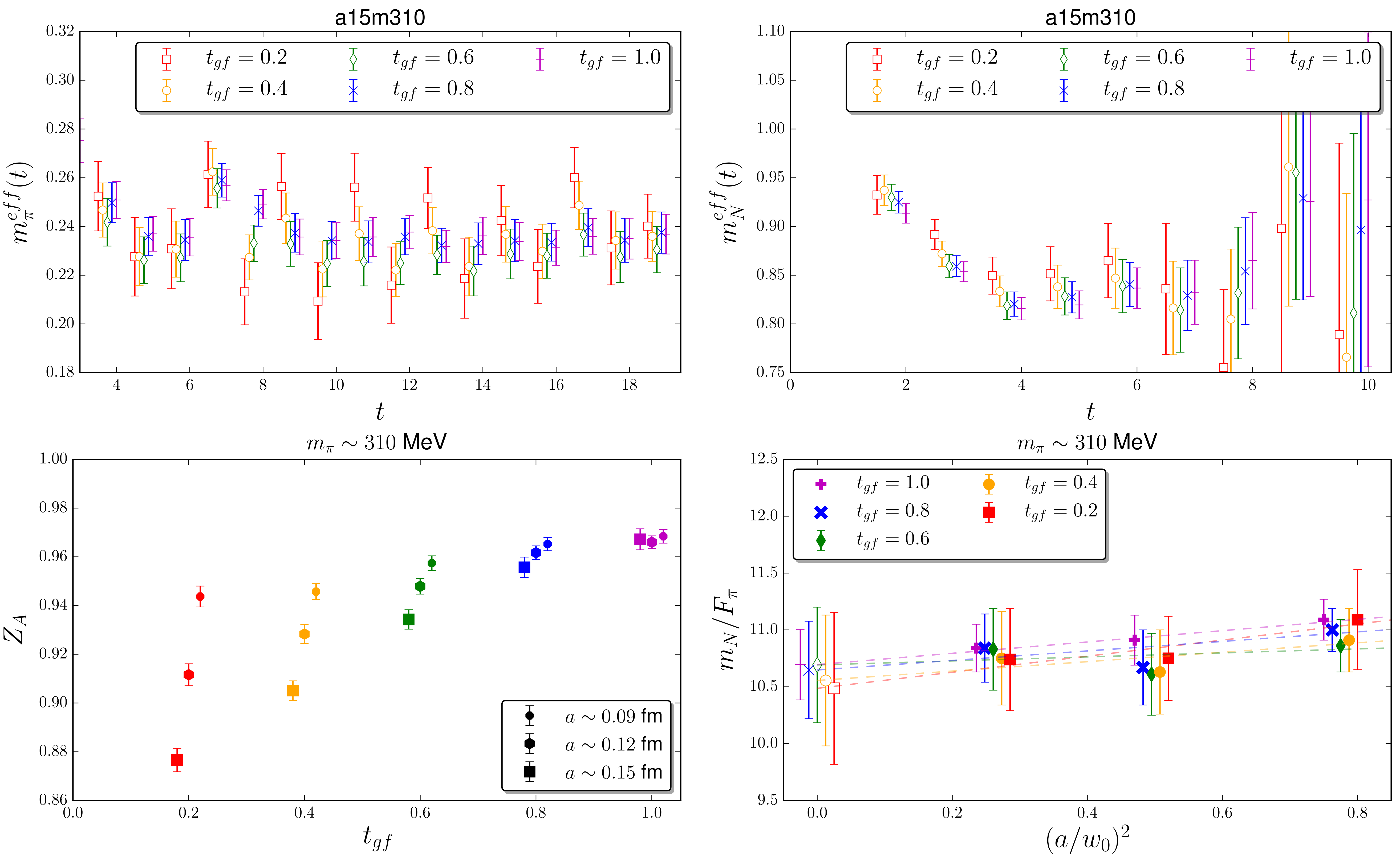




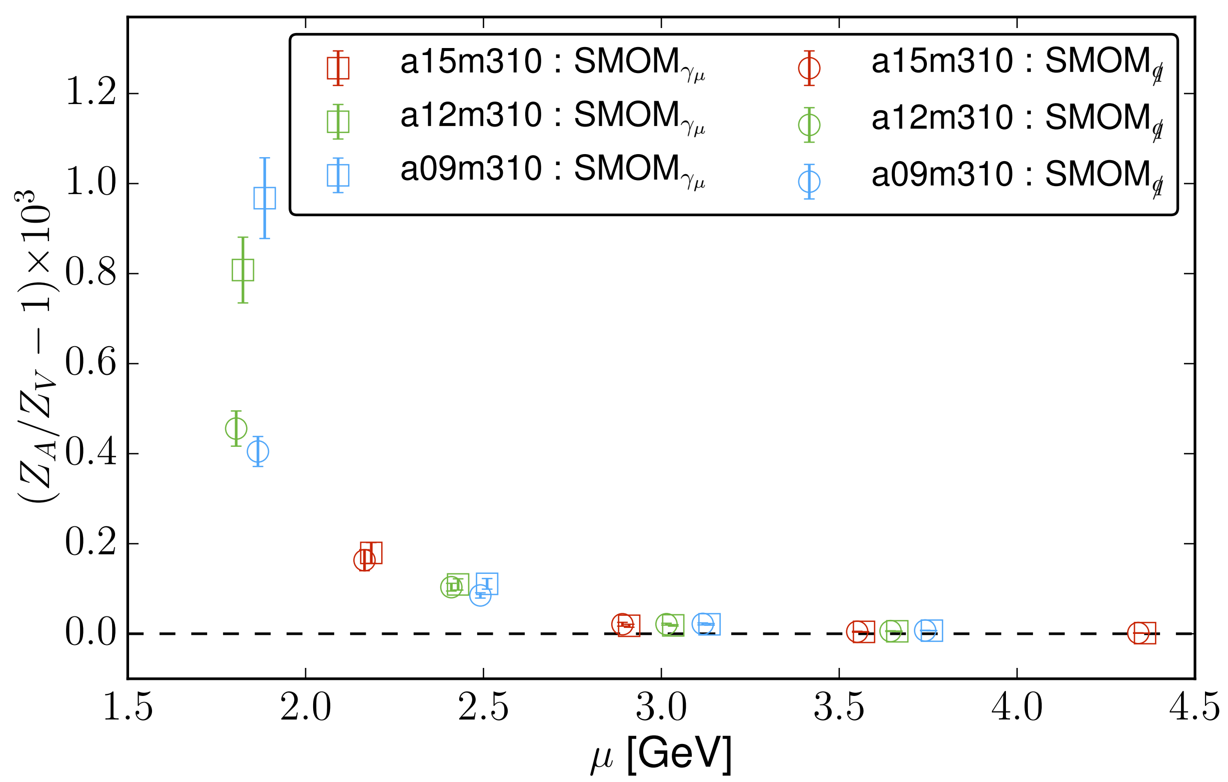


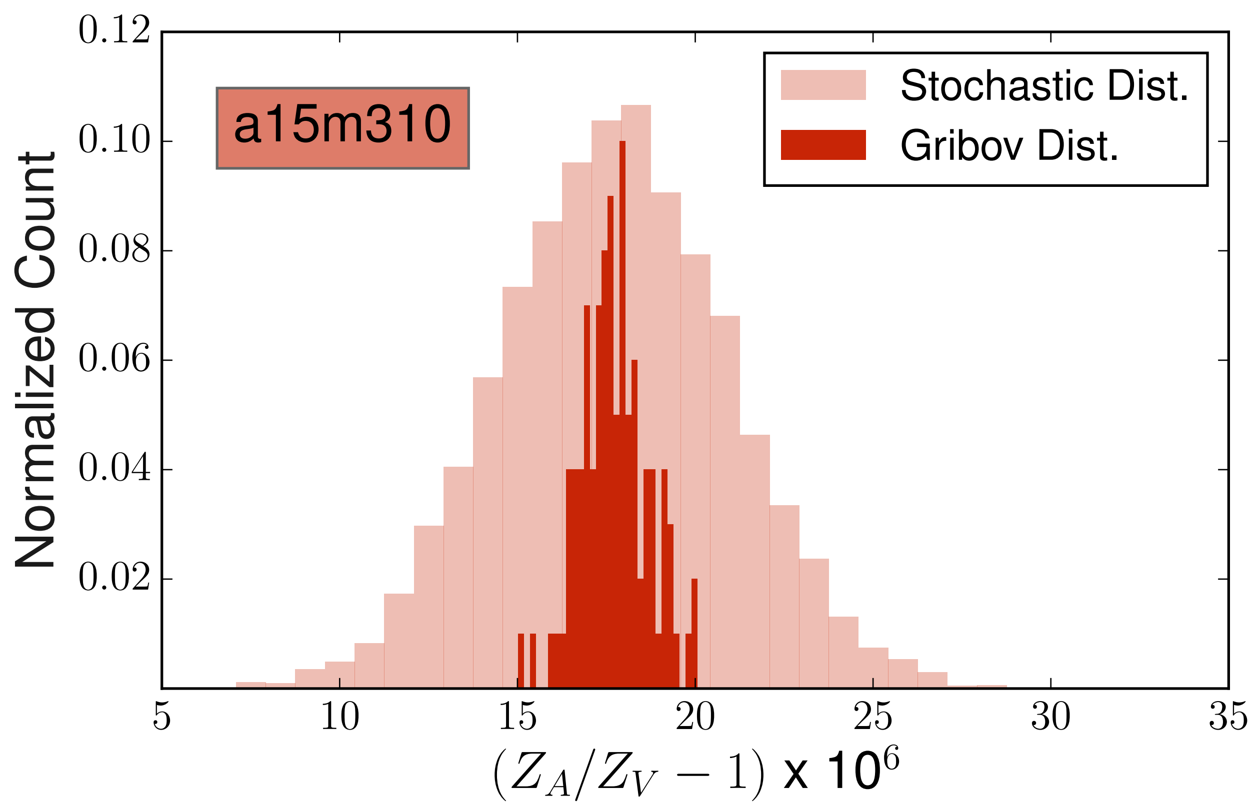
Correlator fit stability

Smearing Study



Nonperturbative Renormalization





What does this have to do with Feynman-Hellman?

Bouchard, Chang, Kurt, Orginos, Walker-Loud arXiv:1612.06963

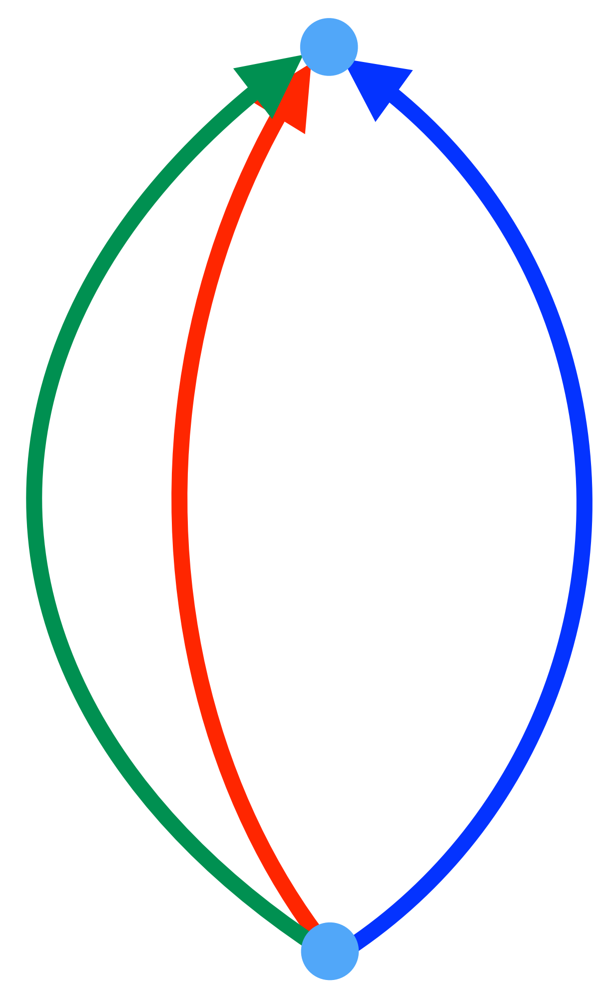
$$C(t) = \langle \mathcal{N}(t)\bar{\mathcal{N}}(0)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}U\ \mathcal{N}(t)\bar{\mathcal{N}}(0)\ e^{-S[U]} = \frac{\mathrm{tr}\left[\mathcal{N}(t)\bar{\mathcal{N}}(0)e^{-\beta H}\right]}{\mathrm{tr}\left[e^{-\beta H}\right]}$$

$$m_{\mathrm{eff}} = \frac{1}{\tau}\ln\left(\frac{C(t)}{C(t+\tau)}\right) \xrightarrow{t\to\infty} E_0$$

FH: $$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \psi_\lambda \left| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \right| \psi_\lambda \right\rangle$$

$$\partial_\lambda E_0 = \text{a matrix element of interest}$$

C(t)



What does this have to do with Feynman-Hellman?

