SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
FROM QUARKS AND GLUONS TO HADRONS AND NUCLEI

PRISMA

JGU JOHANNES GUTENBERG UNIVERSITÄT MAINZ

# Duality sum rules in forward Compton scattering and the proton radius puzzle

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# Thanks to my collaborators:

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T. Londergan (Indiana U.)
T. Hobbs (U. Washington)


MG, Hobbs, Londergan, Szczepaniak, Phys.Rev. C84 (2011) 065202

MG, Llanes-Estrada, Szczepaniak, Phys.Rev. A87 (2013) 052501, [arXiv:1302.2807]

Carlson, MG, Vanderhaeghen, Phys.Rev. A89 (2014) 022504, [arXiv:1311.6512]

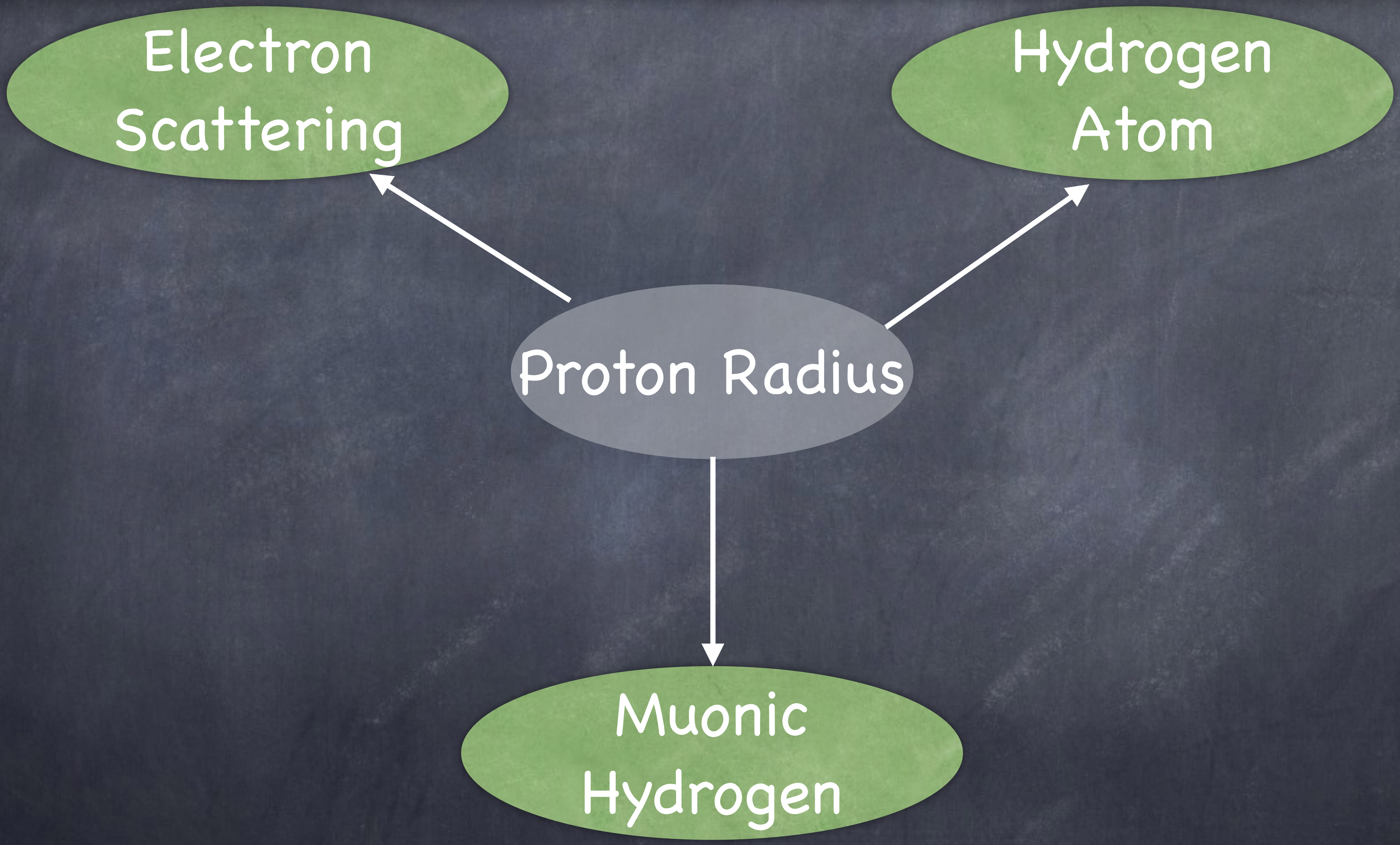
MG, Phys.Rev. C90 (2014) 052201, [arXiv:1406.1612]

MG, Phys.Rev.Lett 115 (2015) 222503, [arXiv:1508.02509]

As a motivation

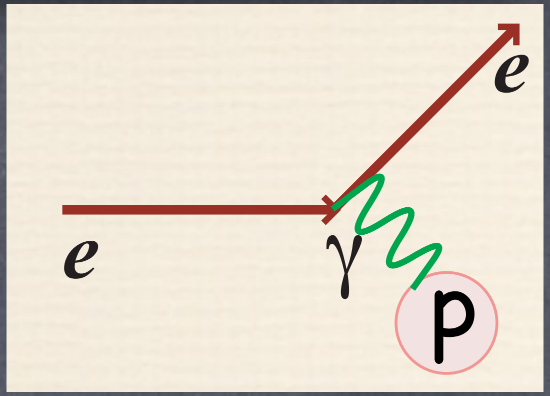
# Proton Radius Puzzle: the Status

# Proton radius puzzle



# Elastic Electron Scattering

Unpolarized cross section




$$\left(\frac{d\sigma}{d\Omega}\right)^{unpol} = \sigma_{\mathrm{Mott}} \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon(1+\tau)}$$

Momentum Transfer $Q^2$ -> $\tau = Q^2/(4M^2)$

Energy E -> $\epsilon$: $0 < \epsilon < 1$ for $E_{min} < E < \infty$

$G_{E,M}(Q^2)$ – electric and magnetic form factors

FFs encode charge, magnetic moment, RMS radii, ...

$$G_E(Q^2) = 1 - (1/6)\, R_{Ch}^2\, Q^2 + \ldots$$

$$G_M(Q^2) = \mu_p[1 - (1/6)\, R_M^2\, Q^2 + \ldots]$$

# Proton Radius from e-scattering

Measure cross section down to low $Q^2$

$$\frac{d\sigma^{exp}}{d\Omega} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\Bigg|_{Q^2\to 0} = 1 + Q^2\left[\frac{\mu_p^2 - 1}{4M^2} - \frac{1}{3}R_{Ch}^2\right] + \ldots$$

The radius is defined as the slope of the FF at origin, data are at finite $Q^2$: extrapolation is unavoidable

How low in $Q^2$ should/can one go?
up to now $Q_{min}^2 = 4 \times 10^{-3}$ GeV$^2$

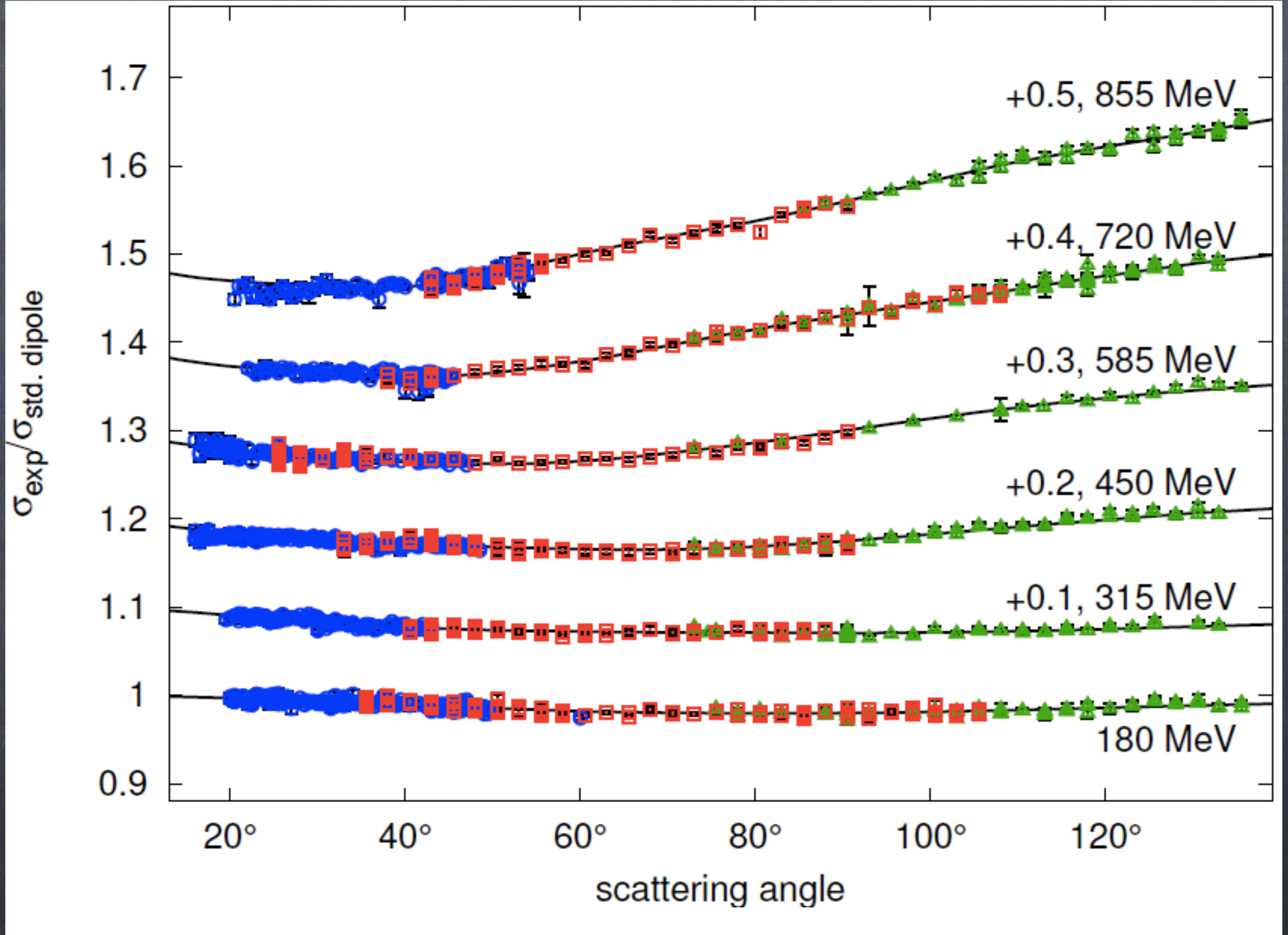
1% uncertainty in $R_{Ch}$ - measure 1 to few x $10^{-4}$ precision!

# Proton Radius from e-scattering

A1 @ MAMI

$R_{Ch} = 0.879(8)$

Bernauer et al., '10




Bernauer et al. (2010)

# Proton Radius from e-scattering

- Individual data points - per cent level accuracy;
- Need large angle coverage to extract the radius to 1%
- Large statistics serves as a lever arm for extracting "1" to 0.05% precision;
- Higher $Q^2$ data influence the extracted radius
- The lower in $Q^2$ one goes, the lesser are higher order terms important - plans with ISR @ Mainz, PRad @ JLab, $Q^2 \geq 10^{-4}$ GeV$^2$

# Proton Radius from e-scattering

- Bernauer et al.: used full statistics (low and moderate $Q^2$)
  studied systematics due to different fit functions
  (polynomial, splines, dipole, double dipole etc.)
  $\chi^2$ close to 1 with 1400 d.o.f.

$R_E^p$ = 0.879(8) fm

- Lorenz '12,13: Dispersion relation fit $G_{E,M}(Q^2) = \int_{4m_\pi^2}^{\infty} \frac{dt\, \rho_{E,M}(t)}{t + Q^2}$

Model of the spectral function: 2π continuum + VDM + QCD asymptotics
Radius mainly sensitive to the lowest states (2π, 3π) which are taken as exact -> fit function might not be flexible enough, $\chi^2$ > 1.1
Consistent with previous DR fits (Höhler '76, Mergell '96, ...)

