

The energy-momentum tensor of nucleon, and new charmonium-pentaquark states at LHCb*

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Outline

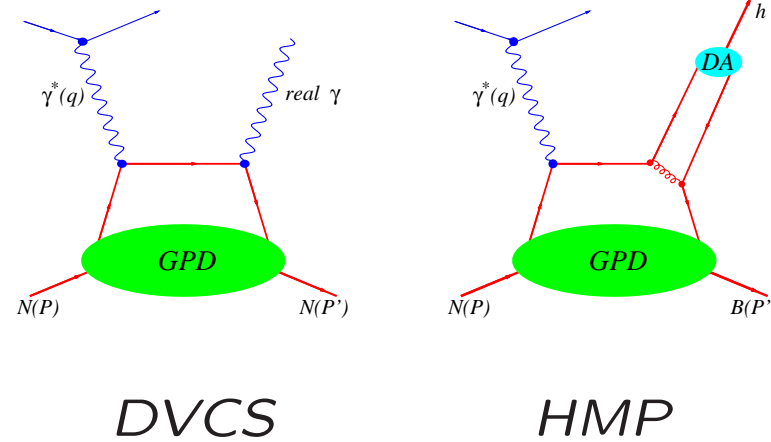
- **Introduction**
via hard-exclusive reactions
we can access GPDs! So what?
- **Energy-momentum tensor**
Ji sum rule & mechanical properties
last unknown global nucleon property(!)
- **Applications**
motivation & vision: study mechanical stability
practical use: from hard-exclusive reactions at JLab ...
... to charmonium pentaquark spectroscopy at LHCb
- **Outlook**
cool and promising future!

* in collaboration with Irina Perevalova, Maxim Polyakov,
based on work supported by JLab, DOE, NSF

Introduction

- hard-exclusive reactions factorization, access to **GPDs**

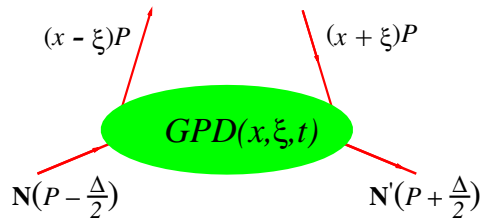
Ji; Radyushkin; Collins, Frankfurt, Strikman



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N'(p') | \bar{\psi}_q(-\frac{\lambda n}{2}) n_\mu \gamma^\mu [-\frac{\lambda n}{2}, \frac{\lambda n}{2}] \psi_q(\frac{\lambda n}{2}) | N(p) \rangle$$

$$= \bar{u}(p') \left[n_\mu \gamma^\mu H^q(x, \xi, t) + \bar{u}(p') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N} E^q(x, \xi, t) \right] u(p)$$

- meaning: “microsurgery”



definitions for completeness:

$$\xi = (n \cdot \Delta) / (n \cdot P), \quad t = \Delta^2$$

$$P = \frac{1}{2}(p' + p), \quad \Delta = p' - p$$

$$n^2 = 0, \quad n \cdot P = 2, \quad k = xP$$

renormalization scale μ

analog gluon GPDs

- what do we learn?

We learn a lot, because GPDs ...

- generalize PDFs & em (axial) form factors

$$\int dx H^q(x, \xi, t) = F_1^q(t)$$

$$\lim_{\Delta \rightarrow 0} H^q(x, \xi, t) = f_1^q(x)$$

- explore impact parameter space & tomography (M. Burkardt, ...)

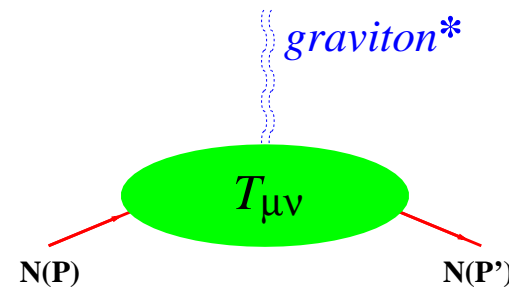
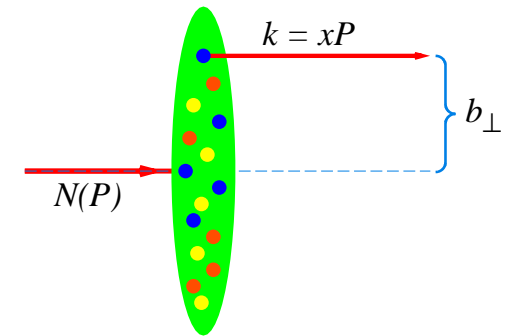
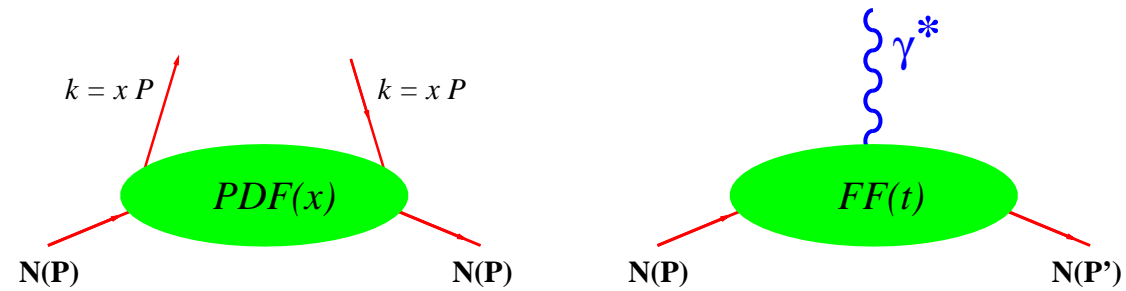
$$H^q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \left[\lim_{\xi \rightarrow 0} H^q(x, \xi, t) \right] e^{i \Delta_\perp b_\perp}$$

- allow to access (polynomiality) **gravitational form factors**

$$\int dx x H^q(x, \xi, t) = A^q(t) + \frac{4}{5} \xi^2 d_1^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - \frac{4}{5} \xi^2 d_1^q(t)$$