Bouchard, Chang, Kurt, Orginos, Walker-Loud arXiv:1612.06963

$$S[U] \to S[U] + \lambda \int_x \mathcal{J}(x)\mathcal{O}(x)$$

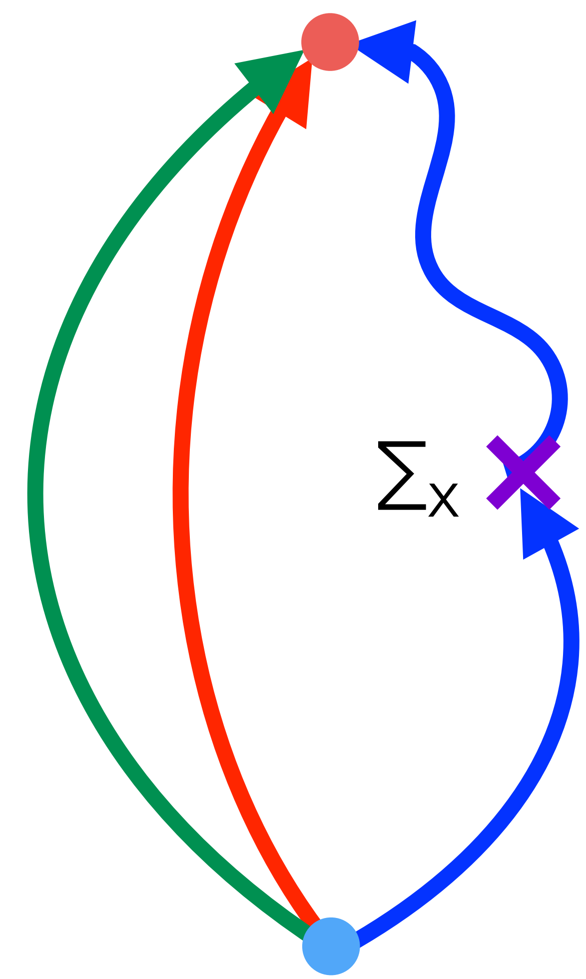
$$\partial_\lambda C(t) = -\left\langle \mathcal{N}(t) \left(\int_x \mathcal{J}(x)\mathcal{O}(x) \right) \bar{\mathcal{N}}(0) \right\rangle$$

$$\left.\frac{\partial m_{\text{eff}}}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t+\tau)}{C(t+\tau)}\right]\Bigg|_{\lambda=0}$$

$$\mathcal{J}_\mu(x) = 1$$

$$\mathcal{O}^\mu(x) = \bar{q}\gamma^\mu\gamma^5\tau^+ q$$

$$\xrightarrow{t\to\infty} g_A + O\left(e^{-E_n t}\right)$$



Details of the spectral representation: 2-point

Bouchard, Chang, Kurt, Orginos, Walker-Loud arXiv:1612.06963 slide courtesy of Enrico Rinaldi

$$
\begin{aligned}
C(t) &= \langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle \\
&= \sum_n \langle\Omega|e^{\hat{H}t}\mathcal{O}(0)e^{-\hat{H}t}\frac{|n\rangle\langle n|}{2E_n}\mathcal{O}^\dagger(0)|\Omega\rangle \\
&= \sum_n Z_n Z_n^\dagger \frac{e^{-E_n t}}{2E_n} \\
&= \sum_n z_n z_n^\dagger e^{-E_n t}
\end{aligned}
$$

$$Z_n \equiv \langle\Omega|\mathcal{O}|n\rangle$$

$$Z_n^\dagger \equiv \langle n|\mathcal{O}^\dagger|\Omega\rangle$$

$$z_n = \frac{Z_n}{\sqrt{2E_n}}$$

Details of the spectral representation: 3-point

Bouchard, Chang, Kurt, Orginos, Walker-Loud arXiv:1612.06963 slide courtesy of Enrico Rinaldi

$$N_J(t) = \sum_{t'} \langle \Omega | T\{O(t) J(t') O^\dagger(0)\} | \Omega \rangle$$

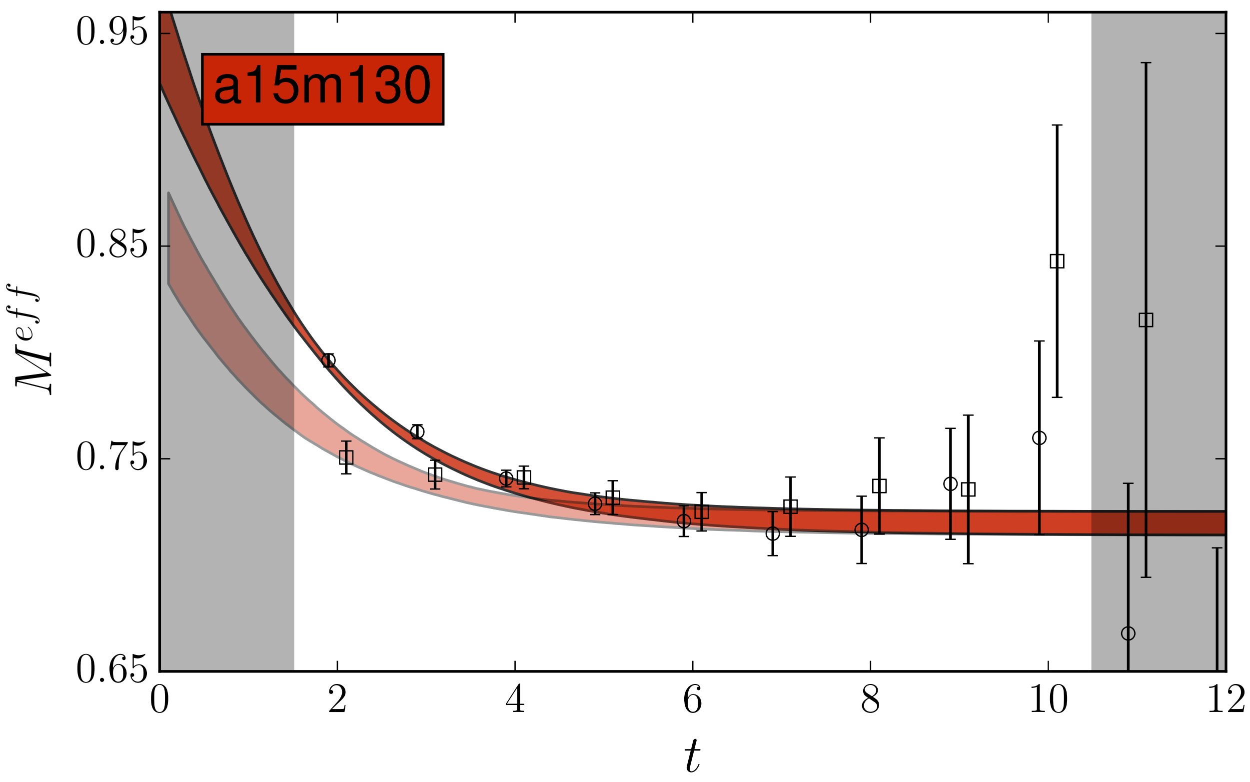
$$N_J(t) = \sum_n \left[(t-1) z_n g^J_{nn} z^\dagger_n + d^J_n \right] e^{-E_n t} + \sum_{n, m \neq n} z_n g^J_{nm} z^\dagger_m \frac{e^{-E_n t + \frac{\Delta_{nm}}{2}} - e^{-E_m t + \frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{mn}}{2}} - e^{\frac{\Delta_{nm}}{2}}}$$

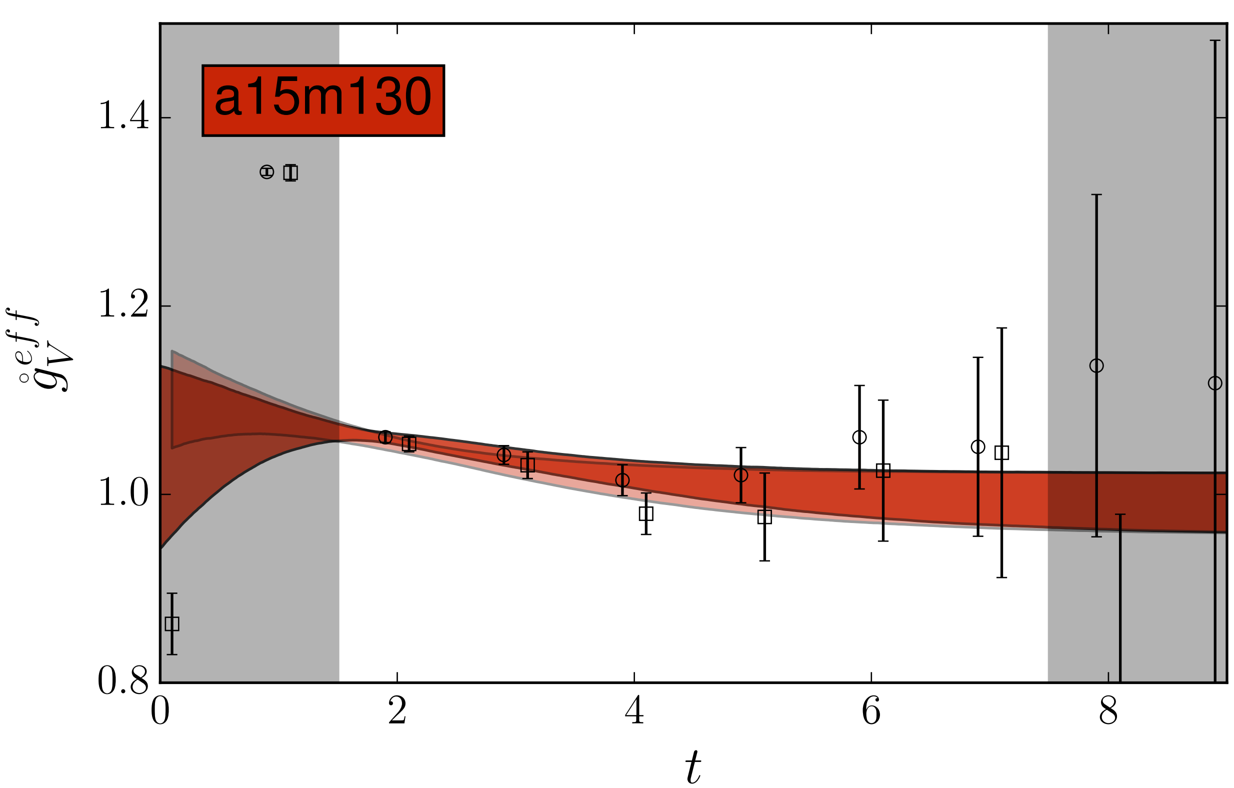
$$g^J_{nn} \equiv \frac{J_{nn}}{2E_n} \qquad J_{nn} = \langle n | J | n \rangle$$

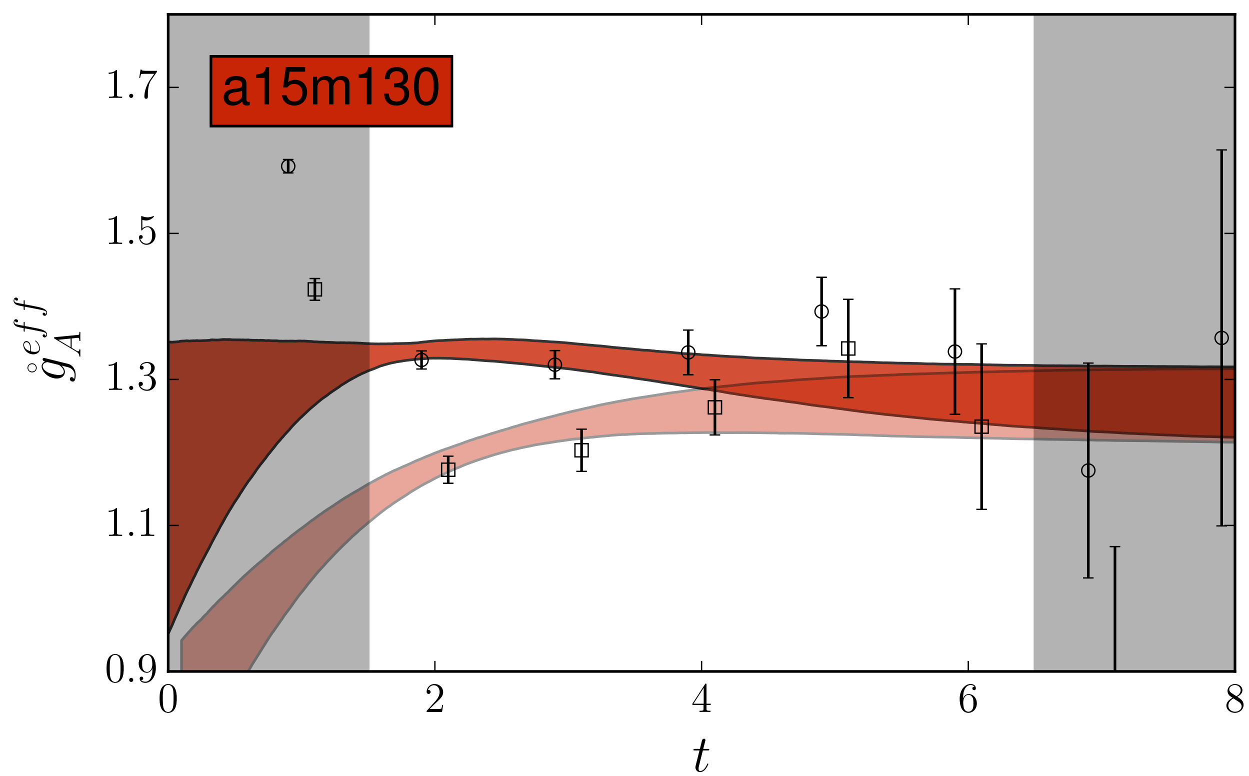
$$g^J_{nm} \equiv \frac{J_{nm}}{\sqrt{4 E_n E_m}} \qquad J_{nm} = \langle n | J | m \rangle$$