$R_E^p$ = 0.84(1) fm

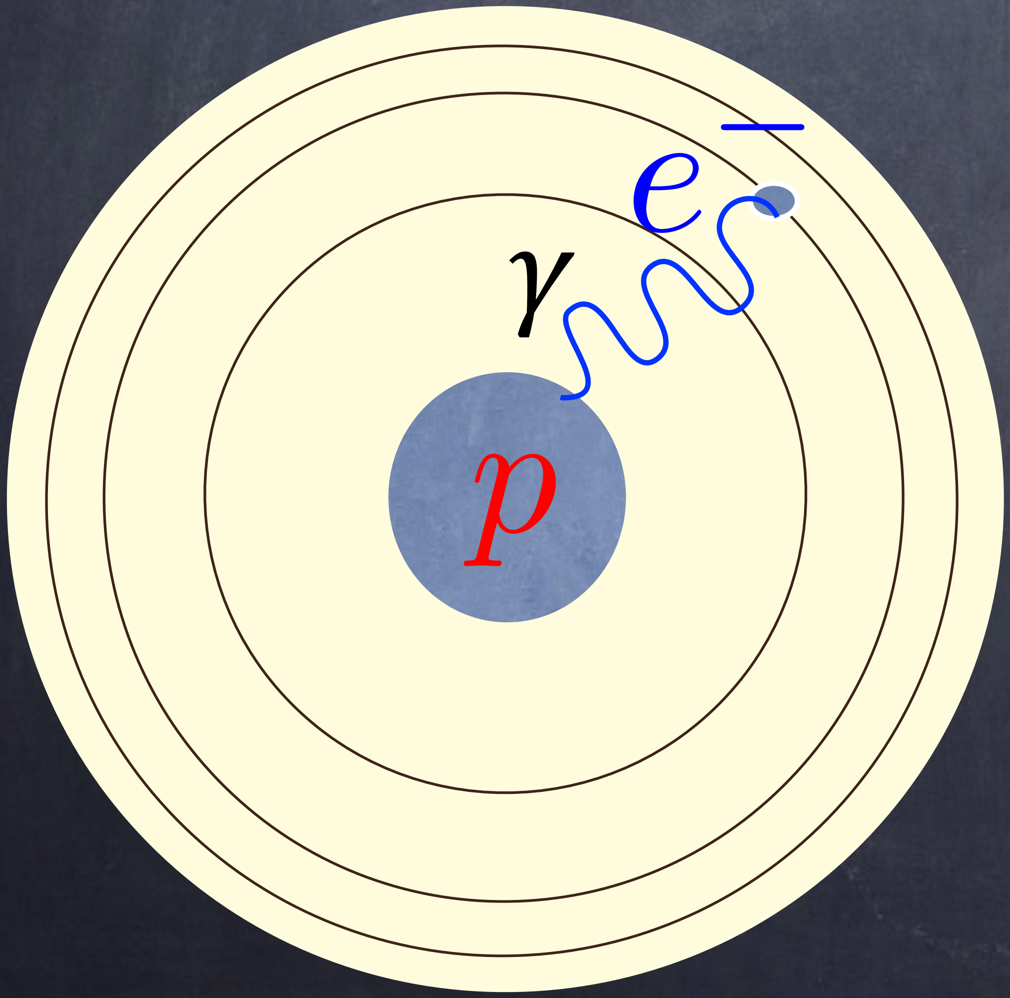
- Hill, Paz '10: Conformal mapping + Fourier series for the spectral fn.

$R_E^p$ = 0.87(2) fm

Data tend to larger radii; Need extra input to get smaller radii

# $R_E^p$ from Lamb Shift in Hydrogen

No extrapolation problem in atoms;
typical momentum transfer in H-atom:
keV$^2$ in e-H, MeV$^2$ $\mu$-H




Electrons occupy stationary orbits

Energy levels $E_{NL}$

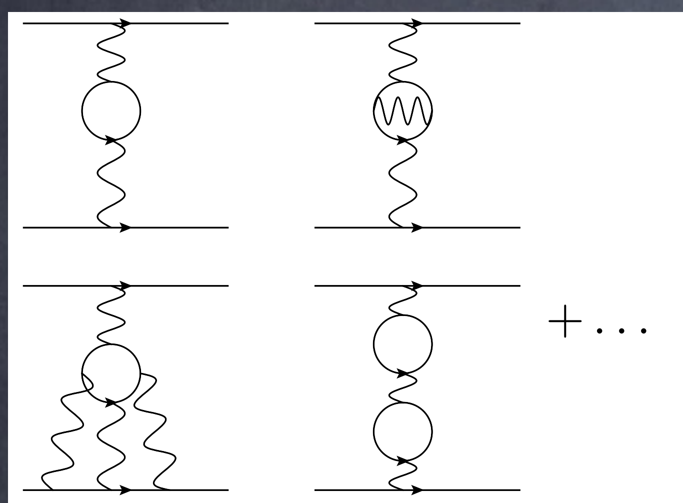
Principal (energy) Q.N.: N=1,2,3...;
Orbital momentum Q.N.: L=S,P,D...;

If only one photon were exchanged:

$$E_{2S} = E_{2P}$$

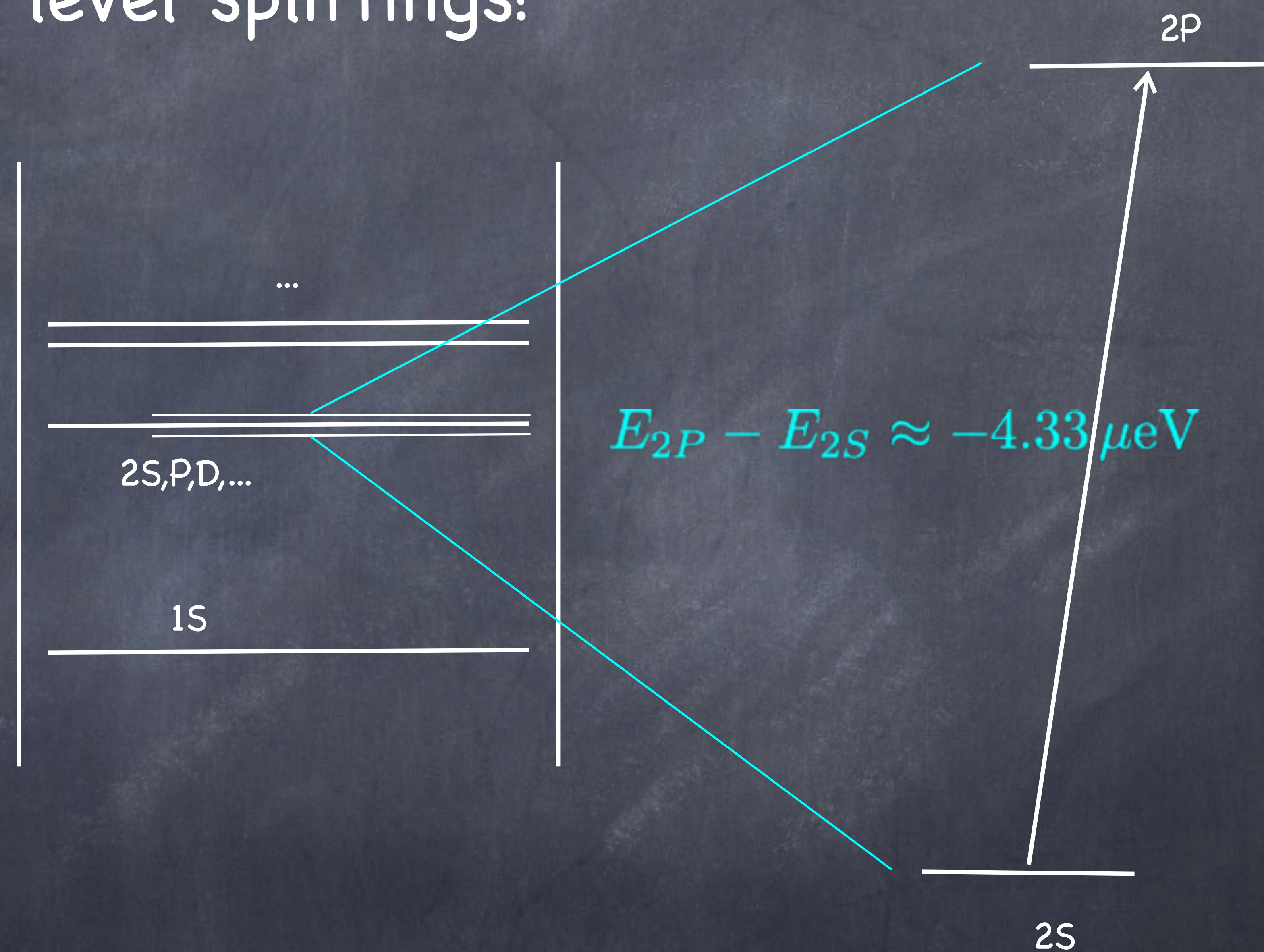
# $R_E^p$ from Lamb Shift in Hydrogen

Radiative corrections: level splittings!




$$E_{2S} - E_{1S} \approx 10.2\,\text{eV}$$

$$E_{1S} \approx -13.6\,\text{eV} = -hc\,R_\infty$$




$$E_{2P} - E_{2S} \approx -4.33\,\mu\text{eV}$$

nS-nP splitting (Lamb shift) - authentic prediction of SM (QED)

Precise calculations of QED corrections: p.p.m. level precision

# $R_E^p$ from Lamb Shift in Hydrogen

- The proton is not a point-like charge - has a finite size - Lamb shift is sensitive to the proton radius

$$\Delta E_{nP-nS} = \Delta E^{QED}_{nP-nS} - \frac{2(Z\alpha)^4}{3n^3} m_r^3 R_E^2 + \mathcal{O}(\alpha_{em}^5)$$

- few p.p.m. correction
  - exceeds the QED precision
  - can be extracted

$$E_{2S} - E_{2P} = 33.7808(1)\,\mu\text{eV} + 0.0008 {R_E^p}^2\,\mu\text{eV}$$

QED                Finite Size

# $R_E^p$ from Lamb Shift in Hydrogen

CODATA

$R_{Ch}$ = 0.8779(94) fm

e-scattering

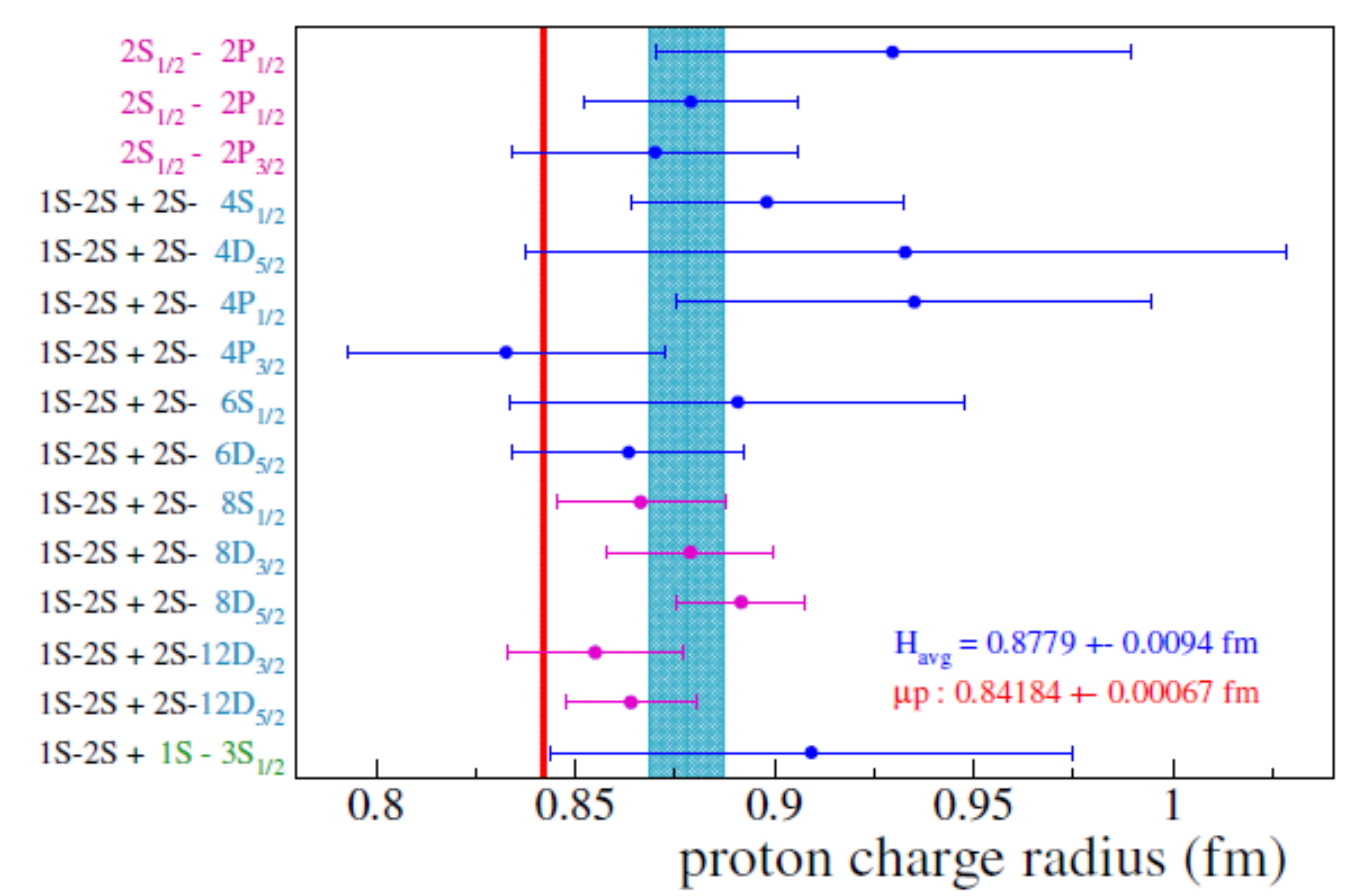
$R_{Ch}$ = 0.879(8) fm

Combined

$R_{Ch}$ = 0.8775(51) fm

$\mu$H data @ PSI

$R_E^p = 0.84087(39)$ fm




Pohl et al [CREMA Coll.] '10, Antognini et al. '13

4% discrepancy for $R_{Ch}$ (0.6% precision from e-p) - 7$\sigma$ away!