- and **gravity** couples to **energy momentum tensor**
probably most fundamental quantity



Energy-momentum tensor (EMT)

- instead of arguing how important EMT is, question: are you aware of introductory QFT text books which *do not* discuss EMT in first chapters?*
- if a theory can be solved:
construct $T_{\mu\nu}$ and generators of Poincaré group
learn what is **mass**, **spin**, **D-term (?)** of the particles
(in introductory text books: free fields, so it can be done & is instructive)
- even if a theory cannot be solved, studies of EMT insightful
3 examples in QCD
(in chronological order)

* interestingly, advanced QFT books discuss EMT in later chapters: *trace anomaly*

$$\hat{T}_\mu^\mu \equiv \frac{\beta}{2g} F^{\mu\nu} F_{\mu\nu} + (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q$$

Adler, Collins, Duncan, PRD15 (1977) 1712;
Nielsen, NPB 120, 212 (1977); Collins, Duncan, Joglekar, PRD 16, 438 (1977)

- **mass**: celebrated Higgs mechanism explains $m_u \approx 2 \text{ MeV}$, $m_d \approx 5 \text{ MeV}$
 $\hookrightarrow 2 \text{ MeV} + 2 \text{ MeV} + 5 \text{ MeV} \stackrel{!?}{=} 938 \text{ MeV}$ 2 orders of magnitude mismatch!
 (“popular propaganda”)

trace anomaly \rightarrow glue! Can we do better? Ji, PRL74, 1071; PRD52, 271 (1995)
 nucleon mass decomposition: how do E_{kin} , E_{pot} of quarks, gluons contribute?

- **spin** structure: **Ji sum rule** X.-D. Ji, Phys. Rev. Lett. 78 (1997) 610
 prominent motivator since its appearance!

$$\int dx x \left(H^q(x, \xi, t) + E^q(x, \xi, t) \right) = A^q(t) + B^q(t) \xrightarrow{t \rightarrow 0} 2J^q(0)$$

- **D-term**, last unknown global property M. V. Polyakov, C. Weiss, PRD60 (1999) 114017
 stress tensor & **mechanical properties** M. V. Polyakov, PLB555 (2003) 57

Energy momentum tensor $T^{\mu\nu}$

• definition

$$\langle P' | \hat{T}_{\mu\nu}^{q,g} | P \rangle = \bar{u}(p') \left[\begin{aligned} & \mathbf{A}^{q,g}(t) \frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} \\ & + \mathbf{B}^{q,g}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{4M_N} \\ & + \mathbf{d}_1^{q,g}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu} \end{aligned} \right] u(p)$$

- $\hat{T}_{\mu\nu}^q$ and $\hat{T}_{\mu\nu}^g$ separately gauge-invariant (but not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}_{\mu\nu}^q + \hat{T}_{\mu\nu}^g$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$
- constraints: **mass** $\Leftrightarrow A^q(0) + A^g(0) = 1$ (100 % of nucleon momentum carried by quarks + gluons)
spin $\Leftrightarrow B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons)
- property: **D-term** $\Leftrightarrow d_1^q(0) + d_1^g(0) \equiv \mathbf{d}_1 \rightarrow$ also conserved **quantity!**
but unconstrained! *Unknown!*

notation: $A^q(t) + B^q(t) = 2 J^q(t)$

\hookrightarrow **D-term** last unknown global property of the nucleon \Rightarrow what does it mean?

- How do we learn about nucleon?

$|N\rangle =$ **strong** interaction particle. Use other forces to probe it!

em: $\partial_\mu J_{\text{em}}^\mu = 0 \quad \langle N' | J_{\text{em}}^\mu | N \rangle \longrightarrow Q, \mu, \dots$

weak: PCAC $\quad \langle N' | J_{\text{weak}}^\mu | N \rangle \longrightarrow g_A, g_p, \dots$

gravity: $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0 \quad \langle N' | T_{\text{grav}}^{\mu\nu} | N \rangle \longrightarrow M, J, d_1, \dots$

1st global properties:

Q_{prot}	=	$1.602176487(40) \times 10^{-19} \text{C}$
μ_{prot}	=	$2.792847356(23) \mu_N$
g_A	=	$1.2694(28)$
g_p	=	$8.06(0.55)$
M	=	$938.272013(23) \text{ MeV}$
J	=	$\frac{1}{2}$
d_1	=	???

$\hookrightarrow d_1 =$ "last" global unknown

2nd partonic structure:

...
...
...
...

- **interpretation** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $\mathbf{T}_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle$

interpretation okay for large- N_c : $M_N \sim N_c$, $t \sim N_c^0 \Rightarrow$ recoil corrections $t/M_N^2 \sim 1/N_c^2$ (formulae correct $\forall N_c$)

$$\int d^3r T_{00}(\vec{r}) = M_N \quad \text{known}$$

$$\int d^3r \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}) = \frac{1}{2} \quad \text{known}$$

$$-\frac{M_N}{2} \int d^3r \left(r^i r^j - \frac{r^2}{3} \delta^{ij} \right) T_{ij}(\vec{r}) \equiv d_1 \quad \text{new!}$$

with: $\mathbf{T}_{ij}(\vec{r}) = \mathbf{s}(\mathbf{r}) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + \mathbf{p}(\mathbf{r}) \delta_{ij} \quad \text{stress tensor}$