$$\Delta_{nm} \equiv E_n - E_m$$

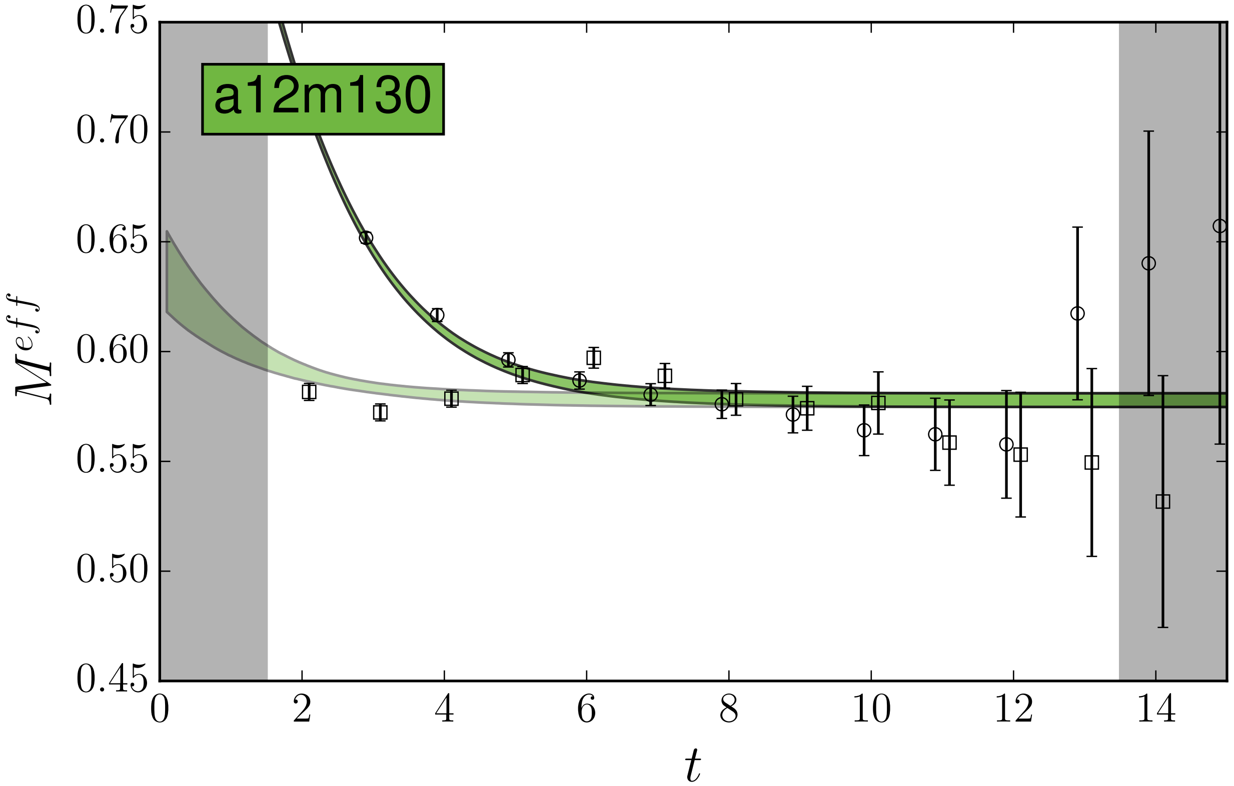
$$d^J_n \equiv Z_n Z^\dagger_{J:n} + Z_{J:n} Z^\dagger_n + Z_n Z^\dagger_n \langle \Omega | J | \Omega \rangle + \sum_j \frac{Z_n Z^\dagger_{nj} J^\dagger_j + J_j Z_{jn} Z^\dagger_n}{2E_j \left(e^{E_j} - 1 \right)}$$

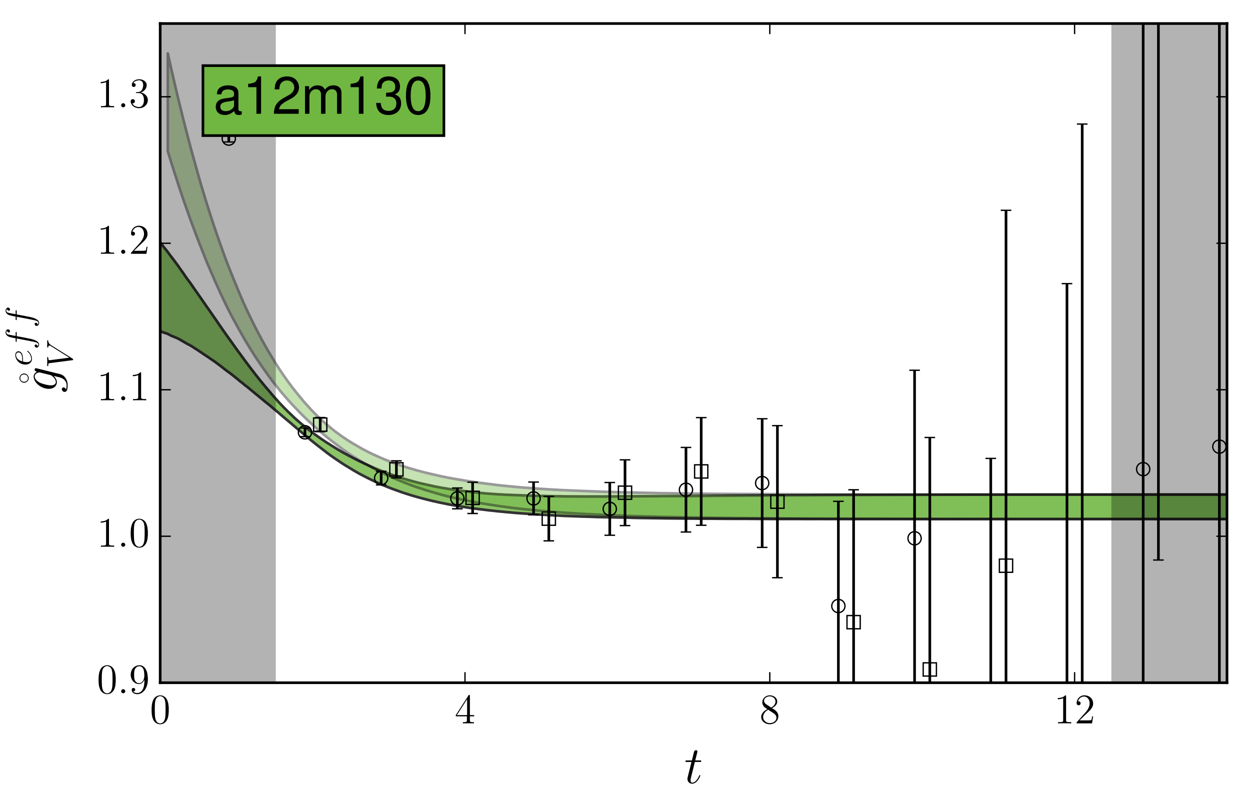

a15m130
M^{eff}
t

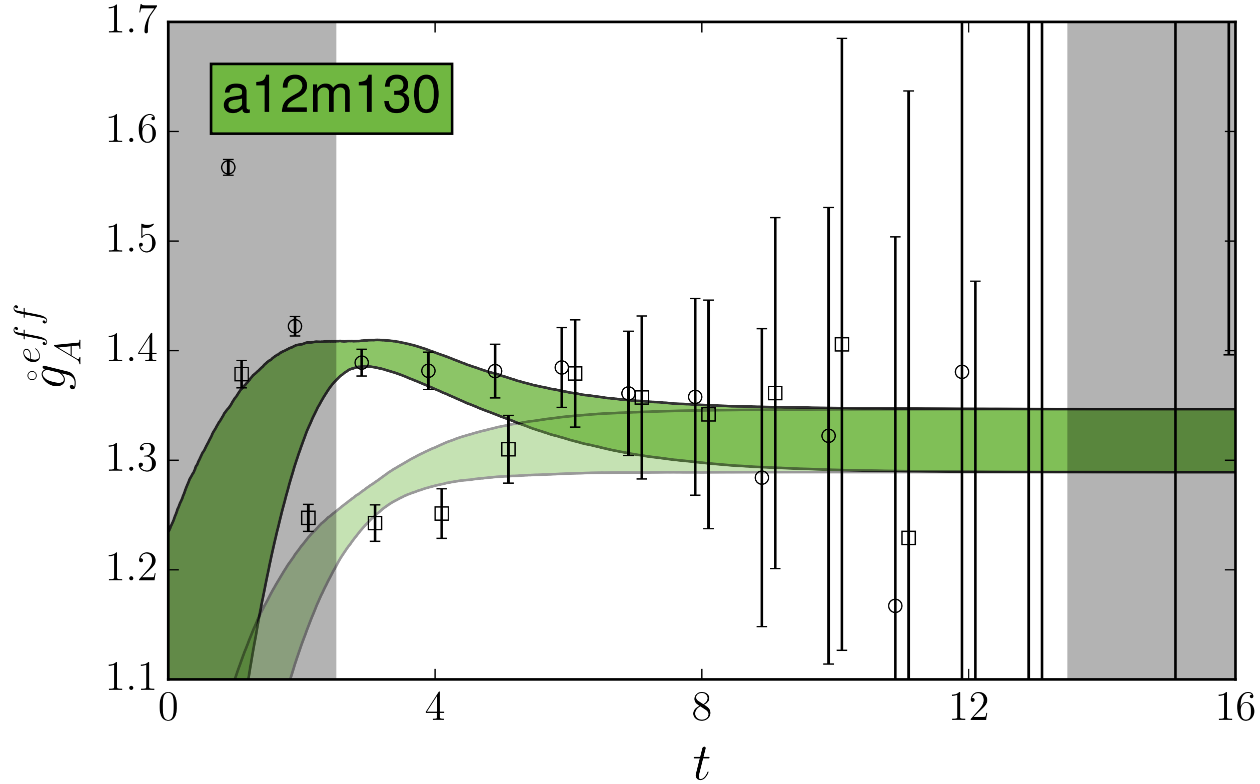

a15m130
$\mathring{g}_V^{eff}$
t


a15m130
$\mathring{g}_A^{eff}$
t

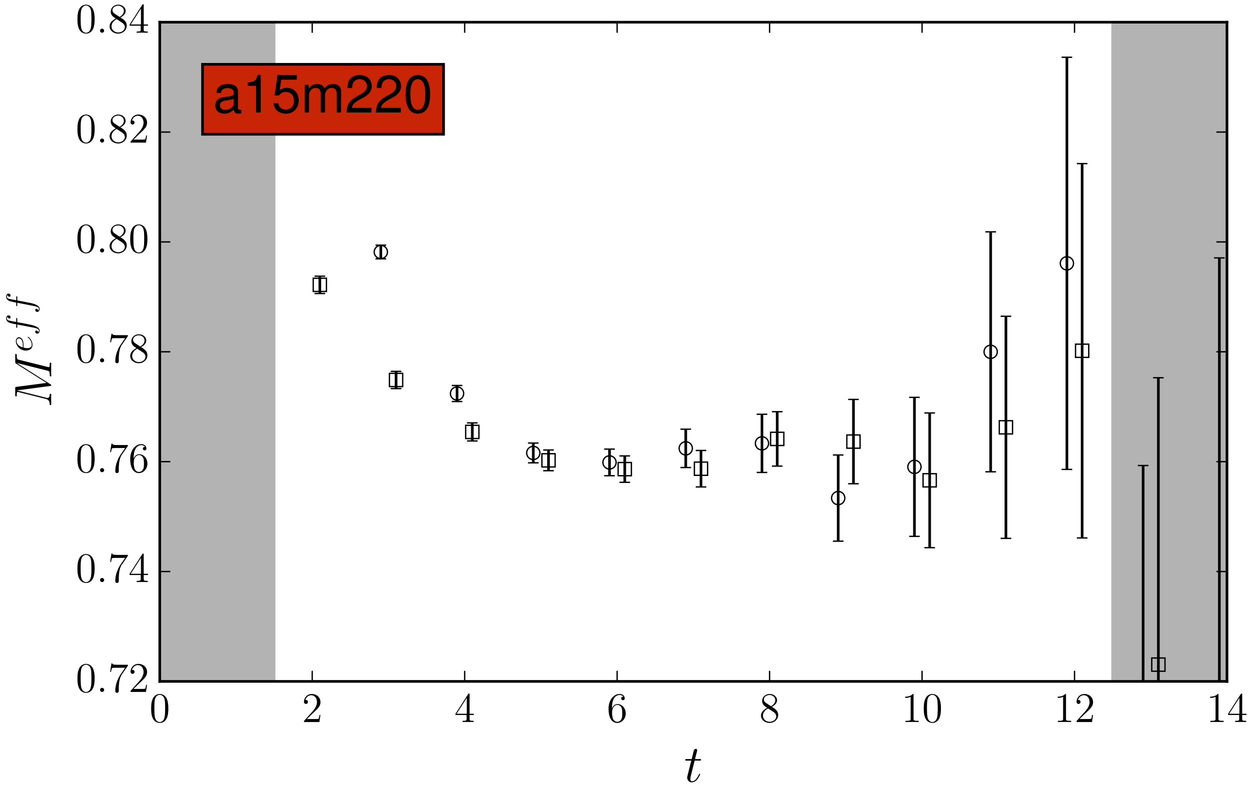
Plateaus

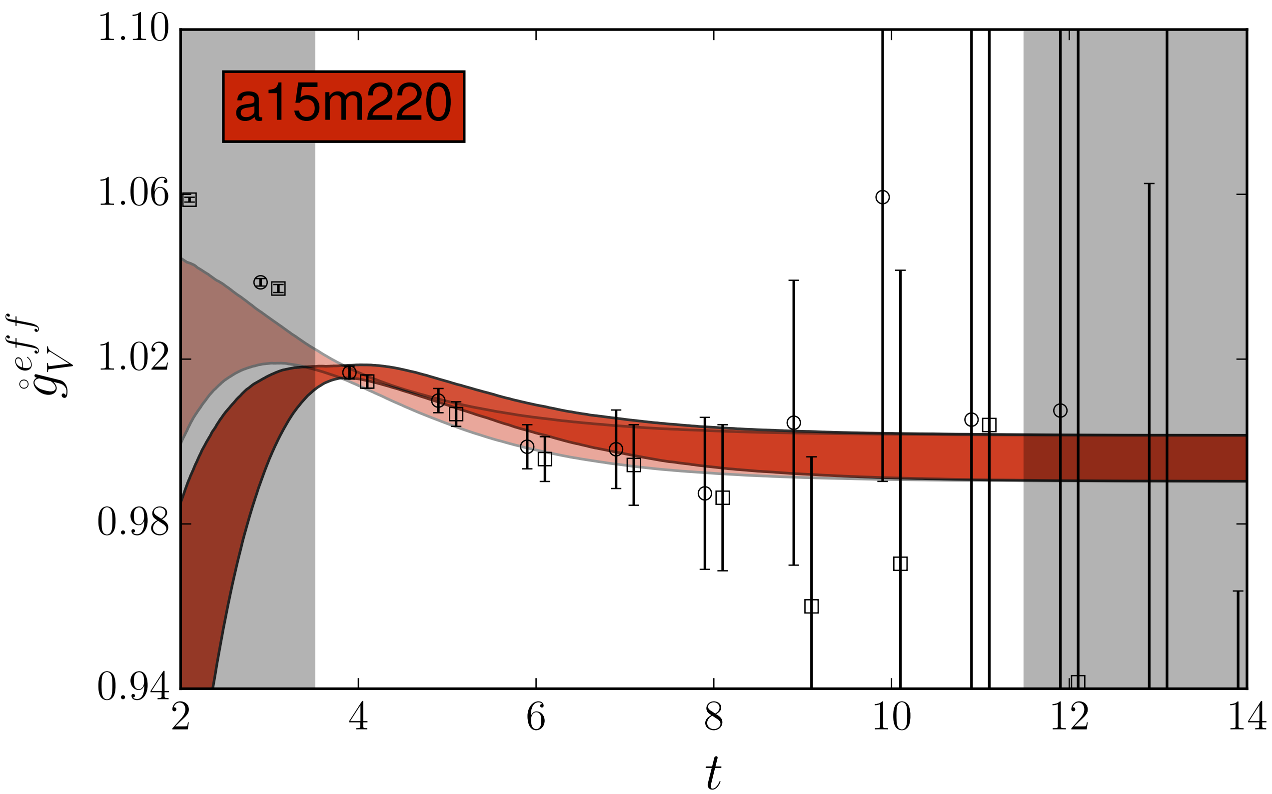

a12m130
M^{eff}
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0
2
4
6
8
10
12
14
t