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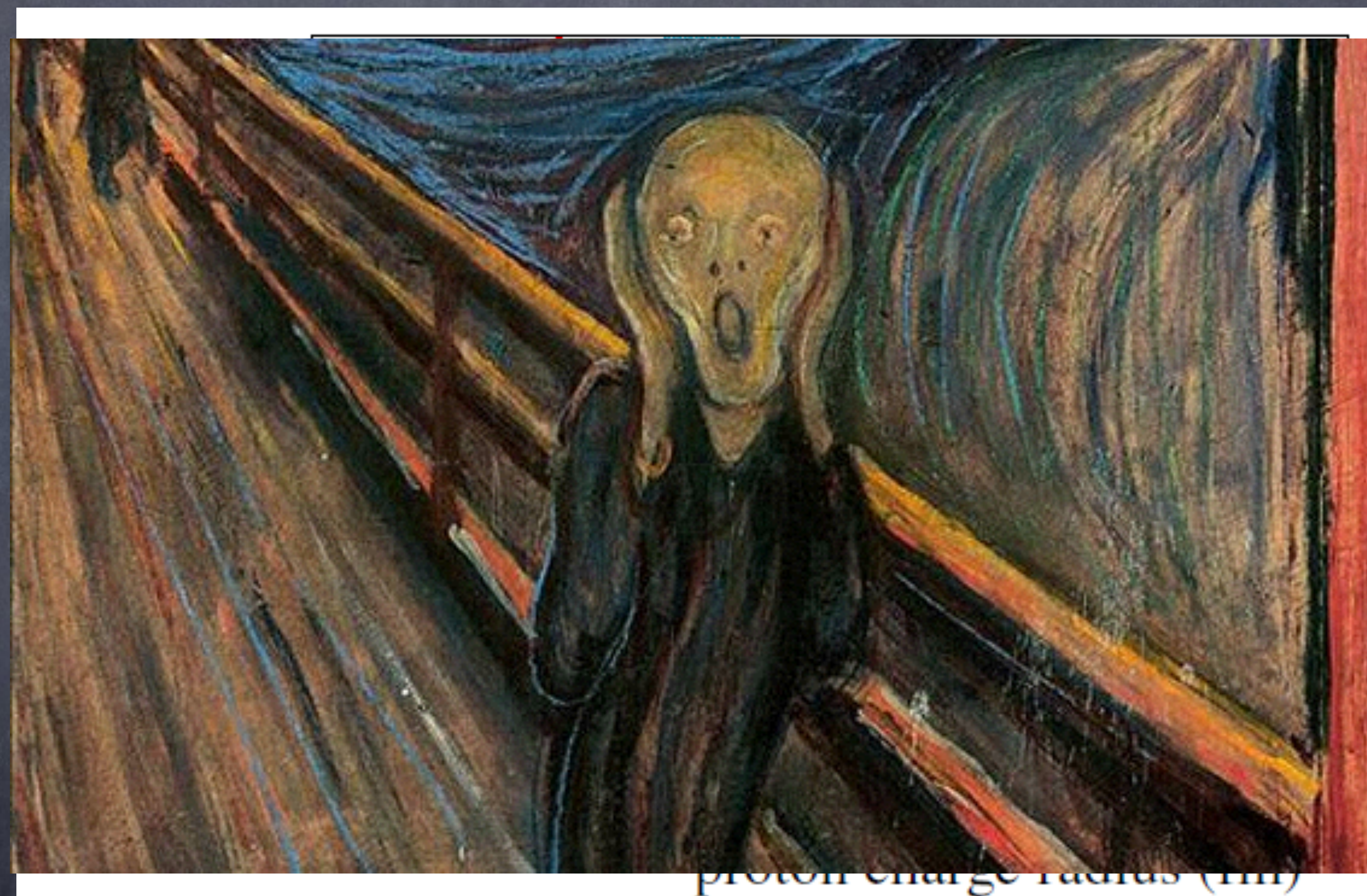

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4% discrepancy for $R_{Ch}$ (0.6% precision from e-p) - $7\sigma$ away!

# $R_E^p$ from e-H

Almost all individual e-H points are within 1.5$\sigma$ from the muonic point

BUT they all lie systematically at larger radii - correlated systematics?

All QED corrections have been studied up to $\alpha^6$ - under control

Electron scattering is the most precise single measurement and is in nice agreement with the statistical average of the e-H data.

Most of the measurements are old - may be a good idea to remeasure

New experiments with projected 1% radius extraction - under way:

2S-2P measurement - York U. (Canada);

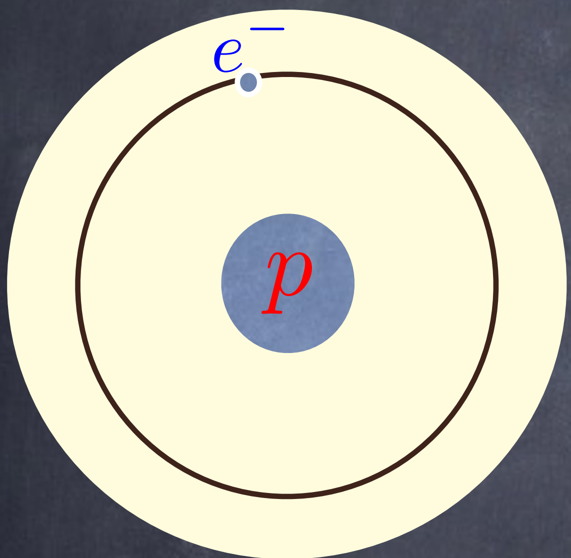
2S-4S measurement - MPI Garching;

1S-3S measurement - Laboratoire Kastler Brossel (Paris);

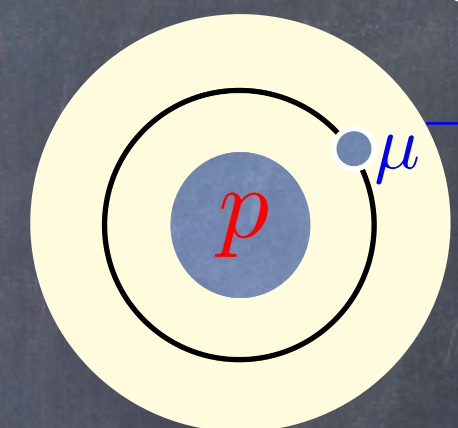
# What's special about μ–H?

QED: the only difference is the mass $m_\mu \approx 200\, m_e$

Hydrogen atom




muonic Hydrogen




Bohr radius

$$R_B \sim \frac{1}{\alpha m_r}$$

Fine structure constant $\alpha \approx 1/137$

Reduced lepton-proton mass $m_r = \dfrac{mM}{m+M}$

Finite size Lamb shift: $\Delta E^{R_E^p}_{2P-2S} \propto \alpha^4 m_r^3$

$$\Delta E^{eH}_{2P-2S} = -8.1 \times 10^{-7}\, R_E^2\, \mathrm{meV} \qquad \Delta E^{\mu H}_{2P-2S} = -5.2275(10)\, R_E^2\, \mathrm{meV}$$

μH unstable ($\tau_{2S}$ ~ μs) - 7 o.o.m. still make it 10 times more precise

# $R_E^p$ from μ-H

Using the proton radius from eH and scattering, expect

$$\left[\Delta E_{2P-2S}^{\text{Measured}} - \Delta E_{2P-2S}^{QED}\right]^{\text{Expected}} \approx -4.0\,\text{meV}$$

Observed splitting - off by 8%, radius off by 4%

$$\left[\Delta E_{2P-2S}^{\text{Measured}} - \Delta E_{2P-2S}^{QED}\right]^{\text{Measured}} \approx -3.7\,\text{meV}$$

What if the μH experiment is wrong?

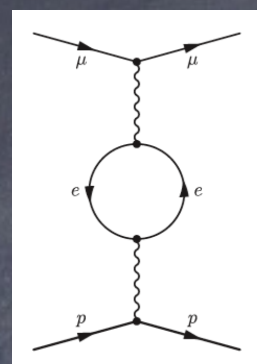
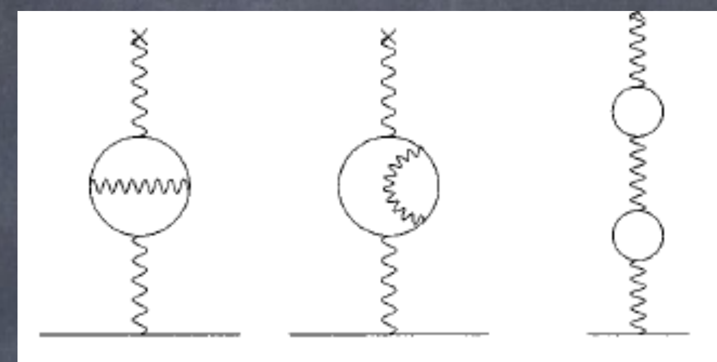
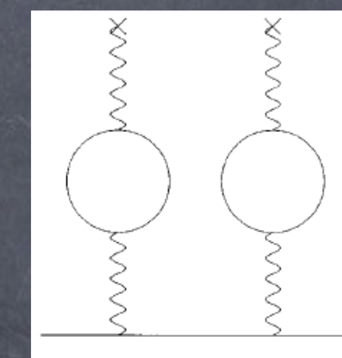
Exp. precision: μeV, much smaller than missing 300 μeV;

Pohl et al. and Antognini et al. measured $2P_{1/2}$ - 2S and $2P_{3/2}$ - 2S transitions, found consistency;

No other facility able to redo the μH experiment exists at the moment.

# What has gone wrong?

QED corrections?