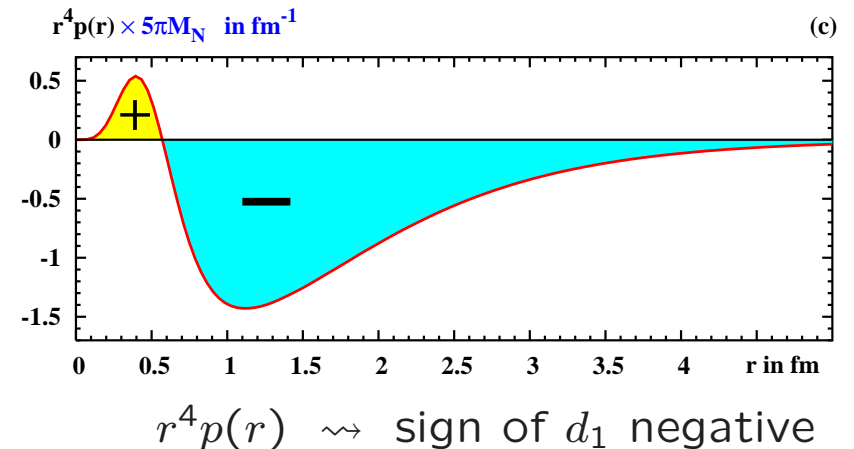
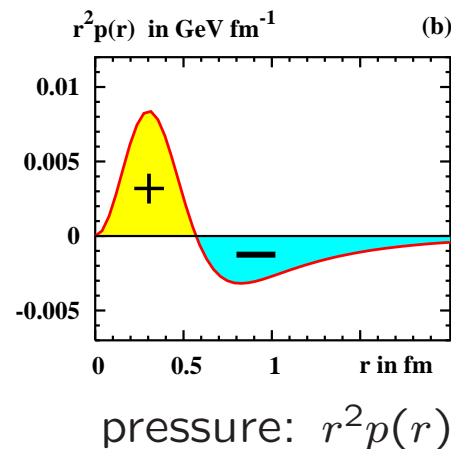
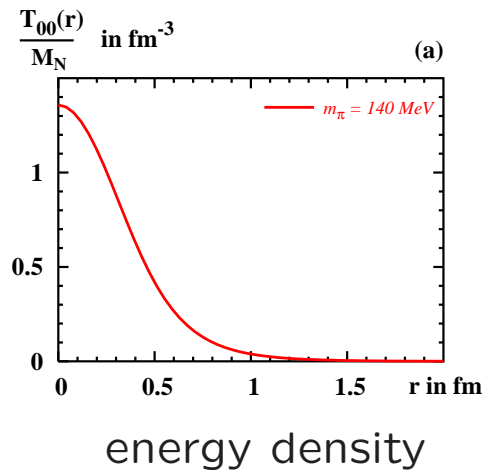
$\mathbf{s}(\mathbf{r})$ related to distribution of *shear forces*
 $\mathbf{p}(\mathbf{r})$ distribution of *pressure* inside hadron } \longrightarrow “mechanical properties”

- **relation to stability:** EMT conservation $\Leftrightarrow \partial^\mu \hat{T}_{\mu\nu} = 0 \Leftrightarrow \nabla^i T_{ij}(\vec{r}) = 0$

\hookrightarrow necessary condition for stability $\int_0^\infty dr r^2 p(r) = 0$ (von Laue, 1911)

$$d_1 = -\frac{4\pi}{3} M_N \int_0^\infty dr r^4 s(r) = 5\pi M_N \int_0^\infty dr r^4 p(r) \quad \hookrightarrow d_1 \text{ shows how internal forces balance}$$

- lessons from model



$$T_{00}(0) = 1.70 \text{ GeV}/\text{fm}^3 \approx 3 \times 10^{15} \rho(\text{H}_2\text{O}) \approx 13 \times (\text{nuclear density})$$

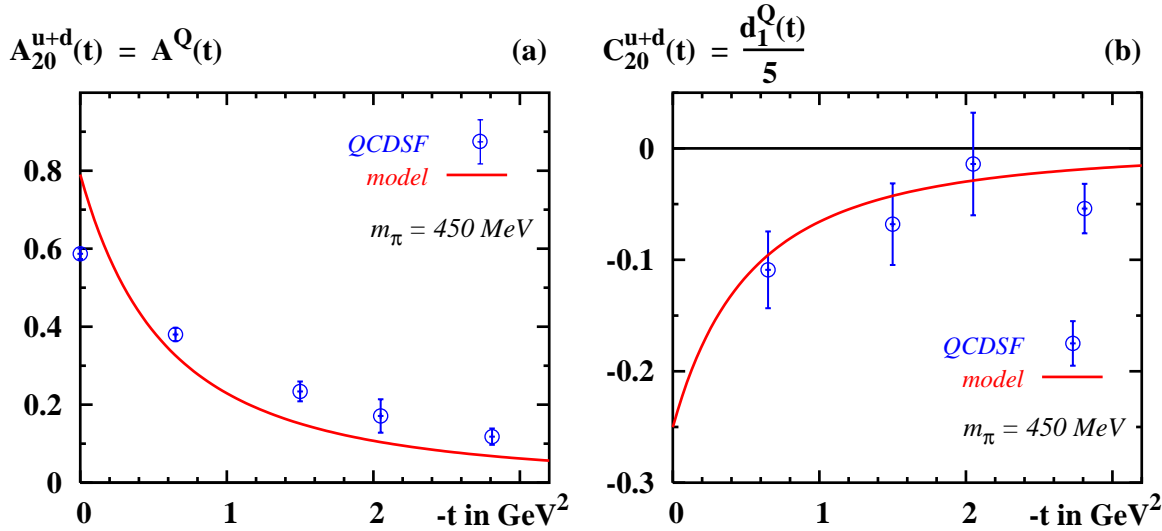
$$p(0) = 0.23 \text{ GeV}/\text{fm}^3 \approx 4 \times 10^{34} \text{ N}/\text{m}^2 \gtrsim 100 \times (\text{pressure in center of neutron star})$$

in chiral quark soliton model (Goeke et al, PRD75 (2007) 094021)

... how does it look like in QCD? We do not know. *Wouldn't it be fascinating to know??*