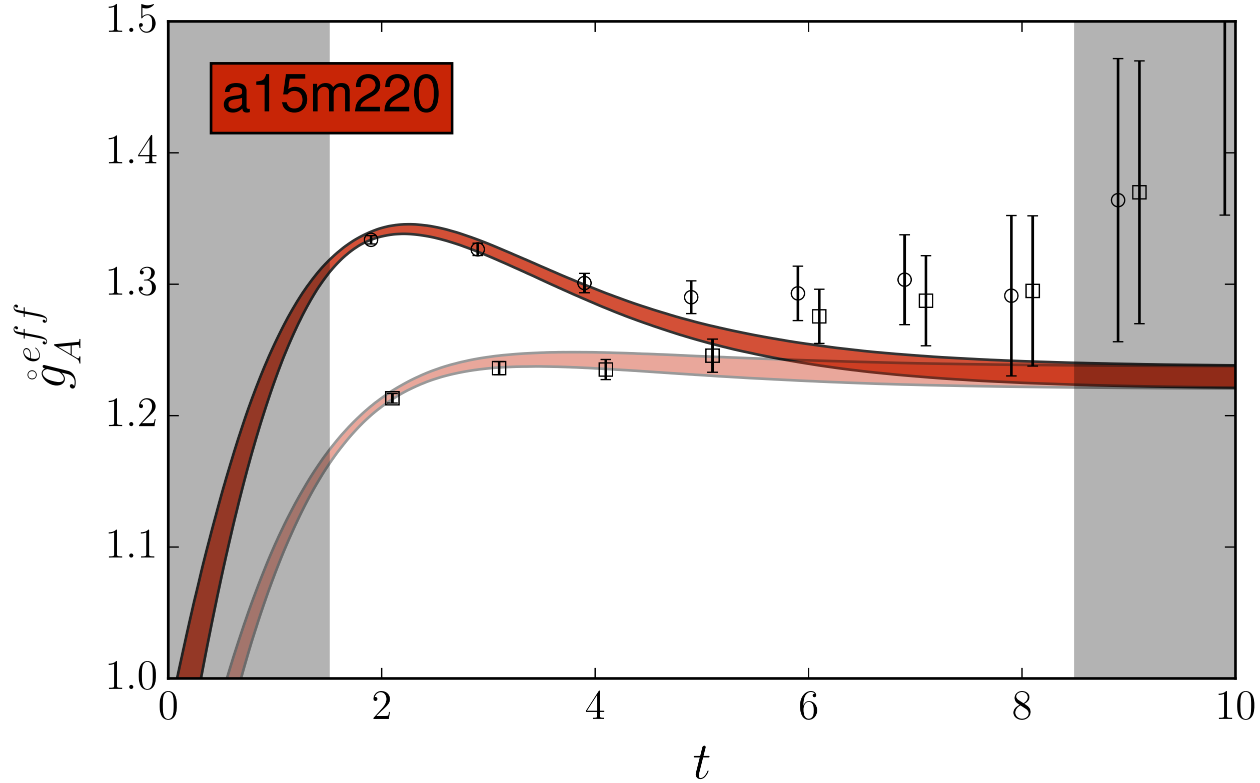

a12m130
$\mathring{g}_V^{eff}$
1.3
1.2
1.1
1.0
0.9
0
4
8
12
t


a12m130
$\mathring{g}_A^{eff}$
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0
4
8
12
16
t