1-loop eVP   2-loop eVP 

$\Delta E$ = 205.0073 meV $\Delta E$ = 1.5081 meV $\Delta E$ = 0.1509 meV

Muon SE + VP $\Delta E$ = -0.6703 meV

QED corrections up to $\alpha^6$ calculated: all < 0.005 meV

Further hadronic structure corrections - start at $(Z\alpha)^5$

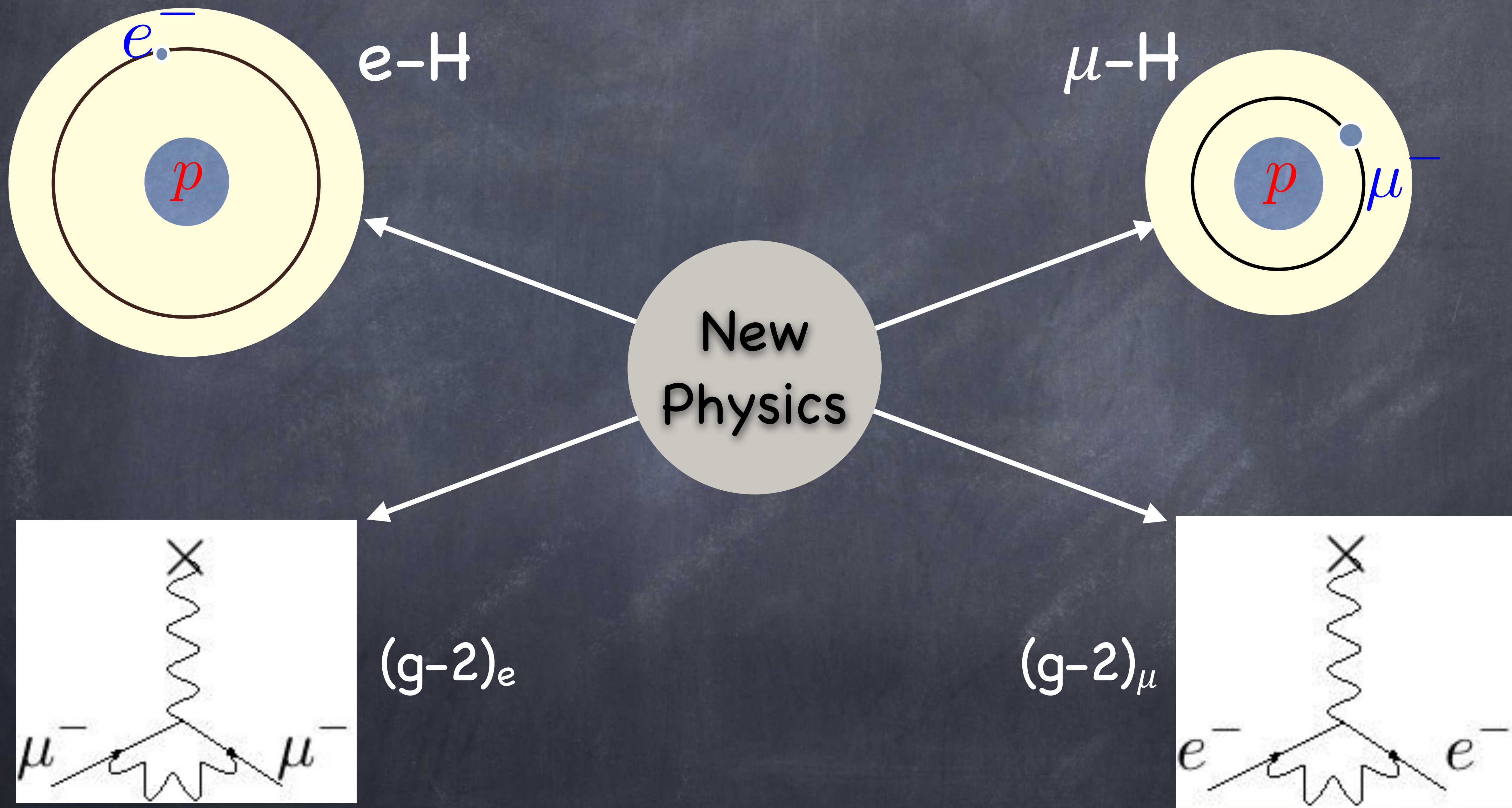
Include the third Zemach radius:

$$\Delta E_{2P-2S}^{\text{Measured}} - \Delta E_{2P-2S}^{QED} = -\frac{(Z\alpha)^4 m_r^3}{12}\left[R_p^2 - \frac{Z\alpha}{2} R_{(2)}^3\right]$$

Correction 0.03 meV - 10 times smaller than the discrepancy

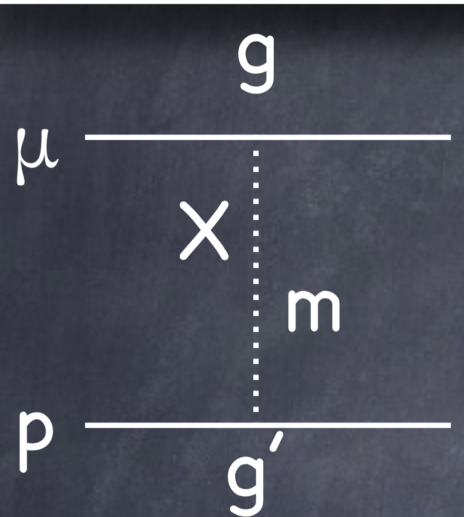
# Proton Radius Puzzle: New Physics?

- Account for all constraints!




Stringent constraints from (g-2)$_e$: substantial $\mu$-e non-universality

# Proton Radius Puzzle: New Physics?

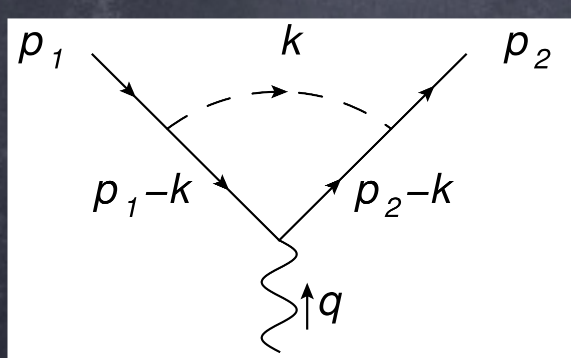



$$\Delta R_E^2 \sim -\frac{6gg'}{4\pi\alpha m^2}$$

Attractive scenario:
scalar exchange would
naturally pick up mass (Yukawa)

Tucker-Smith, Yavin '11; Batell et al, '11;
Brax, Burrage '11; Rislow, Carlson '12, '14; ...

Would contribute to the muon a.m.m.

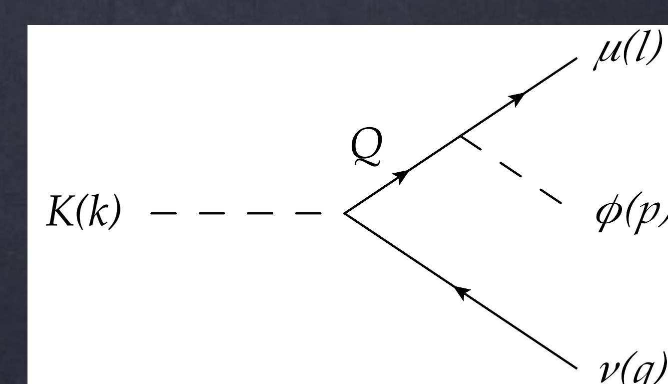



Muon $a_\mu = (g-2)_\mu/2$ has 2 ppm discrepancy

$$a_\mu(\text{data}) = (116\ 592\ 089 \pm 63) \times 10^{-11} \quad [0.5\ \text{ppm}],$$
$$a_\mu(\text{thy.}) = (116\ 591\ 840 \pm 59) \times 10^{-11} \quad [0.5\ \text{ppm}],$$
$$\delta a_\mu = (249 \pm 87) \times 10^{-11} \quad [2.1\ \text{ppm} \pm 0.7\ \text{ppm}].$$

Requires fine-tuned S + PS or V + A exchanges

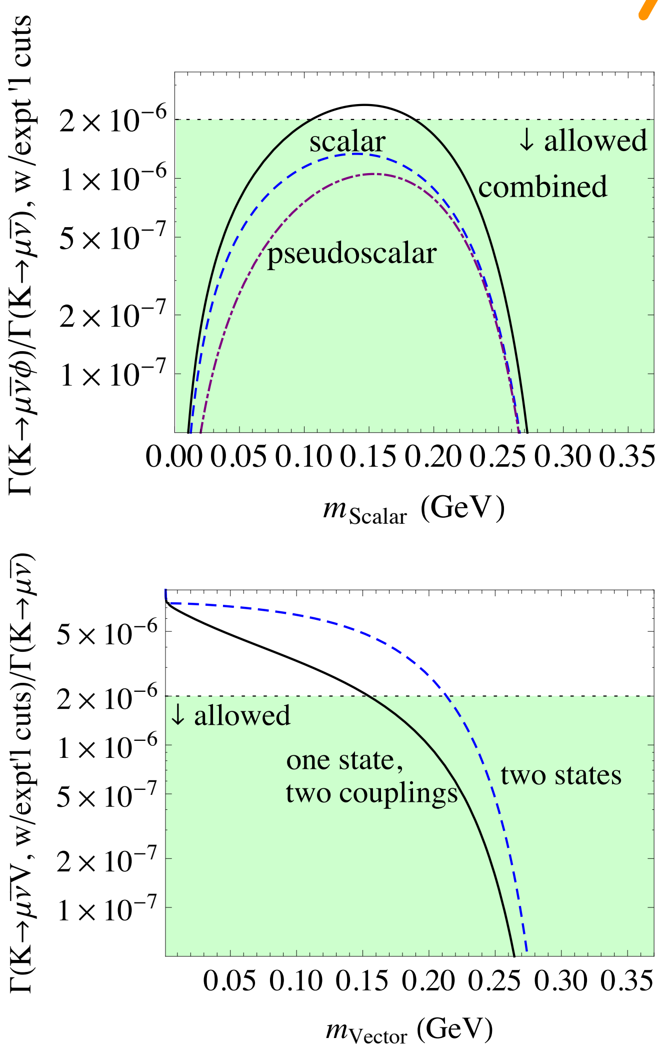
Would contribute to decays K→μ + invisible



# Proton Radius Puzzle: New Physics?

## K-decay constraints

Carlson, Rislow, '12




- Solid line is sum of scalar and pseudoscalar couplings.
- Lower mass or higher mass o.k., but 90-200 MeV excluded.

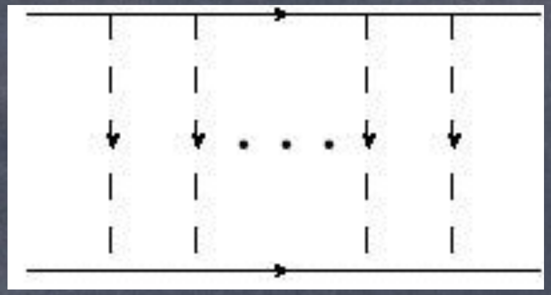
- Same for polar and axial vectors.
- Solid is one particle with both V and A couplings.
- Dashed line is two particles, one polar and one axial vector.
- Lower masses excluded, 160 MeV for PV case, 210 for other case.

Conclusion: BSM explanation possible, requires lepton non-universality, but fine tuned to evade the g-2 constraints

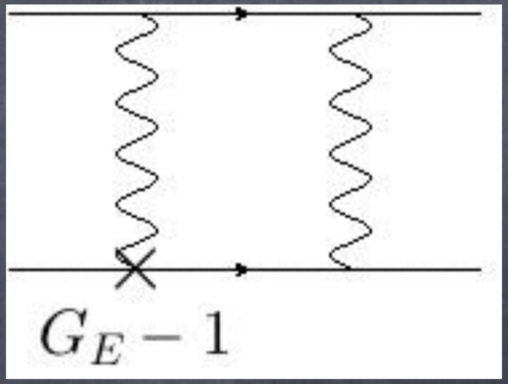
# Further hadronic effects?

Hadronic correction at $(Z\alpha)^5$ - included partially!

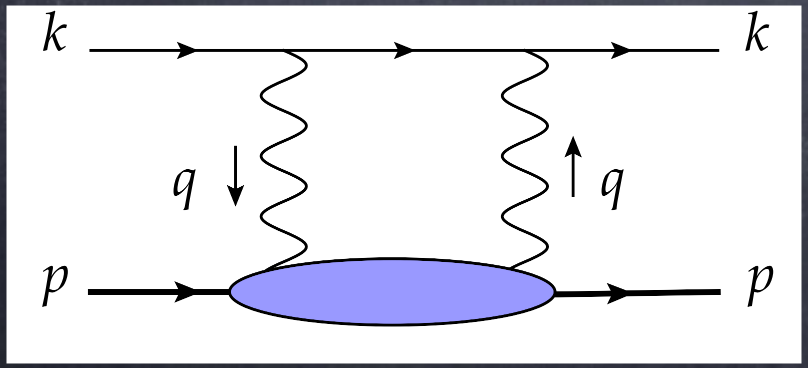
Soft Coulomb:
included in
Schrödinger WF



Hard box:
only part of it
included
(3rd Zemach m.)