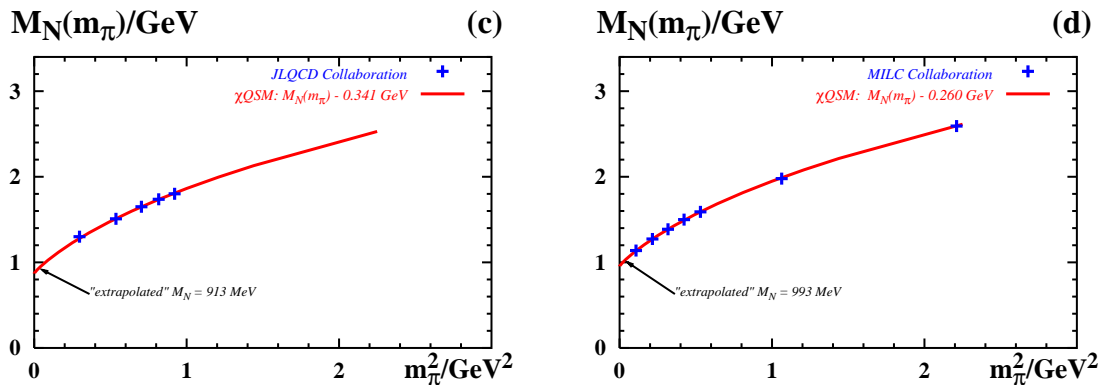
• is the model picture realistic?

chiral quark soliton model well-tested: many nucleon properties vs data: OK within 30% ✓
 no data for EMT, so compare to lattice (available at the time of our calculation)



QCDSF Collaboration
 Göckeler et al, PRL92 (2004) 042002
 Nucl. Phys. Proc. Suppl. 128, 203 (2004)
 VS
 K. Goeke et al, PRC 75, 055207 (2007)

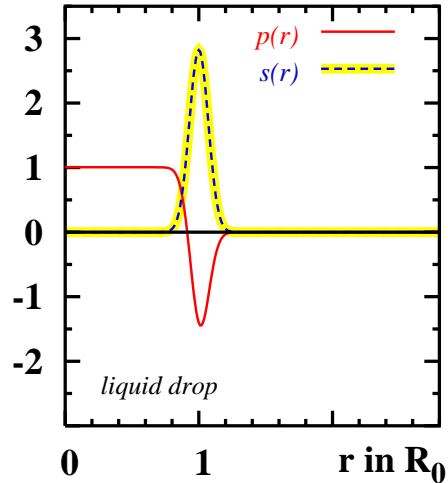
... by the way: can we apply a chiral model to large unphysical pion masses (unvoluntarily probed on lattices until few years ago)?



interestingly, chiral model works also at large unphysical pion masses:
 MILC Collaboration, PRD64 (2001) 054506,
 JLQCD Collaboration, PRD68 (2003) 054502
 VS
 Goeke et al, EPJA 27, 77 (2006)
 model gets "variation of M_N with m_π " right!

- intuition on shear forces and pressure

$p(r)$ & $s(r)$ in units of p_0



liquid drop

radius R_0

inside pressure p_0

surface tension $\gamma = \frac{1}{2}p_0R_0$

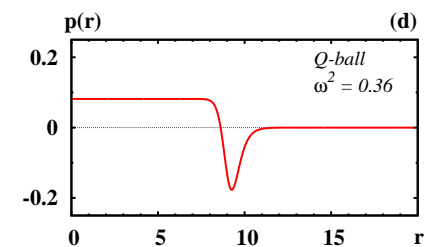
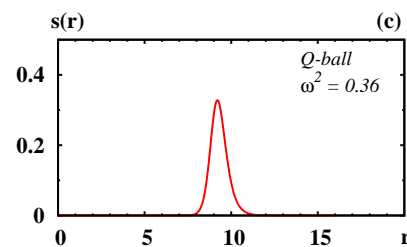
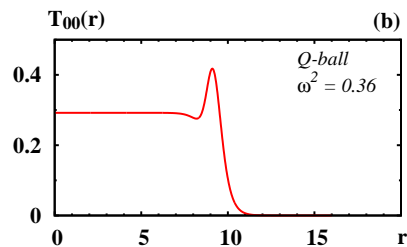
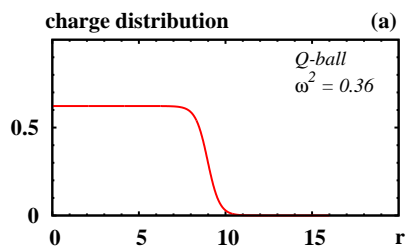
$$s(r) = \gamma \delta(r - R_0)$$

$$p(r) = p_0 \Theta(R_0 - r) - \frac{1}{3}p_0R_0 \delta(r - R_0)$$

application: liquid drop model of nucleus

(M.V.Polyakov, PLB 555 (2003) 57)

realized in field theoretical Q -ball system



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V \text{ with U(1) global symm., } V = A(\Phi^*\Phi) - B(\Phi^*\Phi)^2 + C(\Phi^*\Phi)^3, \quad \Phi(t, \vec{r}) = e^{i\omega t} \phi(r)$$

S. R. Coleman, NPB262 (1985) 263, 269 (1986) 744E; M. Mai, PS PRD86 (2012) 076001

to satisfy $\int_0^\infty dr r^2 p(r) = 0 \rightarrow p(r)$ must have a zero! Could it have more zeros?