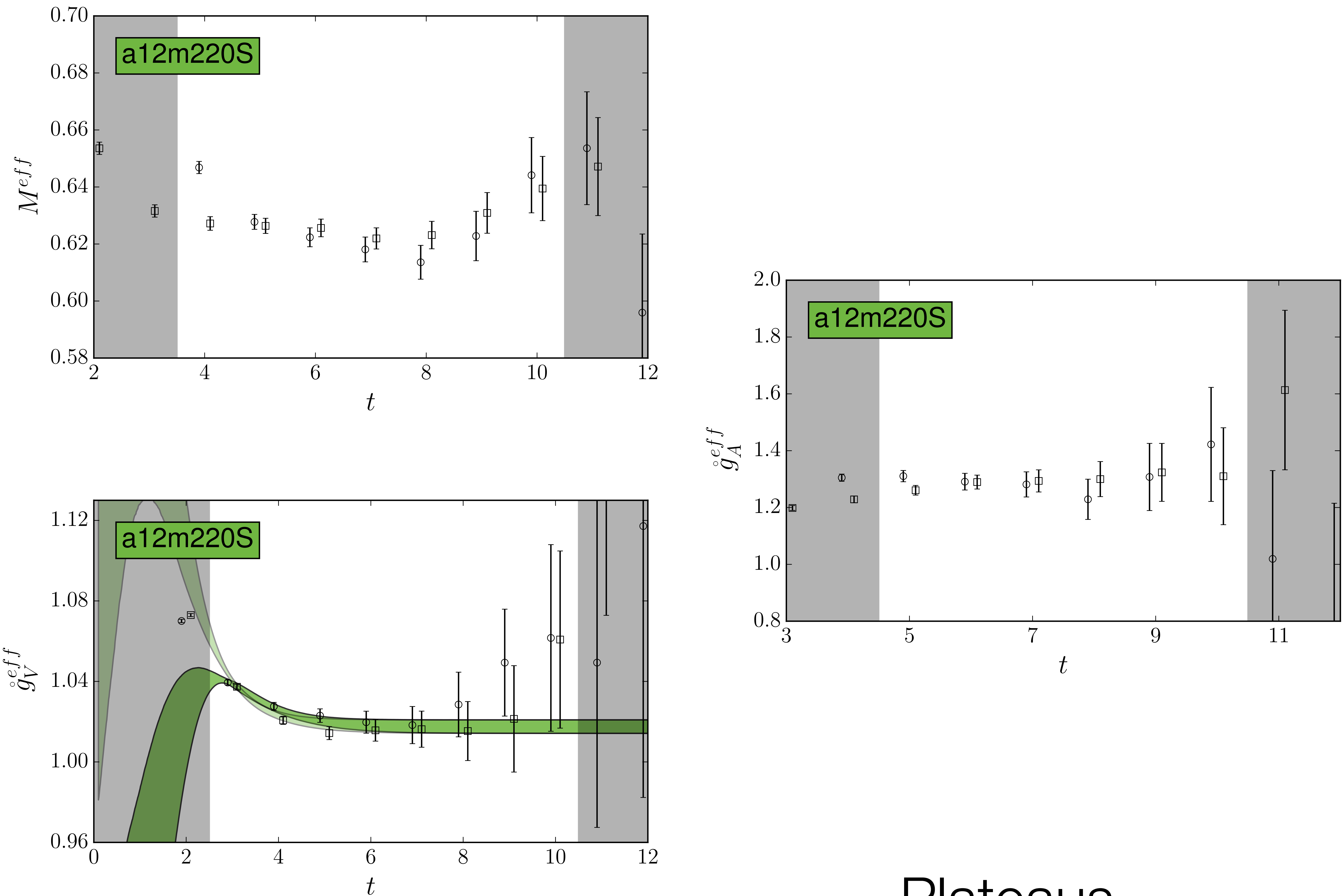
Plateaus


a15m220
M^{eff}
t

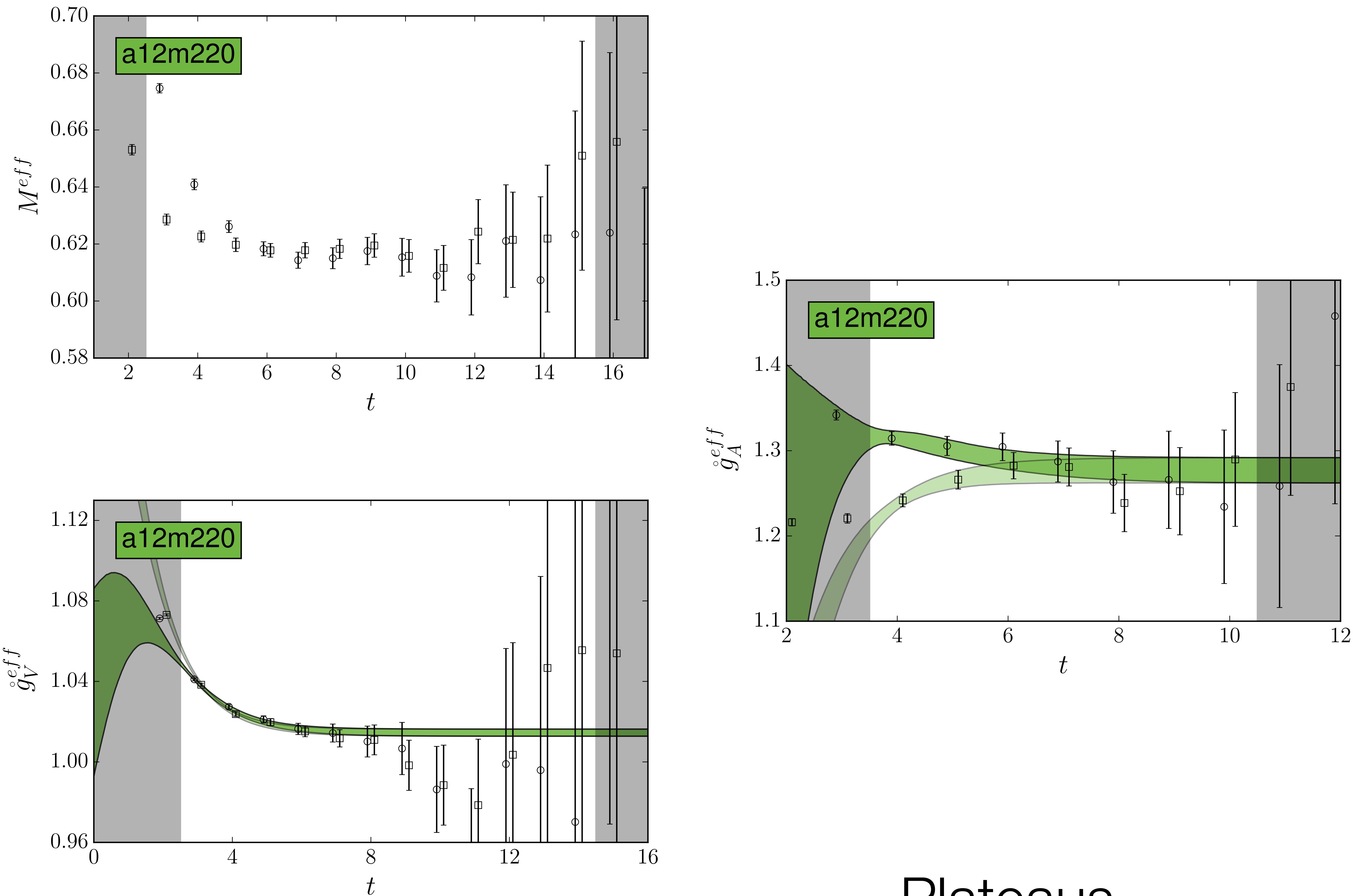

a15m220
$\mathring{g}_V^{eff}$
t


a15m220
$\mathring{g}_A^{eff}$
t

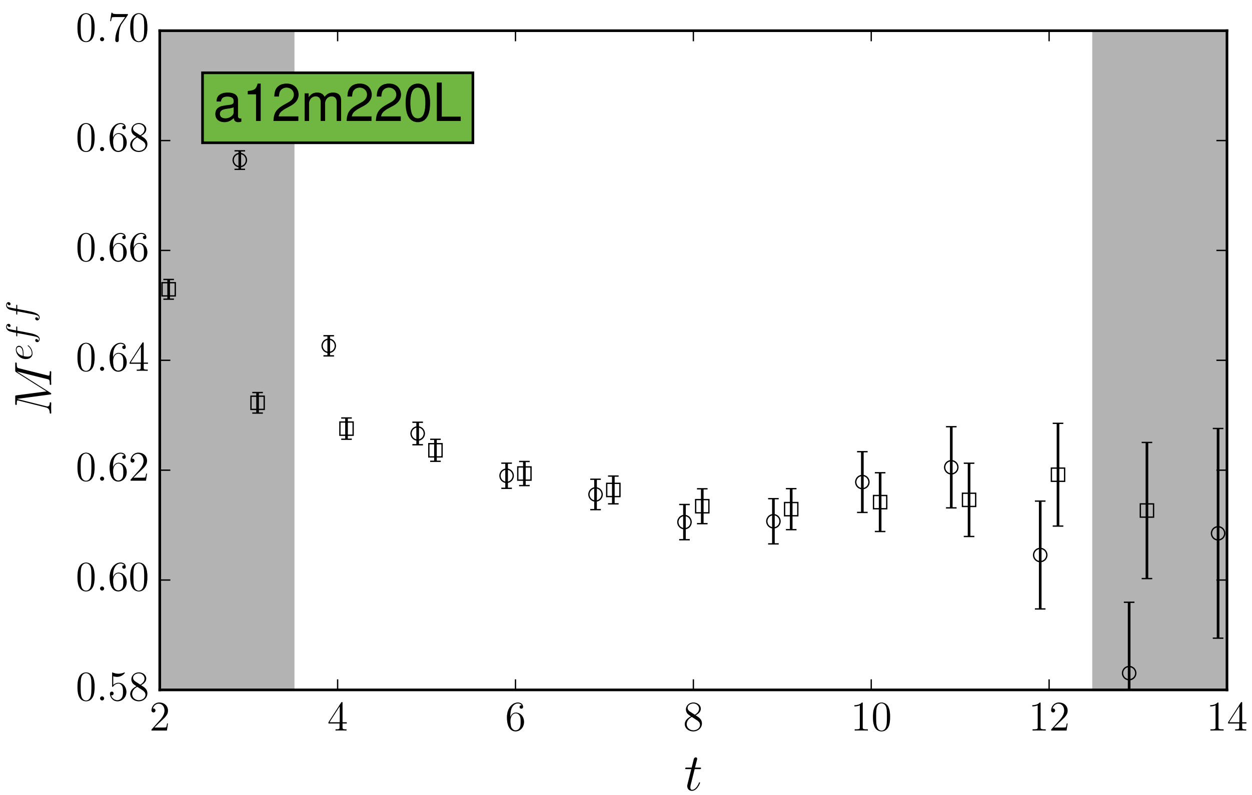
Plateaus

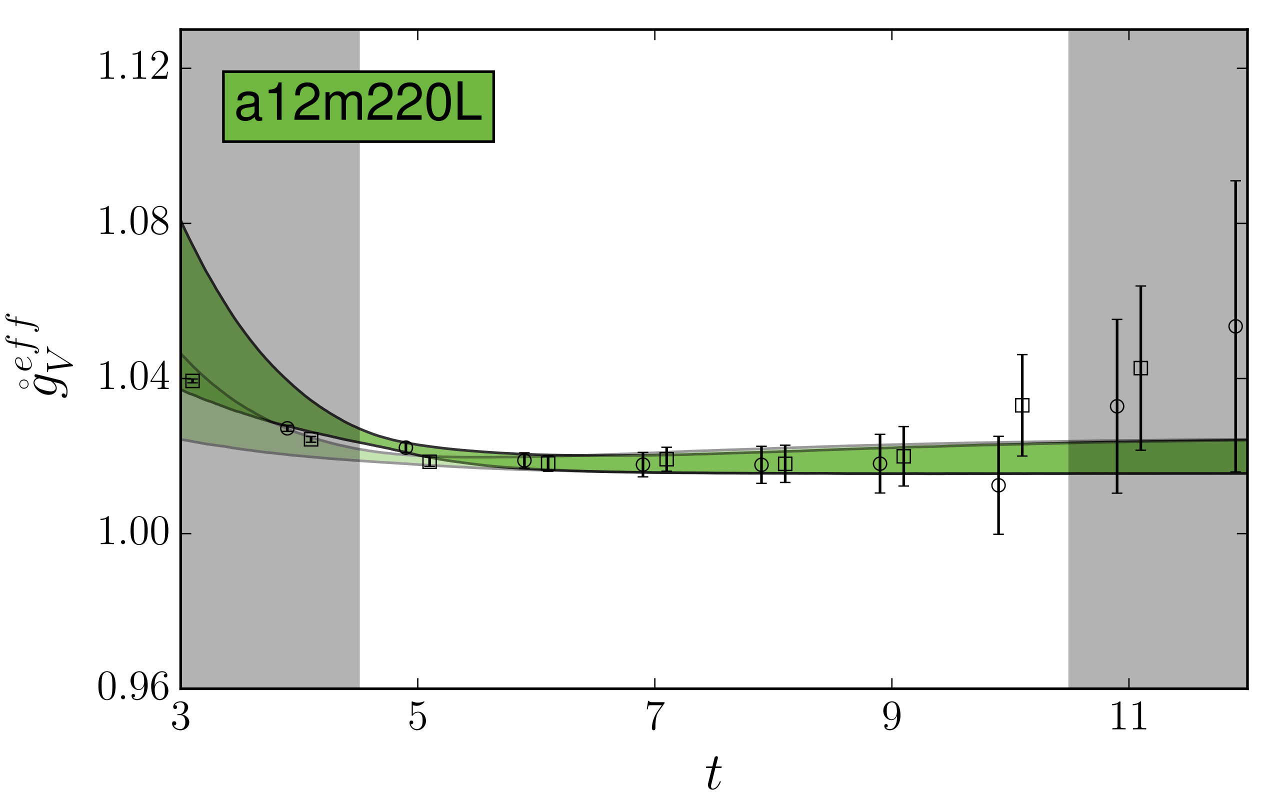

a12m220S
M^{eff}
t
a12m220S
$\mathring{g}_V^{eff}$
t
a12m220S
$\mathring{g}_A^{eff}$
t

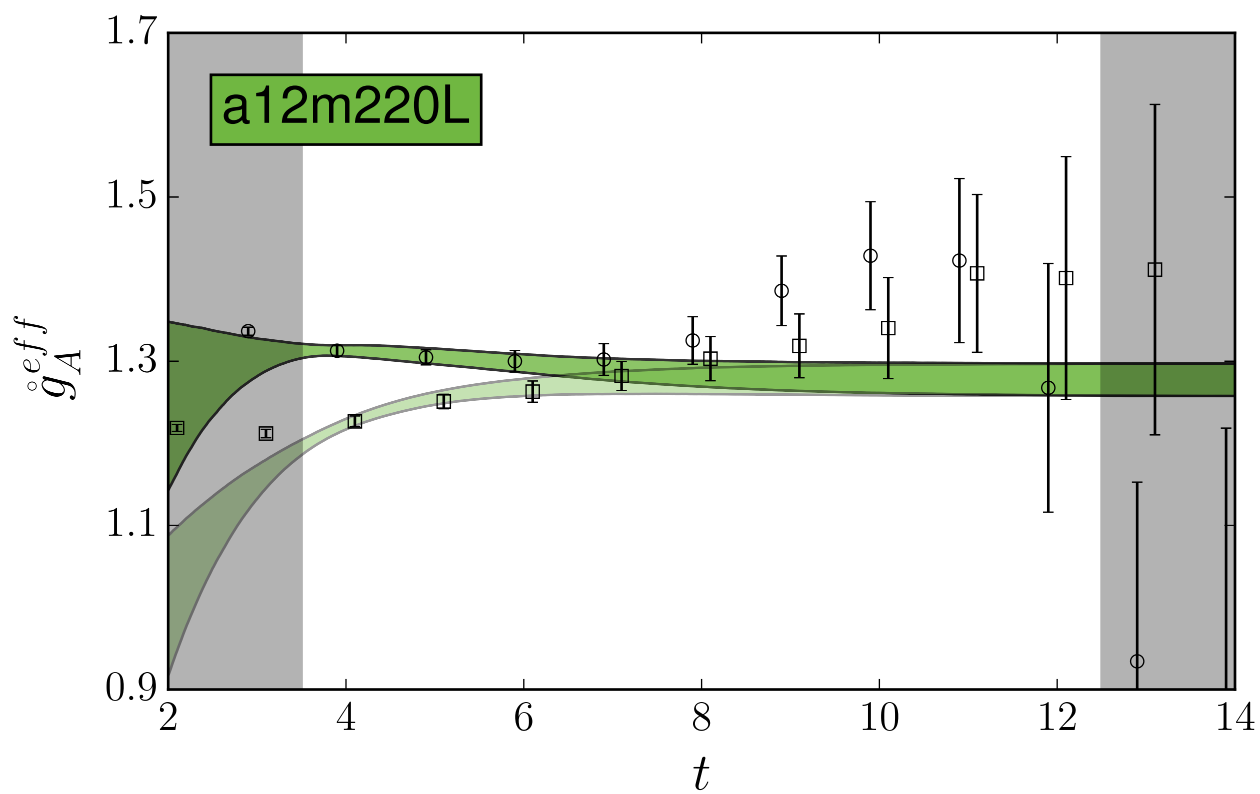
Plateaus


a12m220
0.70
0.68
0.66
0.64
0.62
0.60
0.58
M^{eff}
2
4
6
8
10
12
14
16
t
a12m220
1.12
1.08
1.04
1.00
0.96
$\mathring{g}_V^{eff}$
0
4
8
12
16
t
a12m220
1.5
1.4
1.3
1.2
1.1
$\mathring{g}_A^{eff}$
2
4
6
8
10
12
t