Do the full calculation




Blob: forward virtual Compton tensor

$$T_{\mu\nu} = \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p | T\, j_\mu(x) j_\nu(0) | p \rangle$$

$$M_{2\gamma} = e^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4} \bar{u}(k) \left[ \gamma^\nu \frac{1}{\not{k} - \not{q} - m_l + i\epsilon} \gamma^\mu + \gamma^\mu \frac{1}{\not{k} + \not{q} - m_l + i\epsilon} \gamma^\nu \right] u(k)\, T_{\mu\nu}$$

# Polarizability Correction from DR

$$T_{\mu\nu} = \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p | T\, j_\mu(x) j_\nu(0) | p \rangle$$

T-ordered non-local product of two vector currents - complicated!

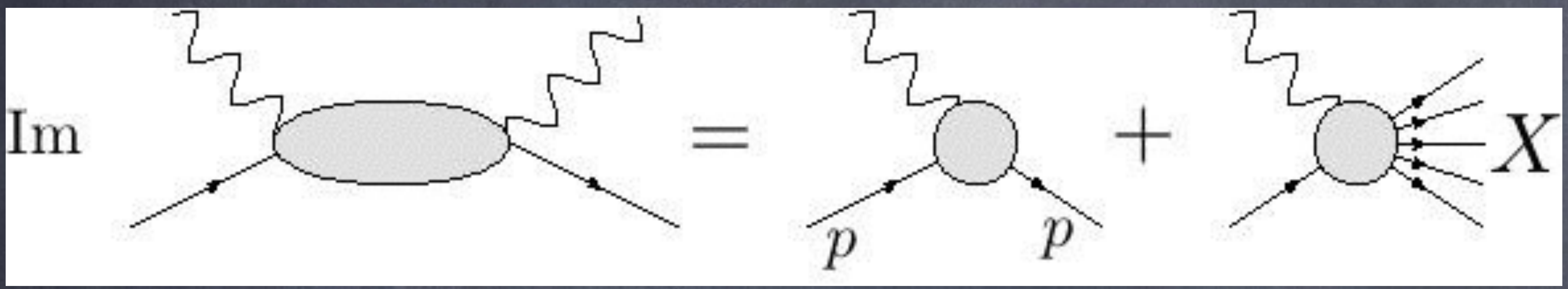
Gauge, Lorentz inv. $T^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{\hat{p}^\mu \hat{p}^\nu}{M^2} T_2(\nu, Q^2)$

(nP - nS) splitting

$$\Delta E = -\frac{\alpha^2}{2\pi m_l M_d} \phi_n^2(0) \int d^4q \frac{(q^2 + 2\nu^2) T_1(\nu, q^2) - (q^2 - \nu^2) T_2(\nu, q^2)}{q^4 [(q^2/2m_l)^2 - \nu^2]}$$

# Polarizability Correction from DR

Optical theorem: absorptive part of $T_{1,2}$ related to data




Form factors

Unpolarized structure functions $F_{1,2}$

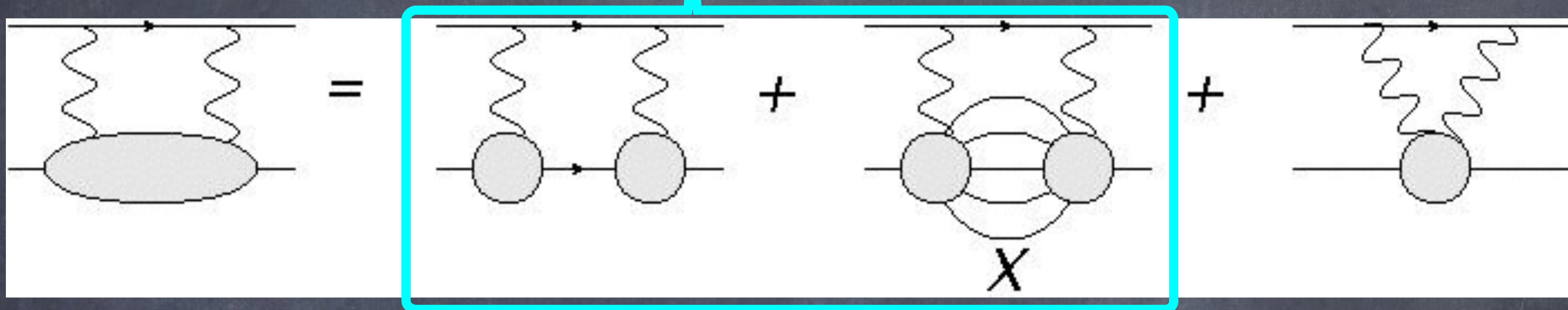
Dispersion relations (subtracted for $T_1$)

$$\mathrm{Re}\, T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{\nu^2}{2\pi M} \mathcal{P} \int_0^\infty d\nu' \frac{F_1(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

$$\mathrm{Re}\, T_2(\nu, Q^2) = \frac{1}{2\pi} \mathcal{P} \int_0^\infty d\nu' \frac{F_2(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

# Polarizability Correction

Dispersion Relation + Data

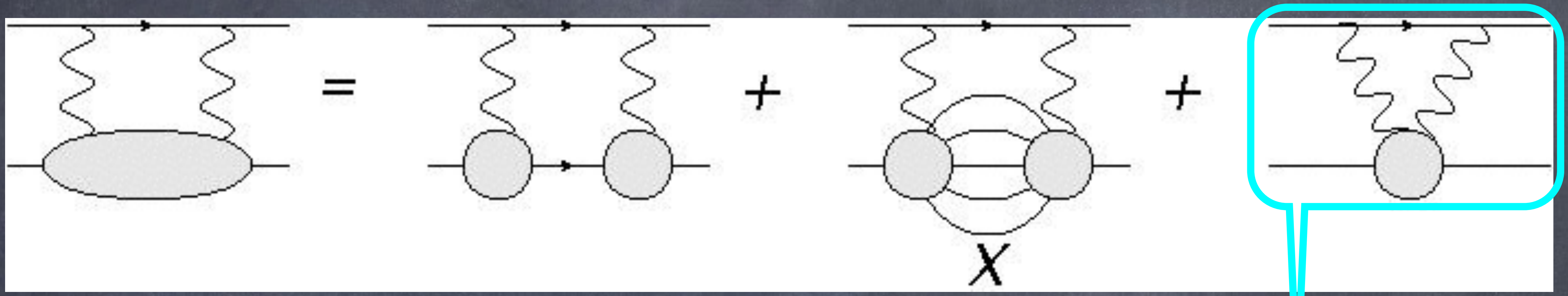



Lamb shift is obtained as

$$\Delta E \sim \alpha_{em}^5 \int_0^\infty dQ^2 \int_0^\infty d\nu \left\{ A(\nu, Q^2)\, F_1 + B(\nu, Q^2)\, F_2 \right\}$$

Good quality data (e.g., JLab) on $F_{1,2}$ $0 < Q^2 < 3$ GeV$^2$, $W < 4$ GeV

# Polarizability Correction



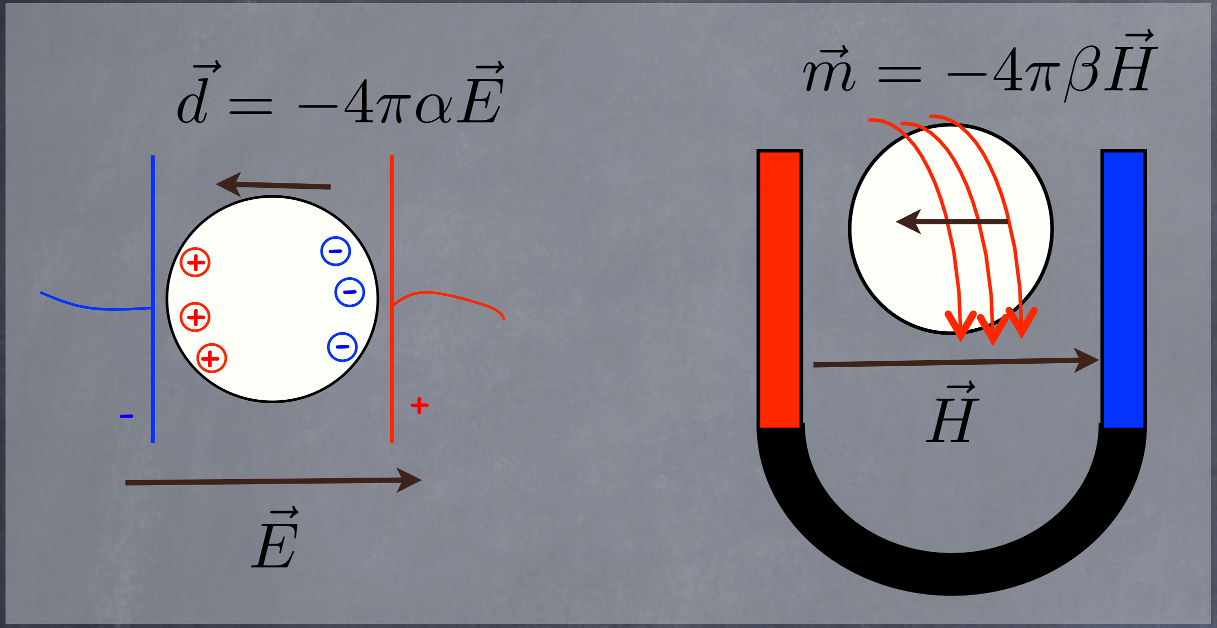

Subtraction function related to proton's magnetic polarizability $\beta_M$

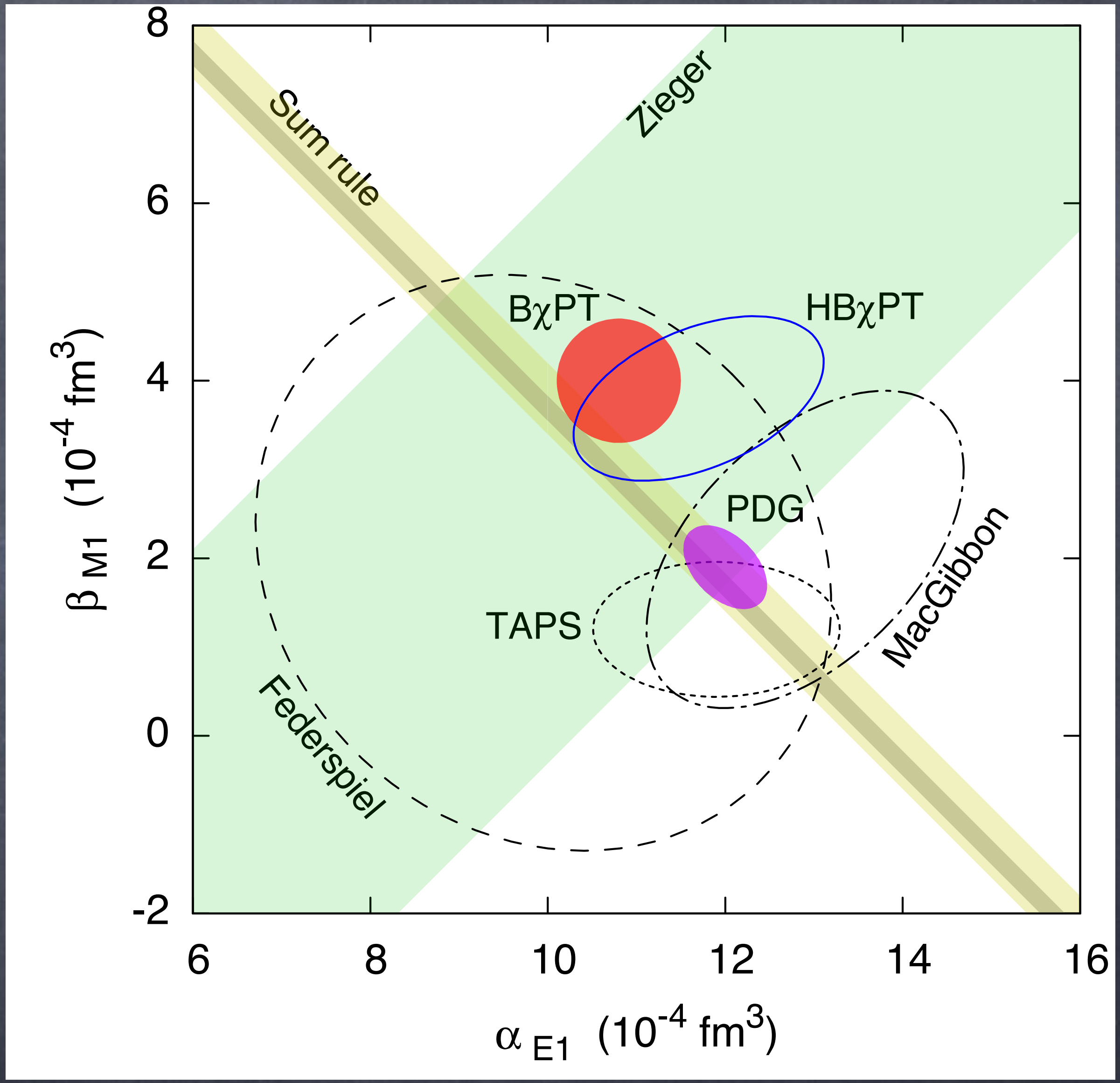
Low-Energy Theorem: $T_1(0, Q^2) = (Q^2/e^2)\,\beta_M$