N^{th} radial excitation of Q -balls

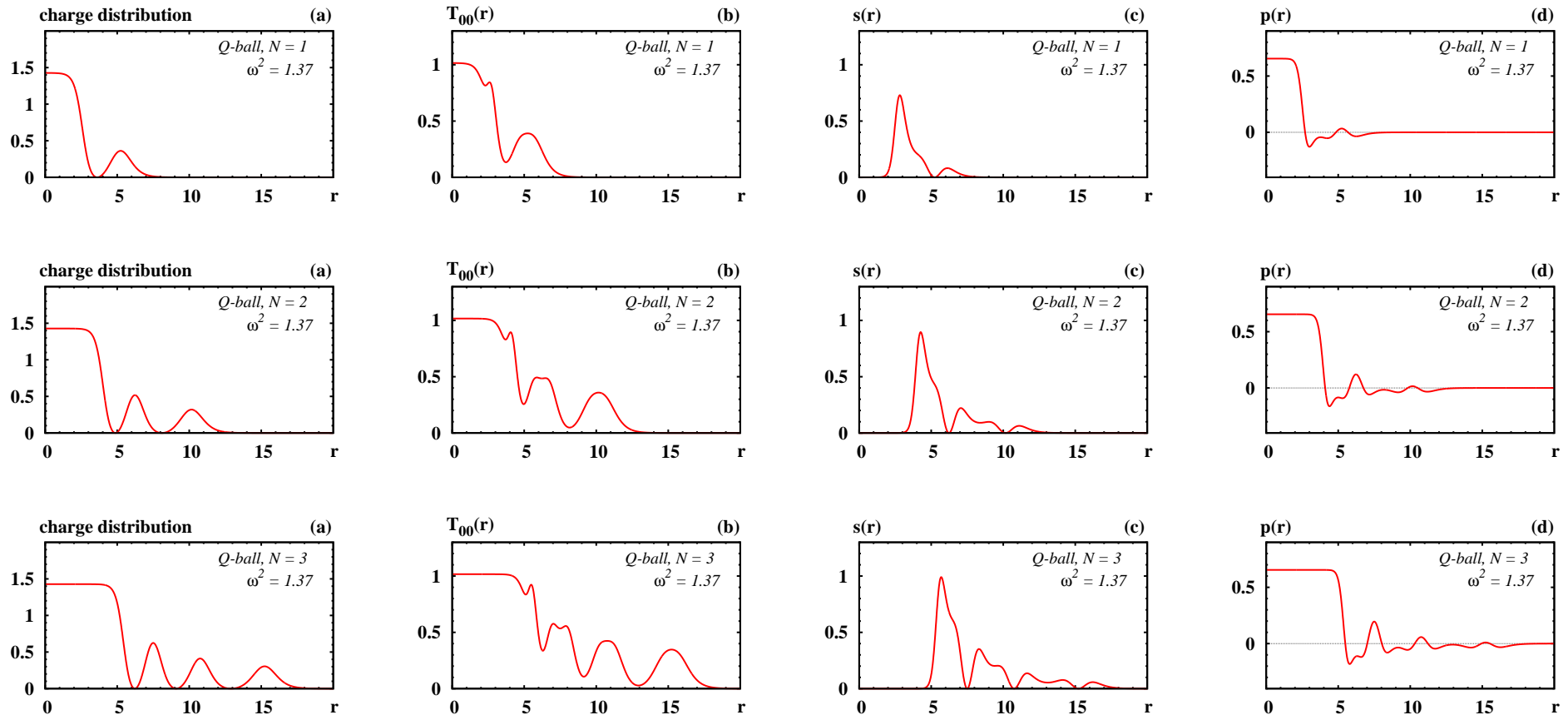
$N = 0$ ground state,

$N = 1$ first excited state, etc

Mai, PS PRD86 (2012) 096002

charge density and $T_{00}(r)$ exhibit N shells

$p(r)$ exhibits $(2N + 1)$ zeros, $s(r)$ exhibits $(N + 1)$ peakses (+ sub-peakes)



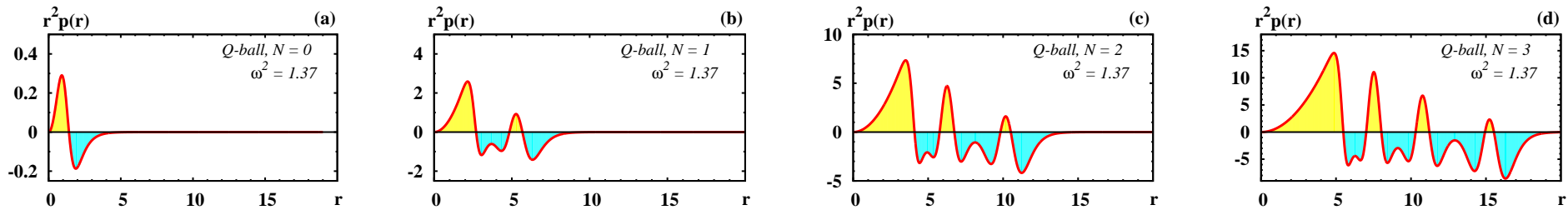
- more lessons from Q -balls

$N > 0$ radial excitations all unstable

→ decay to ground state Q -balls of smaller total energy and same total charge

nevertheless $\int_0^\infty dr r^2 p(r) = 0$ always valid

→ necessary **but not sufficient** condition (local extremum of action, not global)



D -term always negative!

→ is it a theorem?

→ rigorous proof for Q -balls formulated [Mai, PS PRD86 \(2012\) 076001](#)

→ rigorous proof that $d_1 < 0$ for hadrons in QCD still awaiting

all D -terms so far negative (pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q -balls, Q -clouds*)

* what is a **Q -cloud?**

weirdest system:

extreme instability(!)

parametric limit of Q -balls

dissociation in free quanta

$\phi(r) \sim \epsilon \exp(-\epsilon r)$, size $\sim 1/\epsilon$

charge density $\sim \epsilon^2$, pressure $\sim \epsilon^4$

but $Q \sim 1/\epsilon$, mass $\sim 1/\epsilon$, D -term $\sim 1/\epsilon^2$

could this system exhibit a positive D -term?

highly singular limit

but can be controlled!

limiting solution exists

$\lim_{\epsilon \rightarrow 0} (\epsilon^2 d_1) = \text{finite}$

D -term remains negative!

[Cantara, Mai, PS, NPA 953 \(2016\) 1](#)

- **Application I: investigating forces**

prominent property of proton:
life time $\tau_{\text{prot}} > 2.1 \times 10^{29}$ years!

question: how do strong forces
balance to produce stability?

- answer in **model**: strong cancellation of **repulsive forces** due to quark core, and **attractive forces** from pion cloud

- answer in **QCD**: we do not know nice pictures, attractive insights underexplored propaganda(?)

be aware: same for neutron,
 $\tau_{\text{neut}} = 14 \text{ min } 40 \text{ sec} \gg 10^{-23} \text{ sec}$
and even the same picture for Δ ...
 $\tau_{\Delta} \sim 10^{-23} \text{ sec} \rightarrow$ necessary condition!