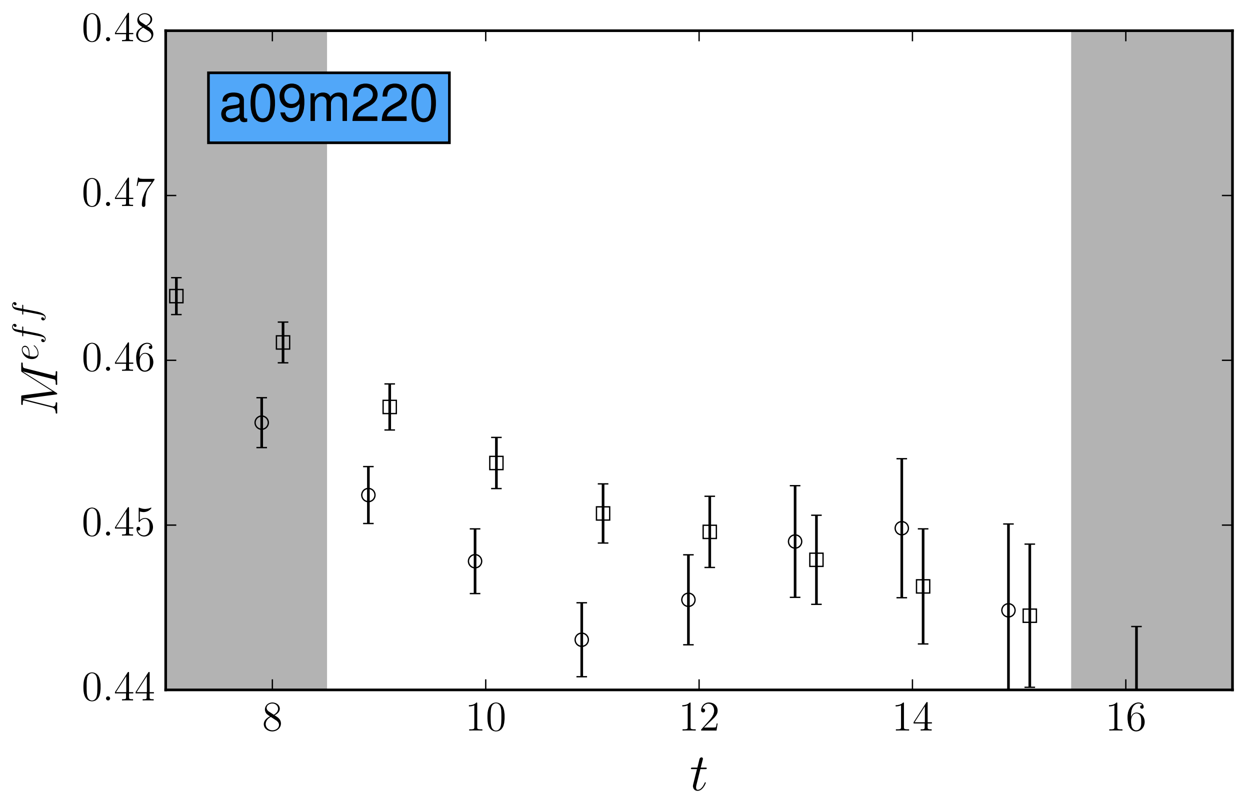
Plateaus

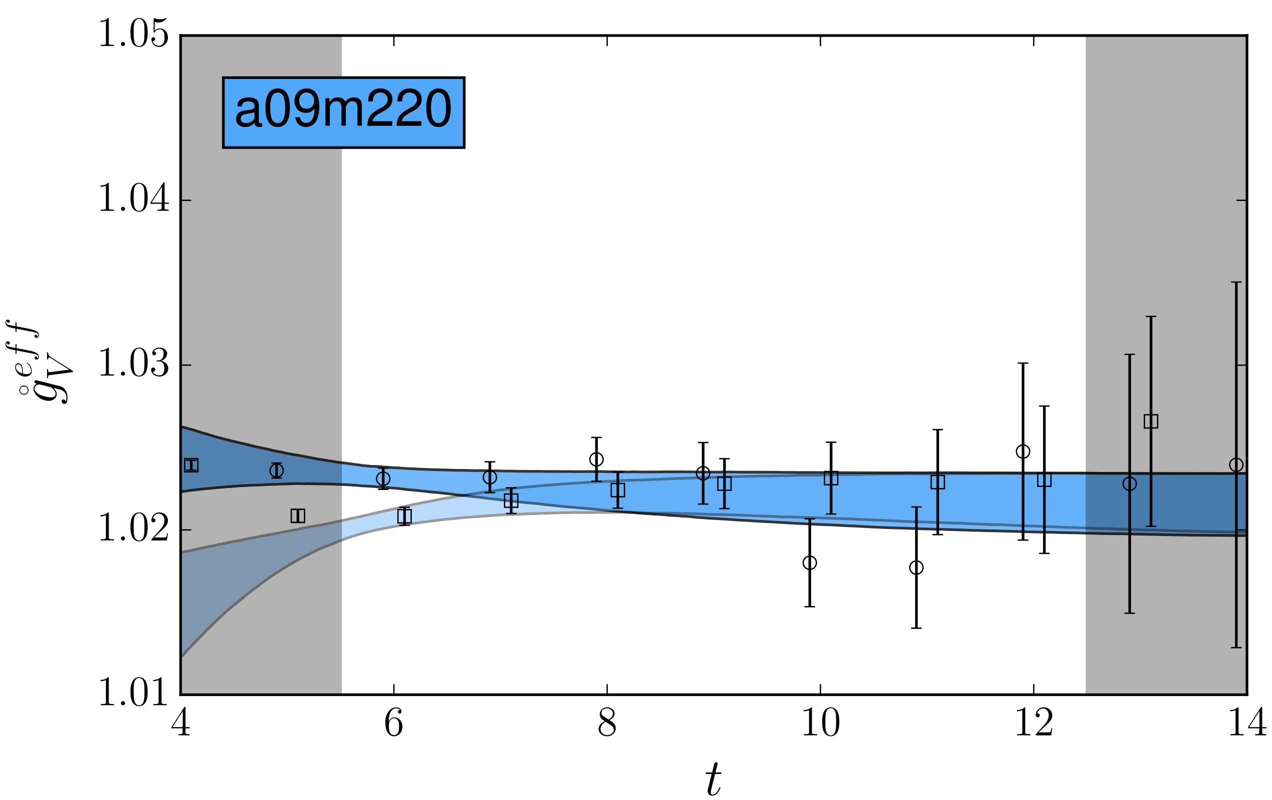

a12m220L
0.70
0.68
0.66
0.64
0.62
0.60
0.58
M^{eff}
2
4
6
8
10
12
14
t

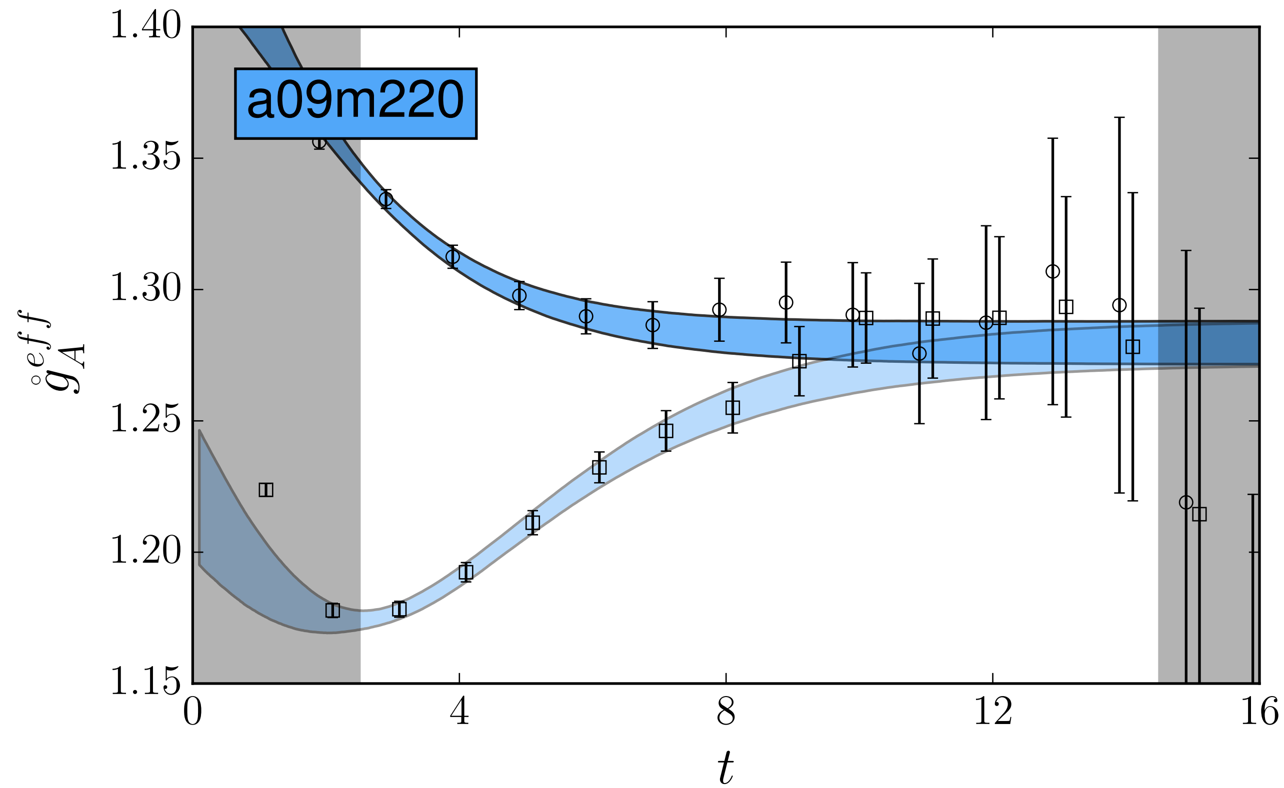

a12m220L
1.12
1.08
1.04
1.00
0.96
$\mathring{g}_V^{eff}$
3
5
7
9
11
t


a12m220L
1.7
1.5
1.3
1.1
0.9
$\mathring{g}_A^{eff}$
2
4
6
8
10
12
14
t

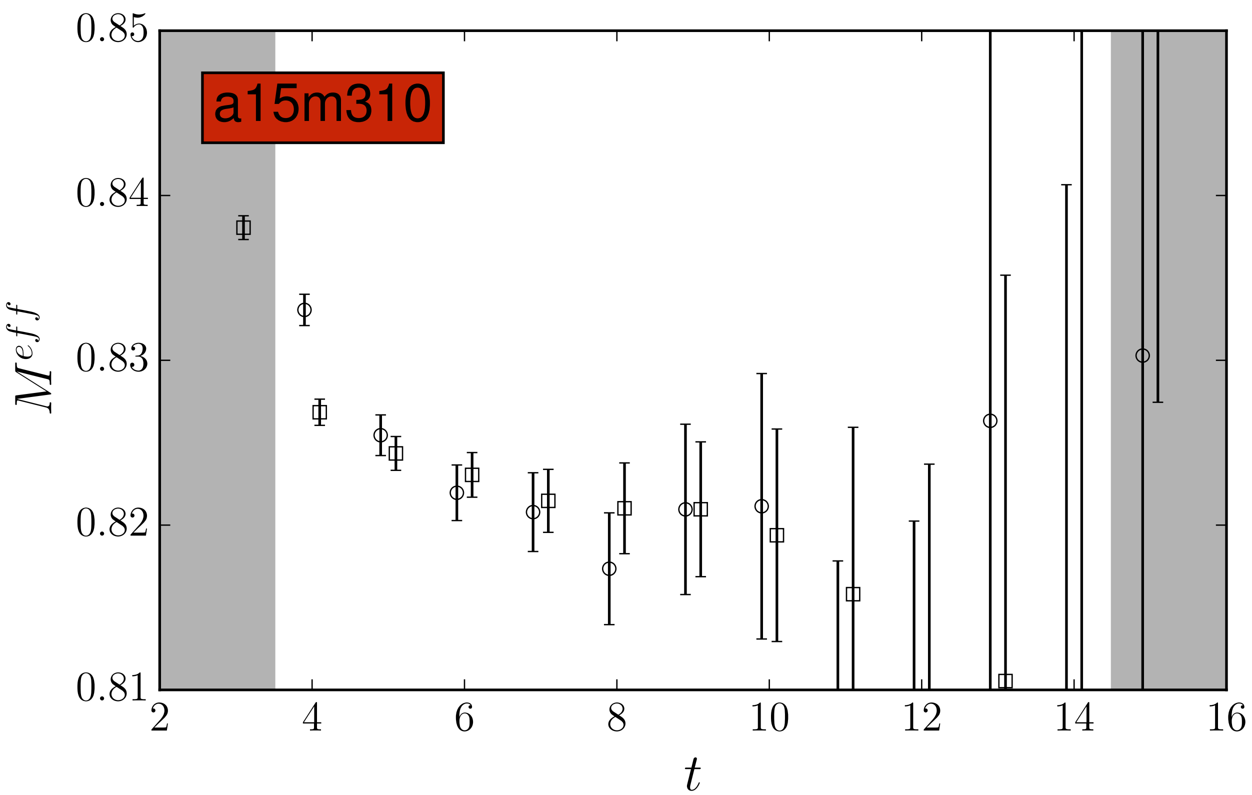
Plateaus

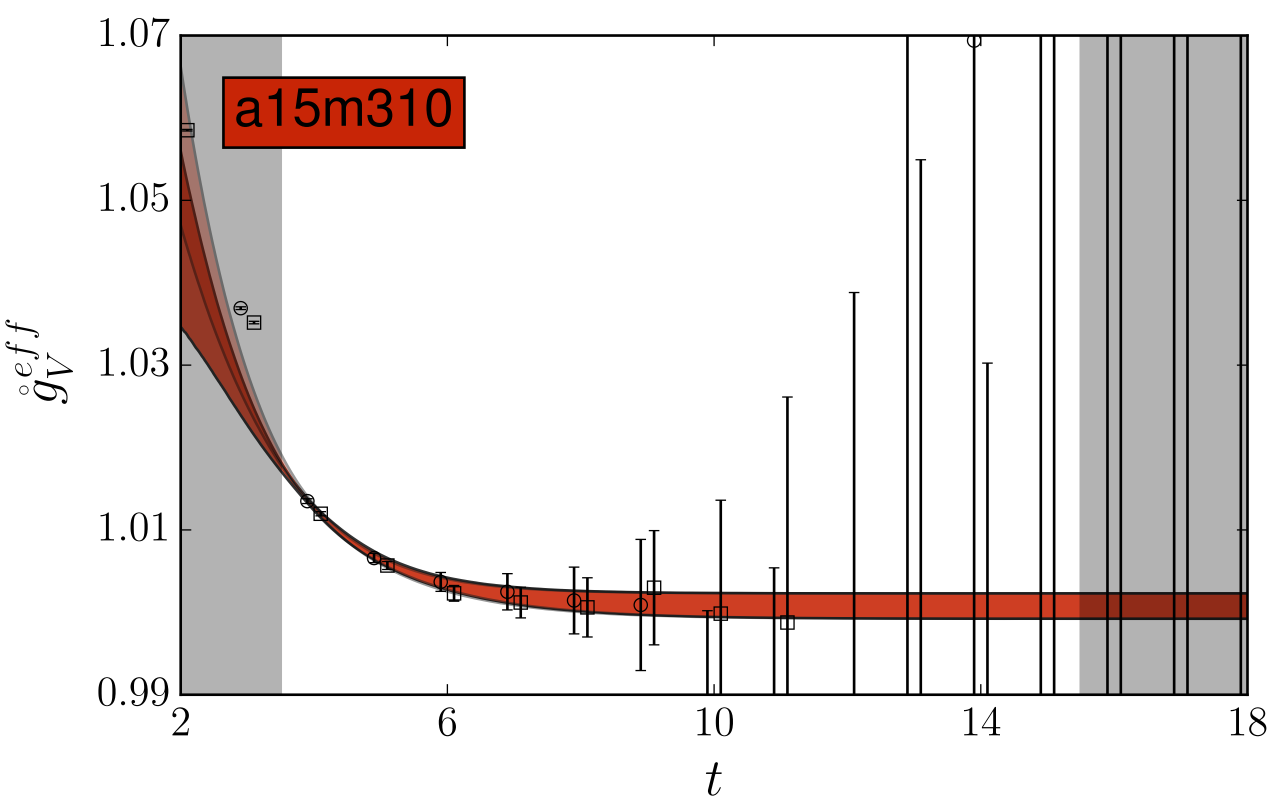

a09m220
0.48
0.47
0.46
0.45
0.44
M^{eff}
8
10
12
14
16
t

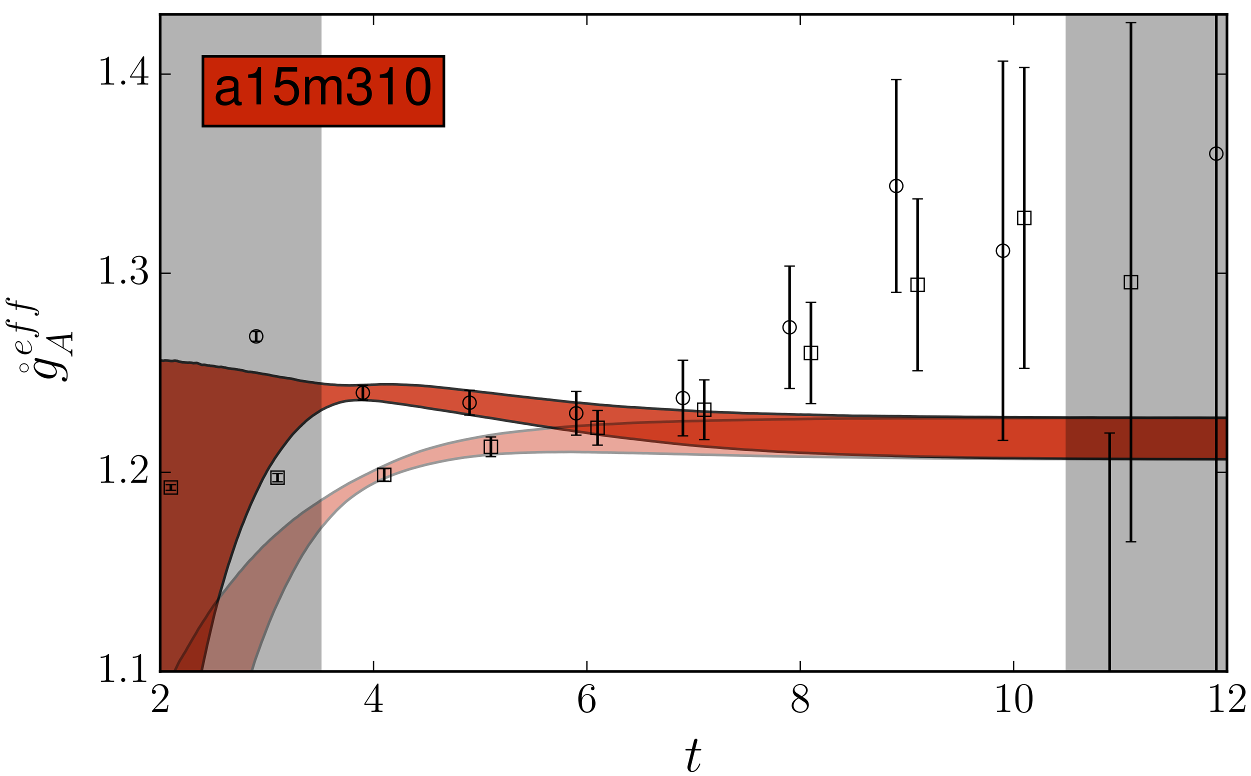