Lamb shift is obtained as $\Delta E^{Sub} \sim \alpha_{em}^5 \int_0^\infty dQ^2 C(Q^2)\,\beta_M F_\beta(Q^2)$

# Subtraction Constant

Proton (dipole) polarizabilities




PDG 2012

$$\alpha_E = 11.2(0.4) \times 10^{-4}\text{fm}^3$$

$$\beta_M = 2.5(0.4) \times 10^{-4}\text{fm}^3$$

# Total polarizability correction

Different approaches to estimate $F_\beta(Q^2)$

Dipole (like FF): Pachucki, 1996
Pion loops: Vanderhaeghen & Carlson, 2011
HBChPT + dipole: Birse & McGovern, 2012
BChPT: Alarcón, Pascalutsa,Lenski 2014
Finite Energy Sum Rule: MG, Llanes-Estrada, Szczepaniak, 2013

Hadronic structure corrections to proton radius puzzle are constrained

$$\Delta E_{2P-2S} = -40 \pm 5\,\mu\text{eV}$$

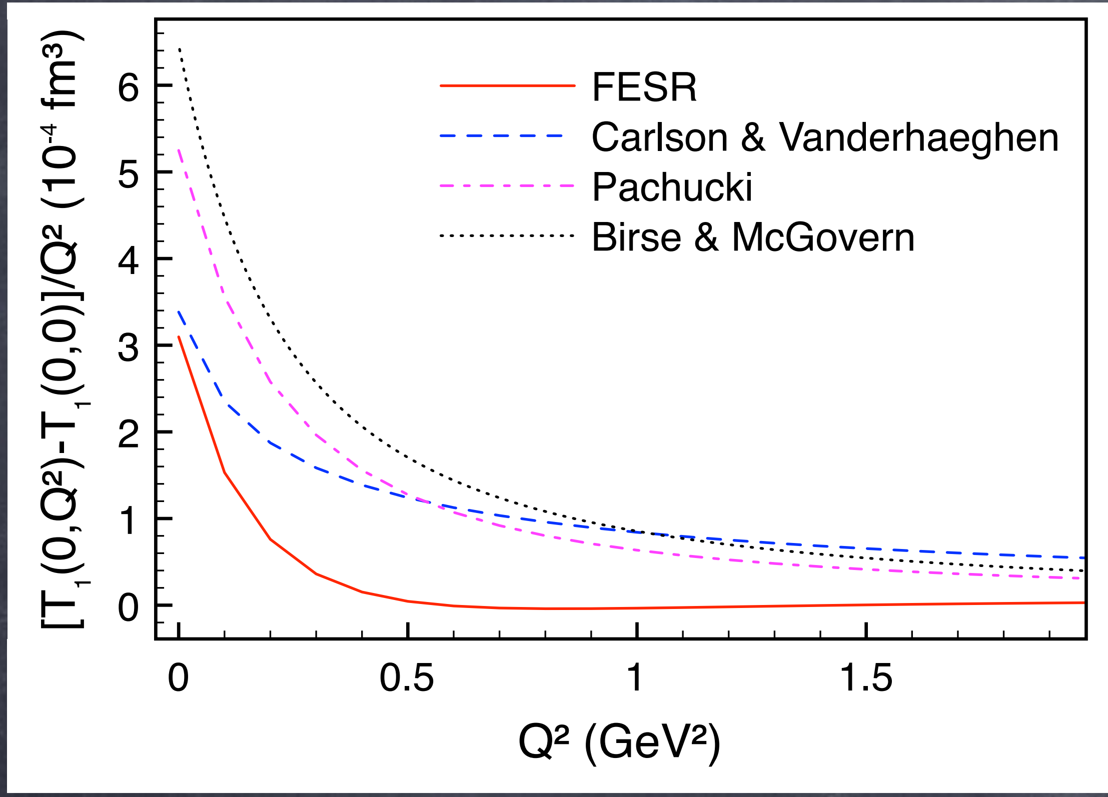
$\updownarrow$

$$\Delta E_{\text{Missing}} \approx -300\,\mu\text{eV}$$

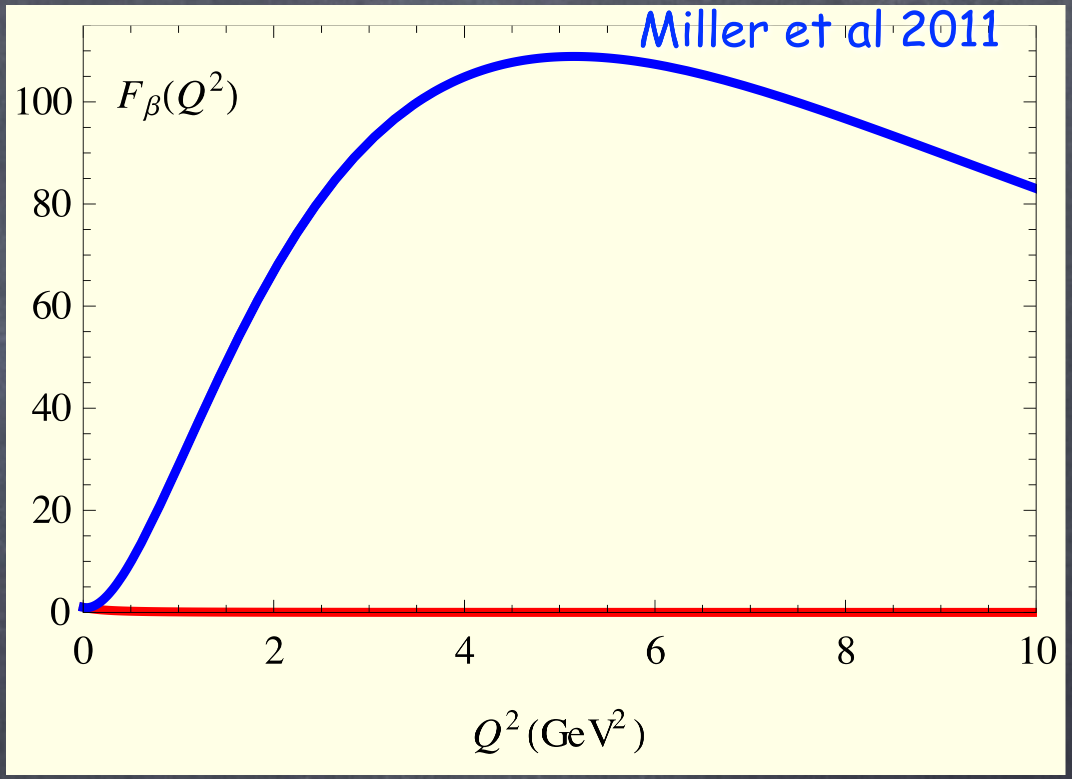
All known constraints built in!

# Exotic Hadronic Contributions?

Reasonable hadronic models

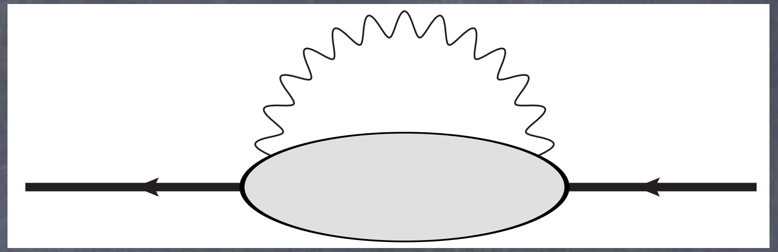



To get ~300 μeV Lamb shift: need something like this



# Exotic Hadronic Contributions?

Cottingham formula (p-n mass difference)



$$M_p - M_n = \frac{\alpha}{2M(2\pi)^3} \int \frac{d^4q}{q^2} \left[T^{p\mu}_{\mu}(\nu, q^2) - T^{n\mu}_{\mu}(\nu, q^2)\right)$$

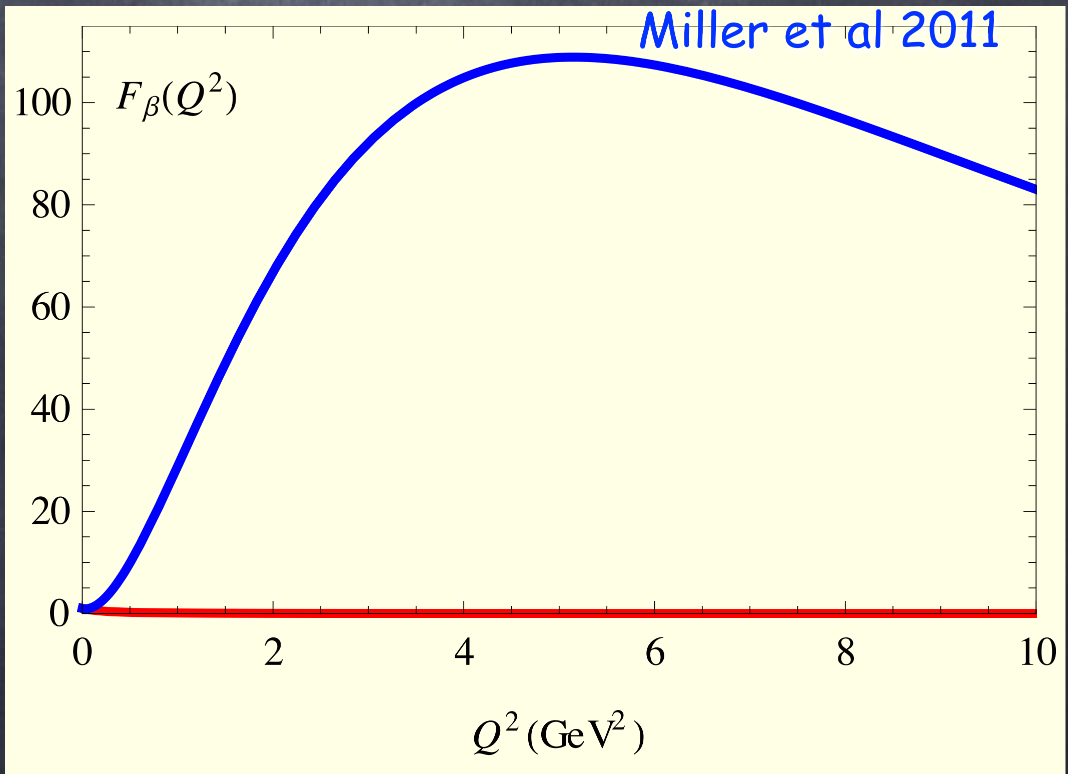
Subtraction function contribution

$$[M_p - M_n]^{Subt} = -\frac{\beta^p_M - \beta^n_M}{(8\pi)^2 M} \int_0^{\Lambda^2} dQ^2 Q^2 F_\beta(Q^2)$$

If the proton radius puzzle
is due to subtraction contribution

$$\delta M^p_{em} \sim 600\, MeV$$

Could be purely isoscalar but...
VERY unnatural!
Should be seen in Deuteron (I=0)



# Muonic deuterium

One further piece of information available - isotope shift: simultaneous 1S-2S splitting measurement in eH and eD

$$R_d^2 - R_p^2 = 3.82007(65)\,\mathrm{fm}^2$$

$R_d^2$-$R_p^2$ from μH, μD @ PSI - in agreement (preliminary)
Exotic hadronic contributions excluded by this finding

Extraction from μD relies on nuclear structure-dependent polarizability correction.