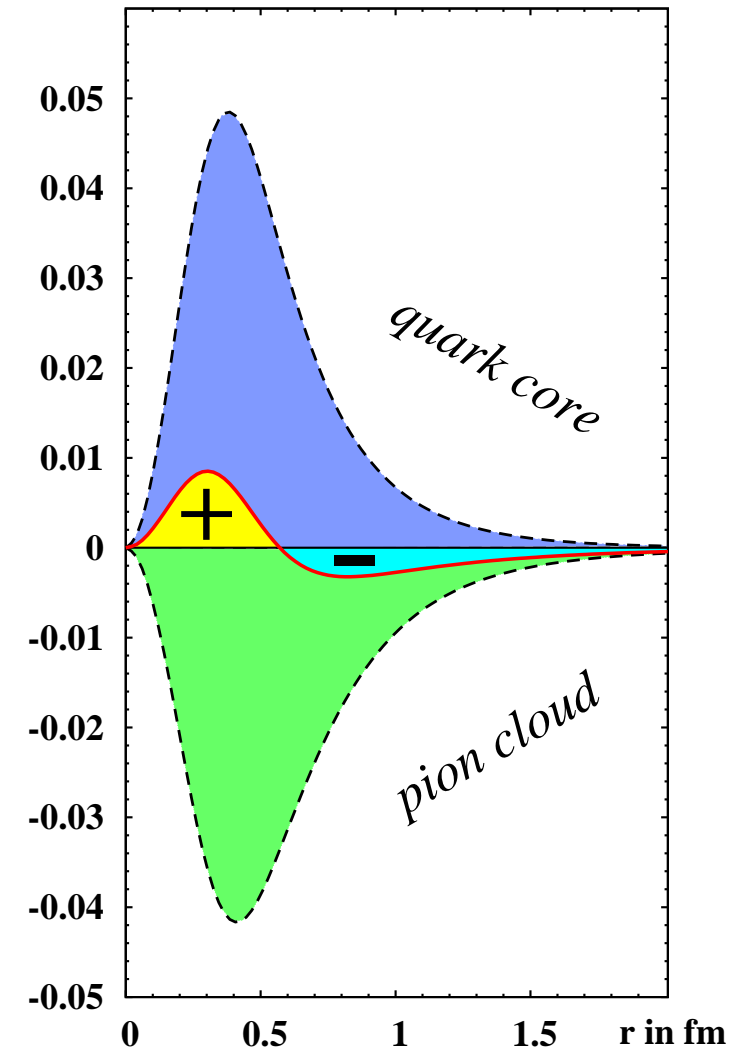
- as mental support for GPD program: okay

... *but is there any practical use of that?*

answer before: *not really* ...

answer today: **Yes!**

$r^2 p(r)$ in GeV fm^{-1}



in chiral quark soliton model

chiral symmetry breaking ✓

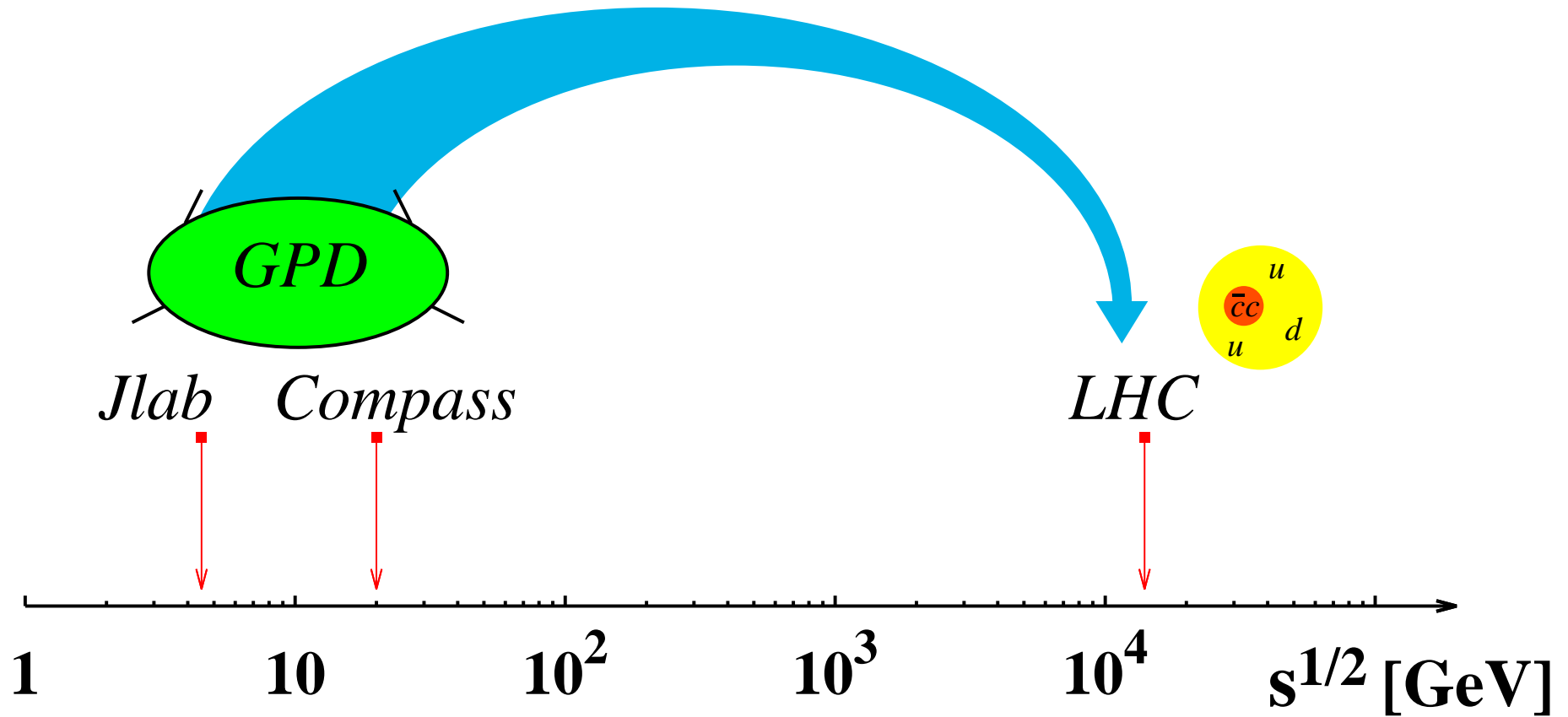
realization of QCD in large- N_c ✓

built on instanton vacuum calculus ✓

not bad, but after all a model ...