a09m220
1.05
1.04
1.03
1.02
1.01
$\mathring{g}_V^{eff}$
4
6
8
10
12
14
t


a09m220
1.40
1.35
1.30
1.25
1.20
1.15
$\mathring{g}_A^{eff}$
0
4
8
12
16
t

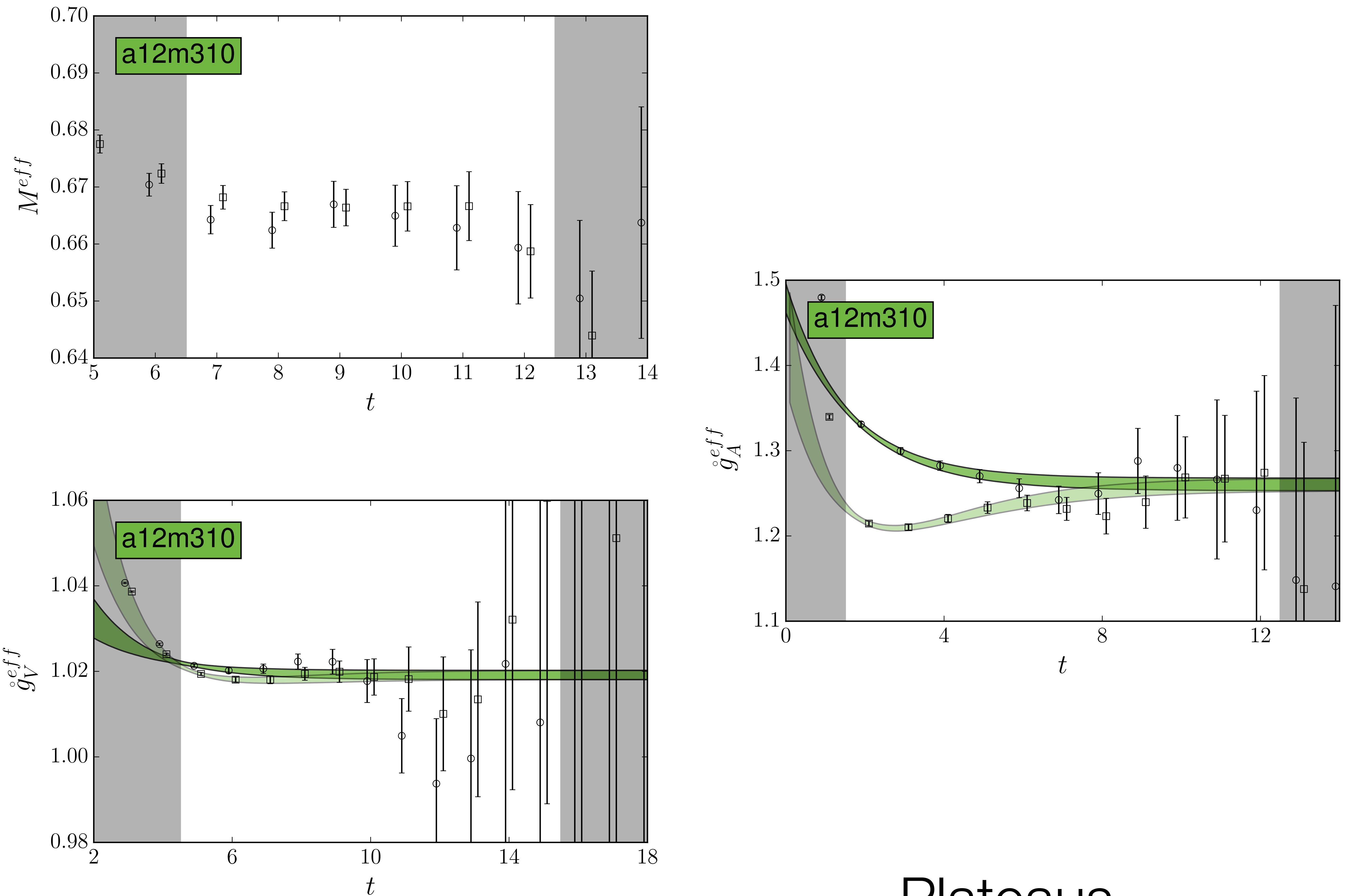
Plateaus


a15m310
M^{eff}
t


a15m310
$\mathring{g}_V^{eff}$
t

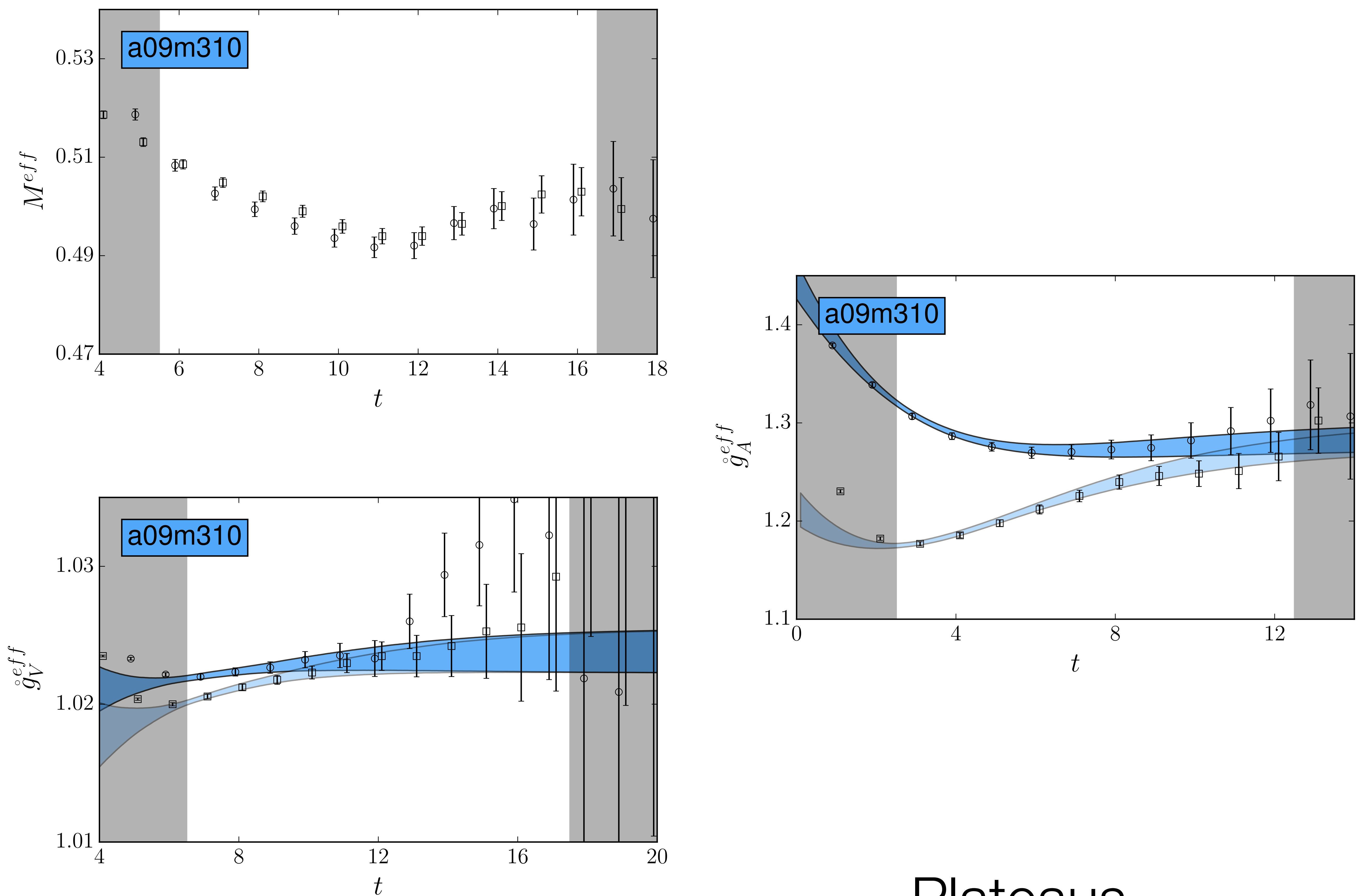

a15m310
$\mathring{g}_A^{eff}$
t

Plateaus

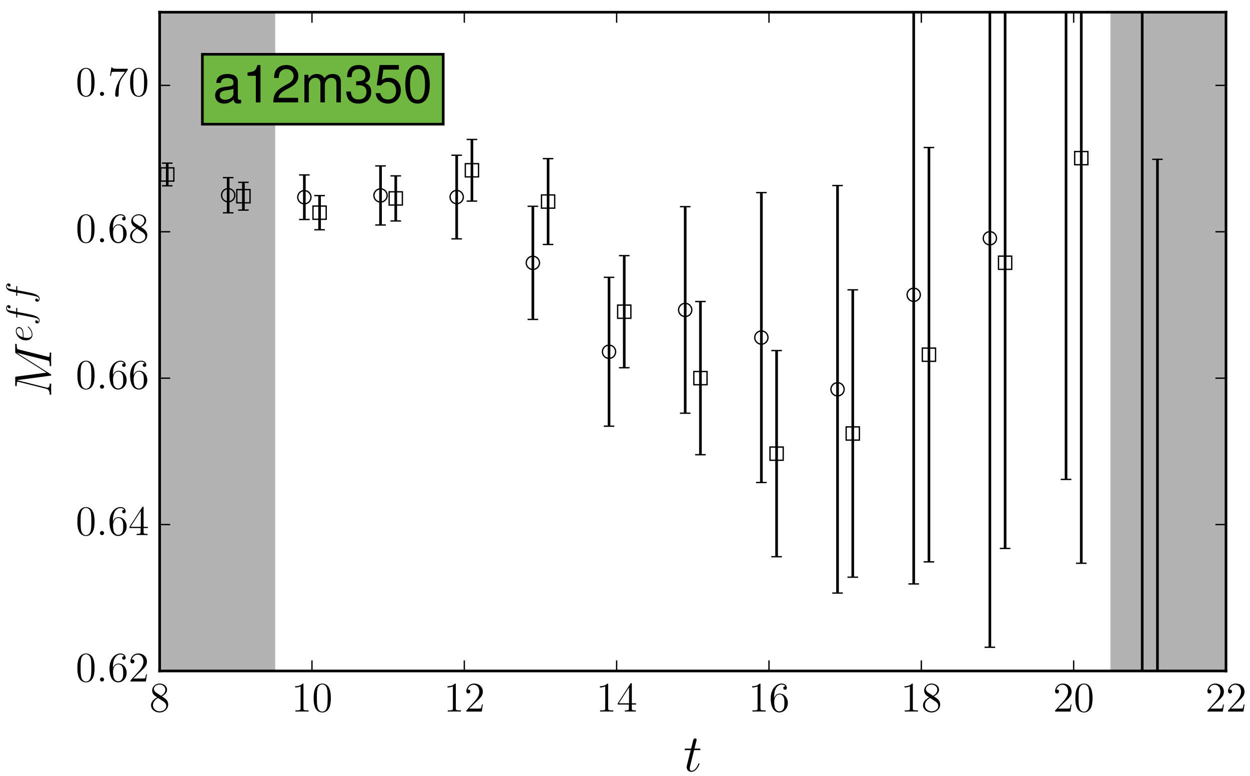


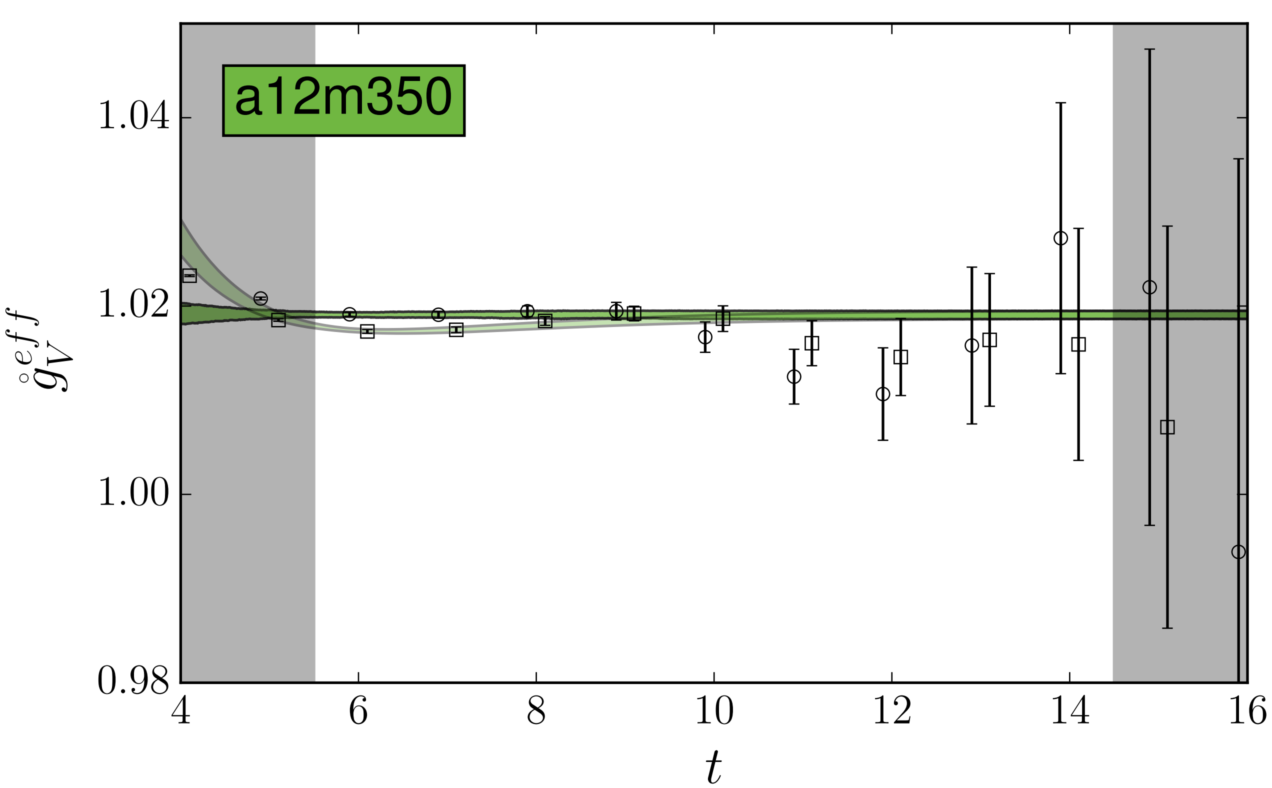
a12m310
M^{eff}
t
a12m310
$\mathring{g}_V^{eff}$
t
a12m310
$\mathring{g}_A^{eff}$
t