Nuclear models vs dispersion relations:

$$\Delta E_{2S}^{Nucl.} = -1.68(16)\,\mathrm{meV}$$

Leidemann, '90; Pachucki '13;
Ji et al, '14; Friar, '14;

$$\Delta E_{2S}^{DR} = -1.75(74)\,\mathrm{meV}$$

Carlson, MG, Vanderhaeghen '14

A simple ansatz for $F_\beta(Q^2)$ used

# Lacking Input to DR for μD

$$\Delta E \sim \alpha_{em}^5 \int_0^\infty dQ^2 \int_0^\infty d\nu \left\{ A(\nu, Q^2)\, F_1 + B(\nu, Q^2)\, F_2 \right\}$$

All kinematics contribute to the dispersive integral;
Not all of them are equally important

The bulk of the correction - quasi elastic data
from $\nu \simeq$ 6-10 MeV and $Q^2 < 0.005$ GeV$^2$
- just below the kinematics of available QE data

New D(e,e')pn data down to $Q^2 = 0.002$ GeV$^2$ A1@MAMI
taken and under analysis;
2% measurement will reduce the uncertainty by a factor 2-4

Once the data are more precise: the model for $F_\beta(Q^2)$
will become the main limitation of the calculation

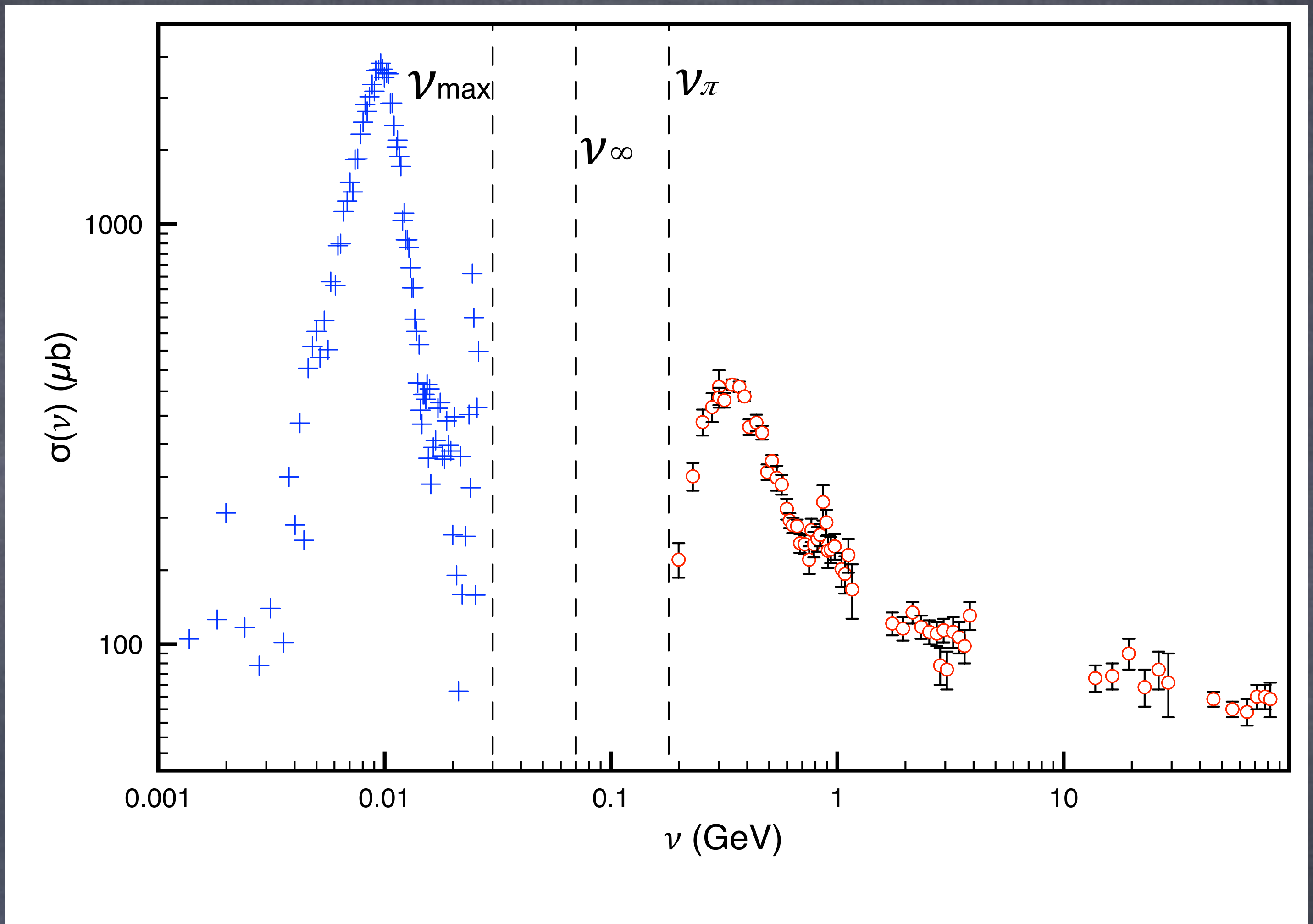
# Subtraction function from finite energy sum rule

$$\mathrm{Re}\, T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{\nu^2}{2\pi M} \mathcal{P} \int_0^\infty d\nu' \, \frac{F_1(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

# FESR (real photons)

Nuclear photoabsorption:
from $\nu_{thr}$ = few MeV
to $\nu_{max}$ = few tens MeV;
"nothing" above that until
$\nu_\pi$ = 150 MeV;
Scale separation:

$\nu_{max} \ll \nu_\infty \ll \nu_\pi$




Evaluate the DR at $\nu = \nu_\infty$

$$\mathrm{Re}\,T_1(\nu_\infty, 0) = \mathrm{Re}\,T_1(0,0) + \frac{\nu_\infty^2}{2\pi M}\mathcal{P}\int_{\nu_{thr}}^{\infty}\frac{d\nu}{\nu(\nu^2-\nu_\infty^2)}F_1(\nu,0)$$

Employ duality

LEX at $\nu$=0: nuclear Thomson term $\mathrm{Re}\,T_1(0,0) = -\frac{Z^2}{4\pi M}$

LEX at $\nu=\nu_\infty$: nucleon Thomson terms + polarizabilities

$$\mathrm{Re}\,T_1(\nu_\infty, 0) = -\frac{Z}{4\pi M_p} + \frac{\nu_\infty^2}{e^2}\left(Z(\alpha^p+\beta^p) + N(\alpha^n+\beta^n)\right)$$

$$\frac{\nu_\infty^2}{2\pi M}\mathcal{P}\int_{\nu_{thr}}^{\infty}\frac{d\nu}{\nu(\nu^2-\nu_\infty^2)}F_1(\nu,0) \approx -\frac{1}{2\pi M}\int_{\nu_{thr}}^{\nu_{max}}\frac{d\nu}{\nu}F_1(\nu,0)$$

Work out the integral

$$+\frac{\nu_\infty^2}{2\pi M}\mathcal{P}\int_{\nu_{max}}^{\nu_\pi}\frac{d\nu}{\nu(\nu^2-\nu_\infty^2)}F_1(\nu,0)$$

$$+\frac{\nu_\infty^2}{2\pi M}\int_{\nu_\pi}^{\infty}\frac{d\nu}{\nu^3}F_1(\nu,0)$$

Balance of coeffs. at $(\nu_\infty)^2$ :

L.H.S. $(\nu_\infty^2/e^2)[Z(\alpha^p+\beta^p)+N(\alpha^n+\beta^n)]$

Baldin sum rule for nucleons: $\alpha_E^{p,n}+\beta_M^{p,n}=\frac{2\alpha}{M}\int_{\nu_\pi}^{\infty}\frac{d\nu}{\nu^3}F_1^{p,n}(\nu,0)$

$$\mathcal{P}\int_{\nu_{thr}}^{\nu_\pi}\frac{d\nu}{\nu(\nu^2-\nu_\infty^2)}F_1(\nu,0)+\int_{\nu_\pi}^{\infty}\frac{d\nu}{\nu^3}\left[F_1(\nu,0)-(Z+N)(ZF_1^p(\nu,0)+NF_1^n(\nu,0))\right]\approx 0$$

Non-interacting nucleons in the nucleus

# Coeffs. at $(\nu_\infty)^0$: Bethe-Levinger photonuclear sum rule

$$-\frac{Z}{4\pi M_p} = -\frac{Z^2}{4\pi M} - \frac{1}{2\pi M}\int_{\nu_{thr}}^{\nu_{max}} \frac{d\nu}{\nu} F_1(\nu,0)$$

$$2\int_{\nu_{thr}}^{\nu_{max}} \frac{d\nu}{\nu} F_1(\nu,0) = ZN$$

Integrated nuclear photoabsorption cross section is given by the number of "elementary" scatterers - nucleons

Thomas - Reiche - Kuhn sum rule in QM:
integrated oscillator strength ~ number of oscillators

# Bethe-Levinger SR: works to 10-20%

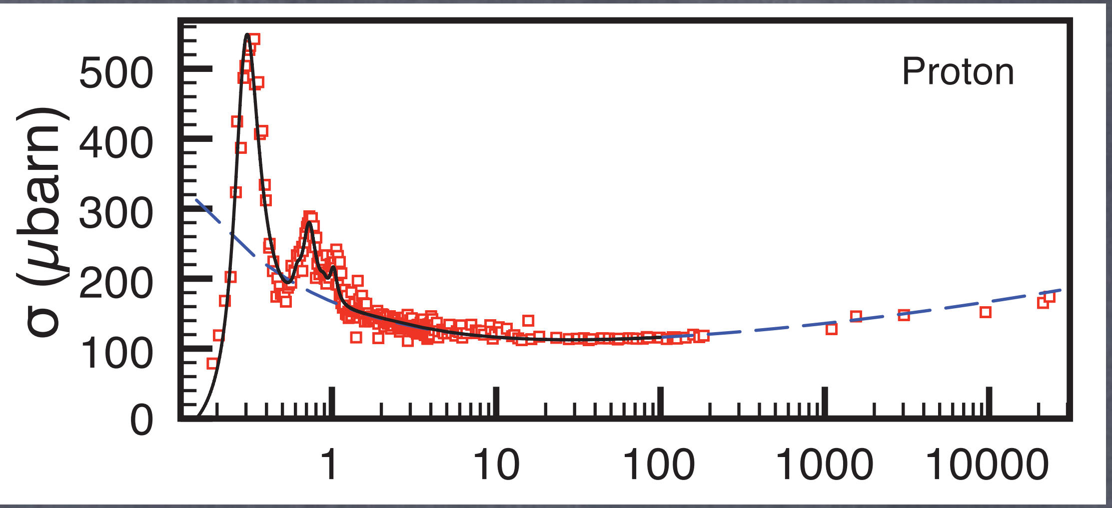


TABLE III. Quantities derived directly from the data—all nuclei.