Goeke et al, PRD75 (2007)

- **Application II:** amazing!



from hard-exclusive reactions at JLab, COMPASS ...

... to spectroscopy of $\bar{c}c$ -pentaquarks at LHCb

not usual hadrons, not just any exotic hadron

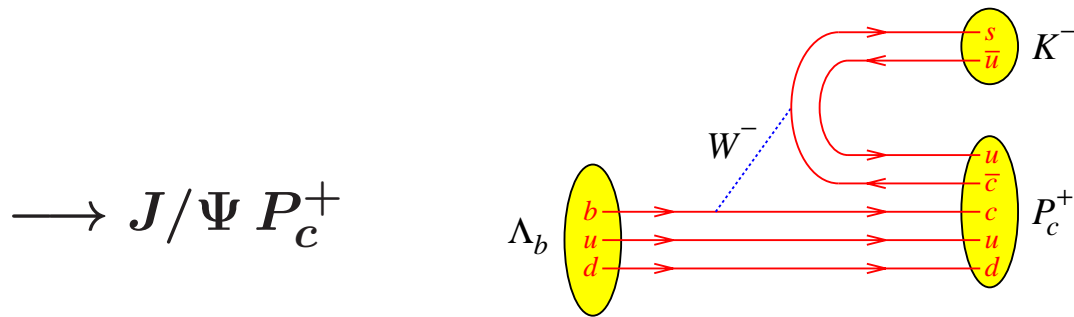
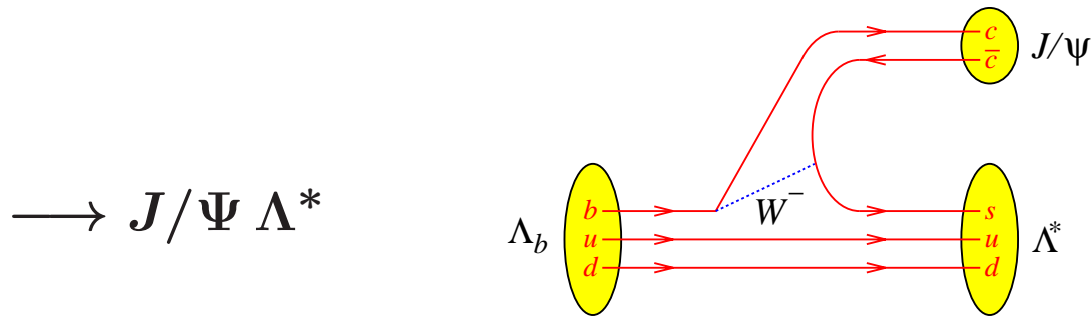
only $\bar{c}c$ -baryon bound states → rich enough!

- discovery of charmonium pentaquarks in Λ_b^0 decays at LHCb

Aaij et al. PRL 115, 072001 (2015)

$\Lambda_b^0 \longrightarrow J/\Psi p K^-$ seen

Λ_b^0 $m = 5.6 \text{ GeV}, \tau = 1.5 \text{ ps}$
 J/Ψ $m = 3.1 \text{ GeV}, \Gamma = 93 \text{ keV}, \Gamma_{\mu^+\mu^-} = 6 \%$
 Λ^* $m = 1.4 \text{ GeV or more}, \Lambda^* \rightarrow K^- p \text{ in } 10^{-23} \text{ s}$



state	m [MeV]	Γ [MeV]	Γ_{rel}	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1) \%$	$J/\psi p$	$\frac{3}{2}^-$ or $\frac{5}{2}^+$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2) \%$	$J/\psi p$	$\frac{3}{2}^{\pm}$ or $\frac{3}{2}^-$

Appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- theoretical approach**

$R_{J/\psi} \ll R_N \Rightarrow$ non-relativistic multipole expansion [Gottfried, PRL 40 \(1978\) 598](#)
 baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

$$V_{\text{eff}} = -\frac{1}{2} \alpha \vec{E}^2 \quad \text{Voloshin, Yad. Fiz. 36, 247 (1982)}$$

- chromoelectric polarizability**

$$\begin{aligned} \alpha(1S) &\approx 0.2 \text{ GeV}^{-3} \text{ (pert),} \\ \alpha(2S) &\approx 12 \text{ GeV}^{-3} \text{ (pert),} \\ \alpha(2S \rightarrow 1S) &\approx \begin{cases} -0.6 \text{ GeV}^{-3} \text{ (pert),} \\ \pm 2 \text{ GeV}^{-3} \text{ (pheno),} \end{cases} \end{aligned}$$

in heavy quark mass limit & large- N_c limit
 \rightsquigarrow “perturbative result” [Peskin, NPB 156 \(1979\) 365](#)

value for $2S \rightarrow 1S$ transition from
 phenomenological analysis of $\psi' \rightarrow J/\psi \pi \pi$ data
[Voloshin, Prog. Part. Nucl. Phys. 61 \(2008\) 455](#)