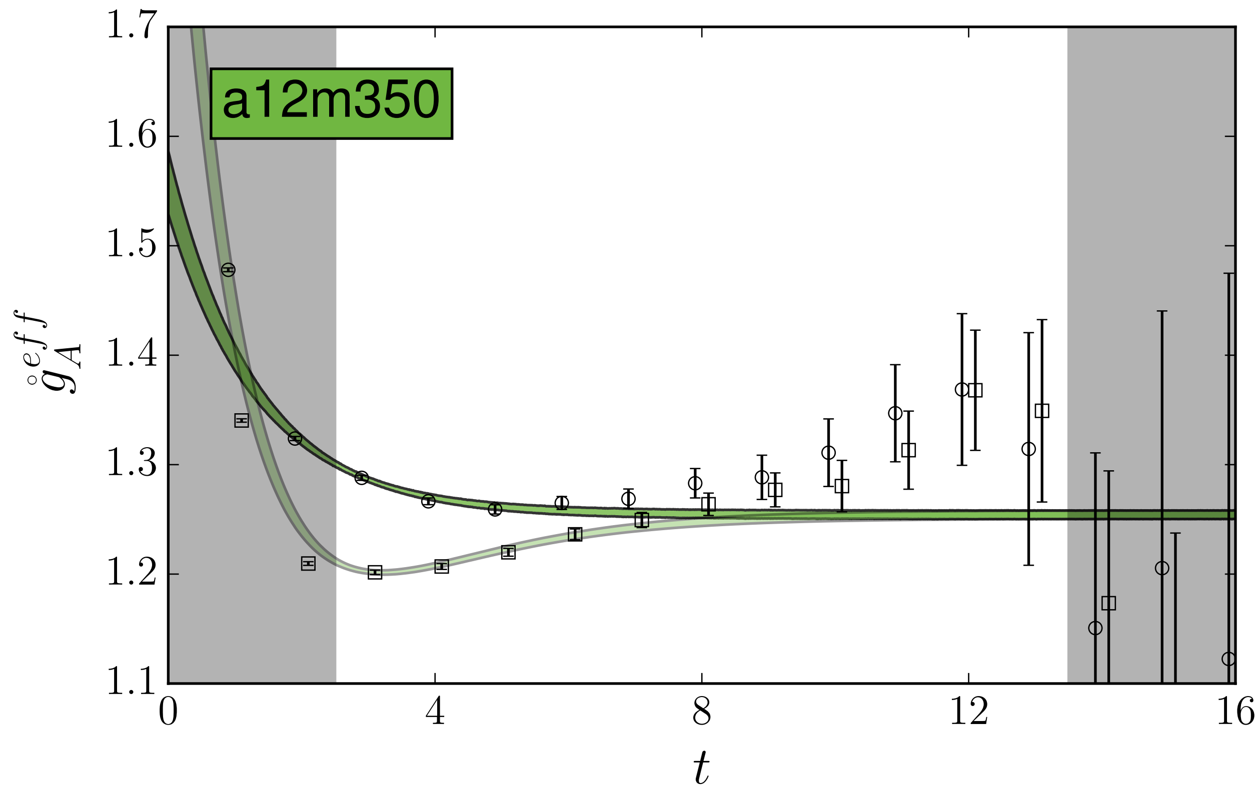
Plateaus


a09m310
0.53
0.51
0.49
0.47
M^{eff}
4
6
8
10
12
14
16
18
t
a09m310
1.4
1.3
1.2
1.1
$\mathring{g}_A^{eff}$
0
4
8
12
t
a09m310
1.03
1.02
1.01
$\mathring{g}_V^{eff}$
4
8
12
16
20
t

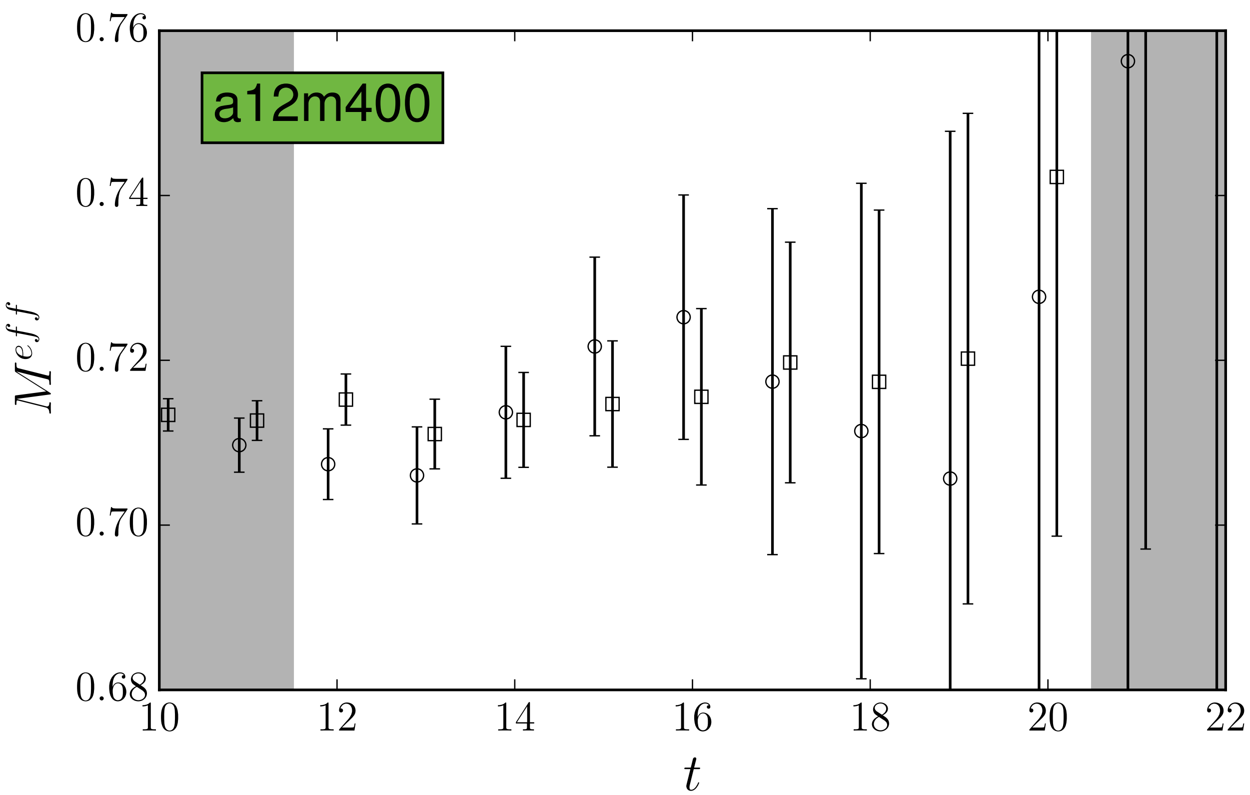
Plateaus

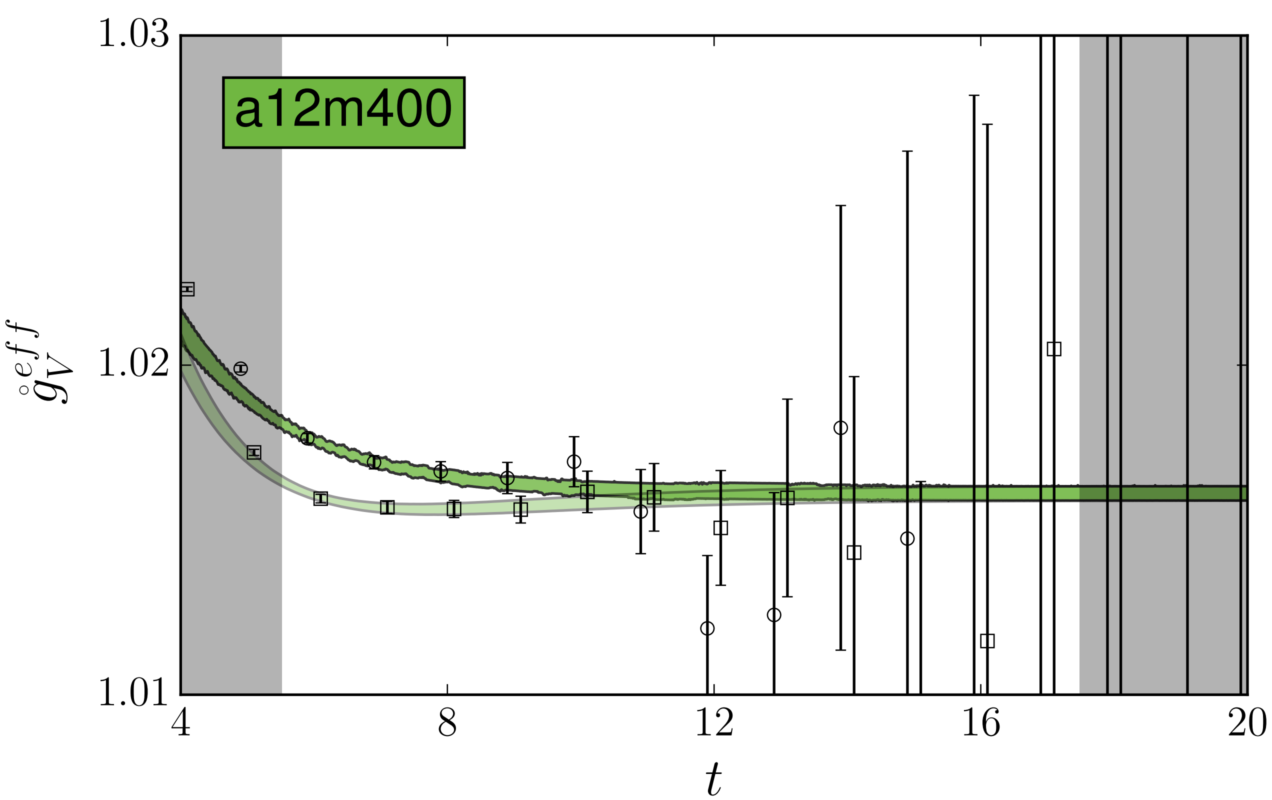

a12m350
0.70
0.68
0.66
0.64
0.62
8 10 12 14 16 18 20 22
M^{eff}
t

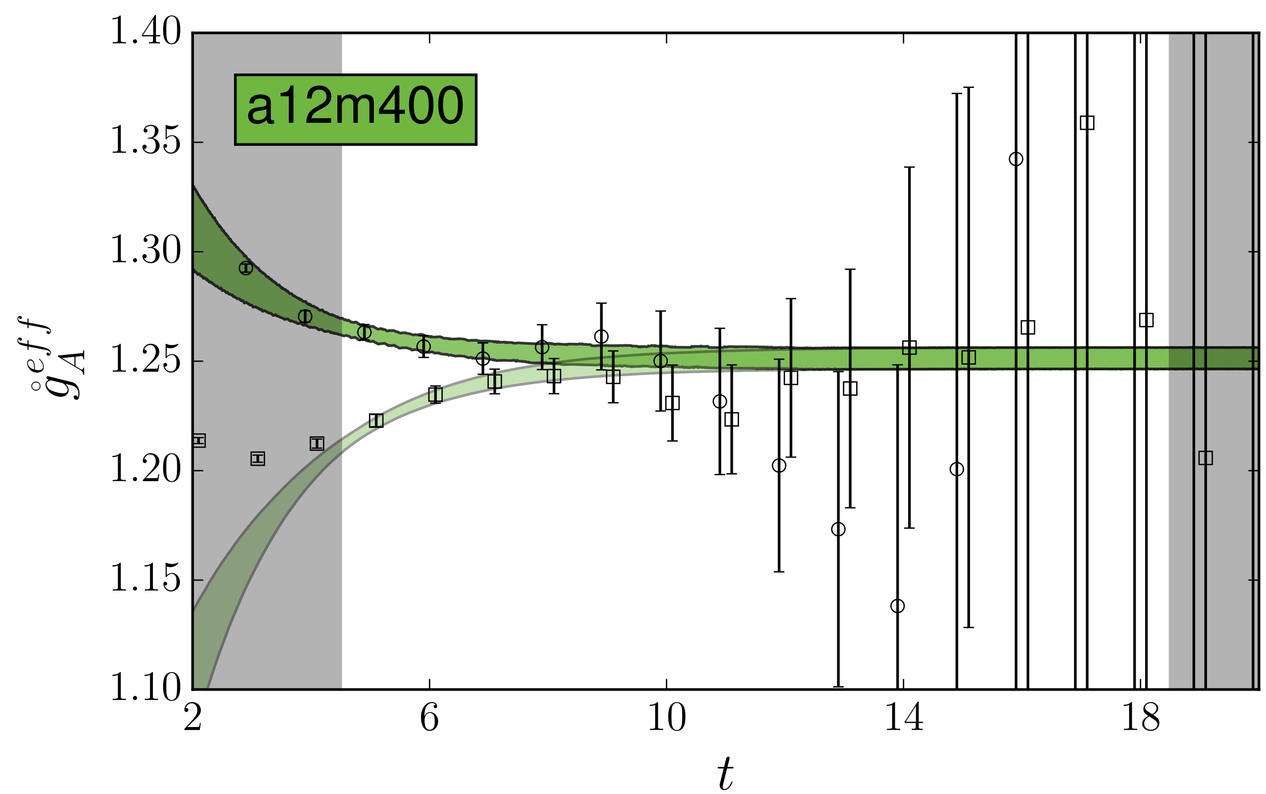

a12m350
1.04
1.02
1.00
0.98
4 6 8 10 12 14 16
$\mathring{g}_V^{eff}$
t


a12m350
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0 4 8 12 16
$\mathring{g}_A^{eff}$
t

Plateaus


a12m400
0.76
0.74
0.72
0.70
0.68
M^eff
10
12
14
16
18
20
22
t


a12m400
1.03
1.02
1.01
g_V^eff
4
8
12
16
20
t


a12m400
1.40
1.35
1.30
1.25
1.20
1.15
1.10
g_A^eff
2
6
10
14
18
t

Plateaus