- chromoelectric field strength:**

$$\vec{E}^2 = g^2 \left(\frac{8\pi^2}{bg^2} T^\mu{}_\mu + T_{00}^G \right)$$

$b = \frac{11}{3} N_c - \frac{2}{3} N_F$ leading coeff. of β -function
 g = strong coupling at low (nucleon) scale $\lesssim 1 \text{ GeV}$
 g_s = strong coupling at scale of heavy quark ($g_s \neq g$)
 $T_{00}^G = \xi T_{00}$ with ξ = fractional contributions of gluon to M_N
 $T^\mu{}_\mu = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r} \frac{r^j}{r} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$

- universal effective potential**

$$V_{\text{eff}} = -\frac{1}{2} \alpha \frac{8\pi^2}{b} \frac{g^2}{g_s^2} \left[\nu T_{00}(r) + 3p(r) \right], \quad \nu = 1 + \xi_s \frac{b g_s^2}{8\pi^2}$$

$\nu \approx 1.5$ estimate by [Eides et al, op. cit.](#)
[Novikov & Shifman, Z.Phys.C8, 43 \(1981\);](#)
[X. D. Ji, Phys. Rev. Lett. 74, 1071 \(1995\)](#)

- **in future GPDs can help:** GPDs \Rightarrow EMT form factors \Rightarrow EMT densities \Rightarrow universal potential V_{eff} for quarkonium-baryon interaction!
- **currently:** use e.g. chiral quark soliton model (done in [Eides et al, op. cit.](#))
- **compute quarkonium-nucleon bound state**

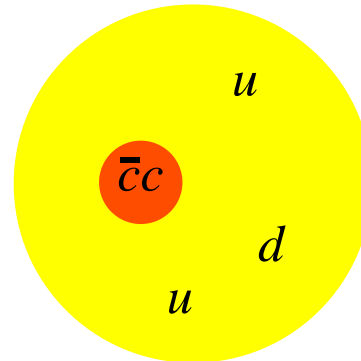
$$\text{solve } \left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\text{eff}}(r) \right) \psi = E_{\text{bind}} \psi$$

$\mu =$ reduced quarkonium-baryon mass

- **results:**

nucleon and J/ψ form no bound state

nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV in $L = 0$ channel, $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ if $\alpha(2S) \approx 17 \text{ GeV}^{-3}$ (consistent with guideline from pert. calc.)



- **decay**

cannot decay directly to $\psi(2S)$ and nucleon, as $M_{\psi(2S)} + M_N > 4450 \text{ MeV}$

instead transition $(2S) \rightarrow (1S)$ governed by the same V_{eff}
but with small $\alpha(2S \rightarrow 1S)$ transition polarizability
 \Rightarrow it “takes time”

after the transition is “completed,”
prompt decay to $J/\psi + \text{nucleon}$
(observed final states)

estimated width is tens of MeV \rightarrow compatible with data!

- based on χ QSM (Eides et al)
model-dependent? Need to confirm in different models!
(Perevalova, Polyakov, PS, in progress)

- **Skyrme model** (Cebulla et al 2007)

incorporates chiral symmetry & also soliton

however, different model, different way to realize stability (Skyrme term)

ideal to provide an independent cross-check

- result:

same conclusions! Confirms all details!

insensitive to model parameters, insensitive to $1/N_c$ corrections

⇒ very **robust predictions** for mass and decay width of $P_c^+(4450)$

- new prediction:

also **Δ and $\psi(2S)$ form a bound state!**

isospin $\frac{3}{2}$, mass = 4.5 GeV, $\Gamma_{\Delta\bar{c}c} \sim 60$ MeV

positive parity, spin $|\frac{3}{2} - 1| \leq J \leq \frac{3}{2} + 1$

(states degenerate in heavy quark limit)

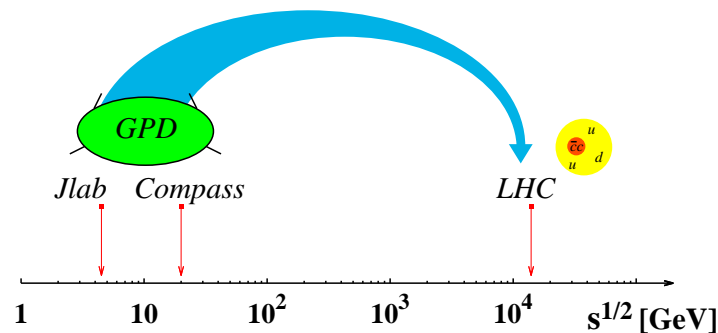
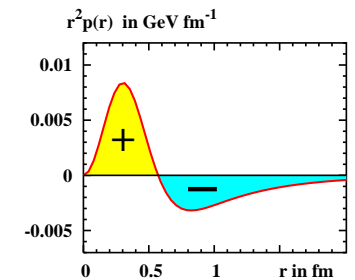
decay $P_c(4500) \rightarrow J/\psi + \underbrace{\text{nucleon} + \text{pion}}_{\Delta\text{-resonance}}$

- what about $P_c^+(4380)$?

broader, more possibilities, under investigation

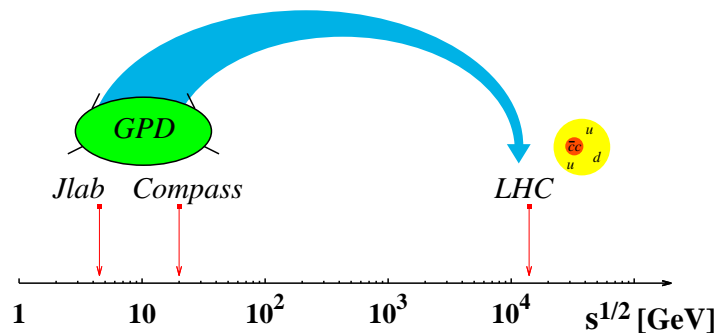
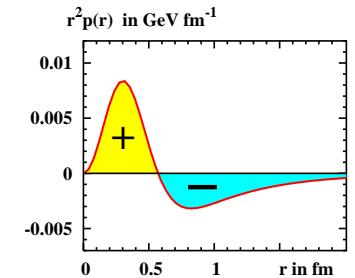
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mass decomposition, spin decomposition, and *D*-term!
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Thank you!