| Nucleus | $E_{\gamma max}$ (MeV) | $\sigma_{int}(\gamma,tot) / 60NZ/A$ | $\sigma_{-1} A^{-4/3}$ (mb) | $\sigma_{-2} / 0.00225 A^{5/3}$ (mb-MeV$^{-1}$) | $\sigma_{int}[(\gamma,2n)+(\gamma,3n)] / \sigma_{int}(\gamma,tot)$ | Reference |
|---|---|---|---|---|---|---|
| $^{91}$Zr | 30.0 | 0.820 | 0.160 | 0.98 | 0.181 | Berman *et al.*, 1967 |
| $^{92}$Zr | 27.8 | 0.804 | 0.154 | 0.93 | 0.414 | Berman *et al.*, 1967 |
| $^{93}$Nb | 24.3 | 0.967 | 0.186 | 1.12 | 0.209 | Leprêtre *et al.*, 1971 |
| $^{94}$Zr | 31.1 | 0.813 | 0.160 | 1.01 | 0.547 | Berman *et al.*, 1967 |
| $^{107}$Ag | 29.5 | 0.858 | 0.155 | 0.89 | 0.194 | Berman *et al.*, 1969a |
| $^{115}$In | 31.1 | 1.111 | 0.202 | 1.17 | 0.278 | Fultz *et al.*, 1969 |
| $^{116}$Sn | 29.6 | 0.978 | 0.175 | 0.99 | 0.248 | Fultz *et al.*, 1969 |
| $^{117}$Sn | 31.1 | 1.102 | 0.199 | 1.16 | 0.271 | Fultz *et al.*, 1969 |
| $^{118}$Sn | 30.8 | 1.072 | 0.190 | 1.07 | 0.297 | Fultz *et al.*, 1969 |
| $^{119}$Sn | 31.1 | 1.145 | 0.202 | 1.17 | 0.334 | Fultz *et al.*, 1969 |
| $^{120}$Sn | 29.9 | 1.185 | 0.209 | 1.19 | 0.330 | Fultz *et al.*, 1969 |
| $^{124}$Sn | 31.1 | 1.123 | 0.200 | 1.16 | 0.361 | Fultz *et al.*, 1969 |
| $^{127}$I | 29.5 | 0.933 | 0.164 | 0.93 | 0.256 | Bramblett *et al.*, 1966b |
| | 24.9 | 1.074 | 0.201 | 1.18 | 0.196 | Bergère *et al.*, 1969 |
| $^{133}$Cs | 29.5 | 1.026 | 0.182 | 1.04 | 0.257 | Berman *et al.*, 1969a |
| $^{138}$Ba | 27.1 | 1.022 | 0.183 | 1.05 | 0.242 | Berman *et al.*, 1970c |
| $^{139}$La | 24.3 | 0.980 | 0.177 | 1.02 | 0.147 | Beil *et al.*, 71 |
| $^{141}$Pr | 29.8 | 1.001 | 0.175 | 0.97 | 0.167 | Bramblett *et al.*, 1966b |
| | 16.9 | 0.691 | 0.138 | 0.85 | | Beil *et al.*, 1971 |
| | 18.1 | 0.678[a] | 0.128[a] | 0.75[a] | | Young, 1972 |
| $^{142}$Nd | 20.2 | 0.901 | 0.170 | 1.00 | 0.024 | Carlos *et al.*, 1971 |
| $^{143}$Nd | 19.8 | 0.910 | 0.176 | 1.08 | 0.094 | Carlos *et al.*, 1971 |
| $^{144}$Nd | 20.2 | 0.896 | 0.170 | 1.01 | 0.299 | Carlos *et al.*, 1971 |
| $^{145}$Nd | 20.2 | 0.965 | 0.193 | 1.26 | 0.323 | Carlos *et al.*, 1971 |
| $^{146}$Nd | 20.2 | 0.905 | 0.173 | 1.05 | 0.347 | Carlos *et al.*, 1971 |
| $^{148}$Nd | 18.8 | 0.795 | 0.155 | 0.97 | 0.491 | Carlos *et al.*, 1971 |
| $^{150}$Nd | 20.2 | 0.931 | 0.178 | 1.09 | 0.416 | Carlos *et al.*, 1971 |
| $^{153}$Eu | 28.9 | 1.022 | 0.181 | 1.03 | 0.311 | Berman *et al.*, 1969b |
| $^{159}$Tb | 28.0 | 0.997 | 0.175 | 1.00 | 0.386 | Bramblett *et al.*, 1964 |
| | 27.4 | 1.109 | 0.198 | 1.15 | 0.243 | Bergère *et al.*, 1968 |
| $^{160}$Gd | 29.5 | 1.099 | 0.195 | 1.14 | 0.448 | Berman *et al.*, 1969b |
| $^{165}$Ho | 28.9 | 1.057 | 0.183 | 1.04 | 0.312 | Berman *et al.*, 1969b |
| | 26.8 | 1.202 | 0.215 | 1.24 | 0.272 | Bergère *et al.*, 1968 |
| $^{175}$Lu | 23.0 | 0.990 | 0.177 | 1.02 | 0.253 | Bergère *et al.*, 1969 |
| $^{181}$Ta | 24.6 | 0.835 | 0.146 | 0.82 | 0.404 | Bramblett *et al.*, 1963 |
| | 25.2 | 1.142 | 0.201 | 1.14 | 0.269 | Bergère *et al.*, 1968 |
| $^{186}$W | 28.6 | 1.123 | 0.191 | 1.06 | 0.449 | Berman *et al.*, 1969b |
| $^{197}$Au | 24.7 | 1.045 | 0.179 | 0.98 | 0.262 | Fultz *et al.*, 1962b |
| | 21.7 | 1.080 | 0.190 | 1.06 | 0.156 | Veyssière *et al.*, 1970 |
| $^{206}$Pb | 26.4 | 0.982 | 0.167 | 0.93 | 0.183 | Harvey *et al.*, 1964 |

Include hadronic photoabsorption

Complication: c.s. increases at high energies




$$F_1(\nu \geq 2\,\mathrm{GeV}, 0) \to F_1^R(\nu, 0) = C_M \left(\frac{\nu}{\nu_0}\right)^{\alpha_M} + C_P \left(\frac{\nu}{\nu_0}\right)^{\alpha_P}$$

$$\nu_0 \approx 1\,\mathrm{GeV}, \quad \alpha_M \approx 0.5, \quad \alpha_P \approx 1.09$$

Build a Regge-behaved analytic function

$$\mathrm{Re}\, T_1^R(\nu, 0) = 0 + \frac{\nu^2}{2\pi M} \mathcal{P} \int_0^\infty \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)} F_1^R(\nu', 0)$$

Subtract Regge behavior: the integral runs up to finite energy N

$$\mathrm{Re}\, [T_1(\nu, 0) - T_1^R(\nu, 0)] = -\frac{Z^2}{4\pi M} + \frac{\nu^2}{2\pi M} \mathcal{P} \int_{\nu_{thr}}^{N} \frac{d\nu' [F_1(\nu', 0) - F_1^R(\nu', 0)]}{\nu'(\nu'^2 - \nu^2)}$$

The remaining amplitude - at most constant asymptotically

The asymptotic constant - (hypothetical) J=0 fixed pole

$$C_\infty = \mathrm{Re}\left[T_1(\nu,0) - T_1^R(\nu,0)\right]\Big|_{\nu\to\infty}$$

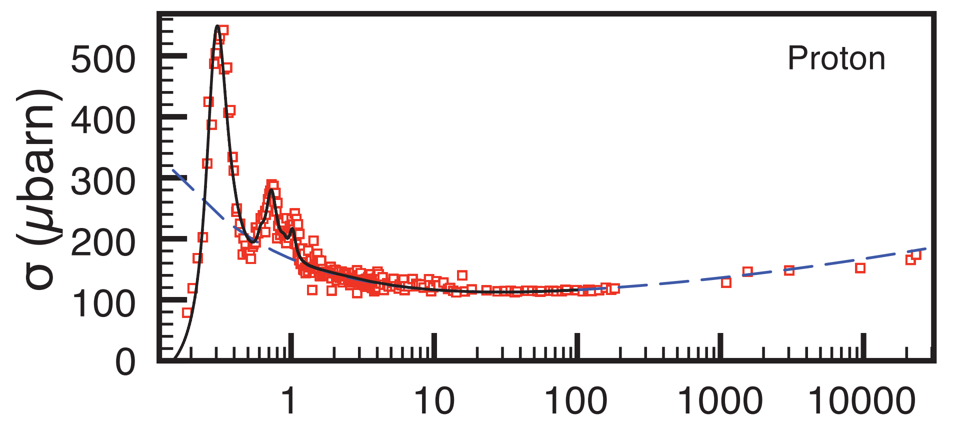
Analyticity: the J=0 pole is not a free constant

$$C_\infty = -\frac{Z^2}{4\pi M} - \frac{1}{2\pi M}\int_{\nu_{thr}}^{N} \frac{d\nu}{\nu} F_1(\nu,0) + \frac{1}{2\pi M}\left[\frac{C_M}{\alpha_M}\left(\frac{N}{\nu_0}\right)^{\alpha_M} + \frac{C_P}{\alpha_P}\left(\frac{N}{\nu_0}\right)^{\alpha_P}\right]$$

Damashek and Gilman, 1969

Exact duality: integrated c.s. = integrated Regge

-> J=0 pole = Thomson term




Deviation of J=0 pole from Thomson term
= duality violation

cf. Müller, Polyakov, Semenov 2015

# Duality of resonance structure with constituent quarks

Choose $\nu_\infty$ ~ few GeV ($\nu_\infty$ ">>" N) - sum of CQ Thomson terms

$$\mathrm{Re}\left[T_1(\nu_\infty,0) - T_1^R(\nu_\infty,0)\right] = -\sum_{q=u,d\in A} \frac{e_q^2}{4\pi M_q} = -\frac{3Z+2N}{4\pi M_p} \qquad M_q \approx M_p/3$$

"Identify" meson Regge exchange as quark-antiquark exchanges

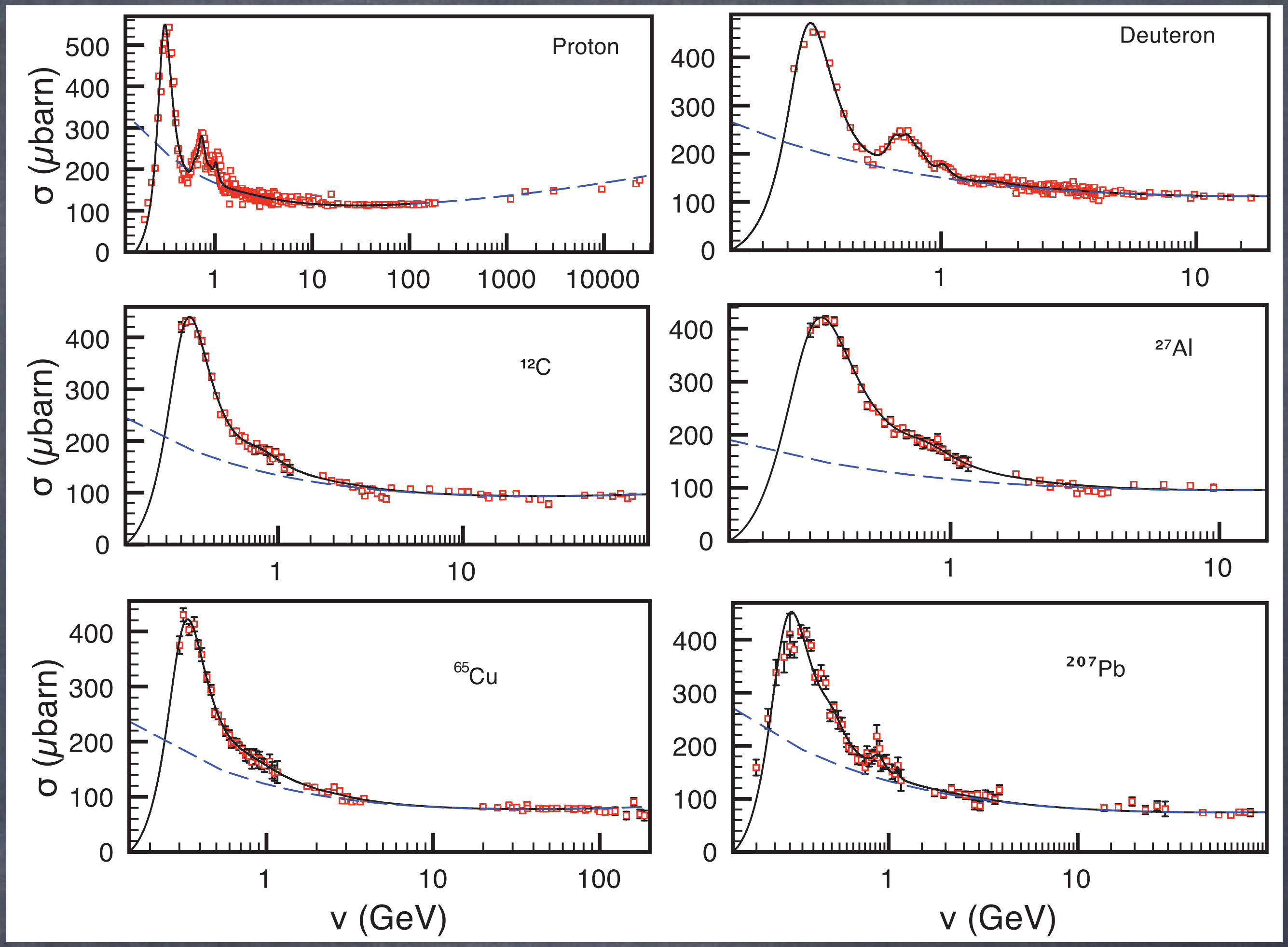
CQM sum rule

$$(Z+N)^2 + \frac{ZN}{2} = \int_{\nu_{thr}}^{\nu_{max}} \frac{d\nu}{\nu} F_1(\nu,0) - \frac{C_M}{\alpha_M}\left(\frac{\nu_{max}}{\nu_0}\right)^{\alpha_M}$$

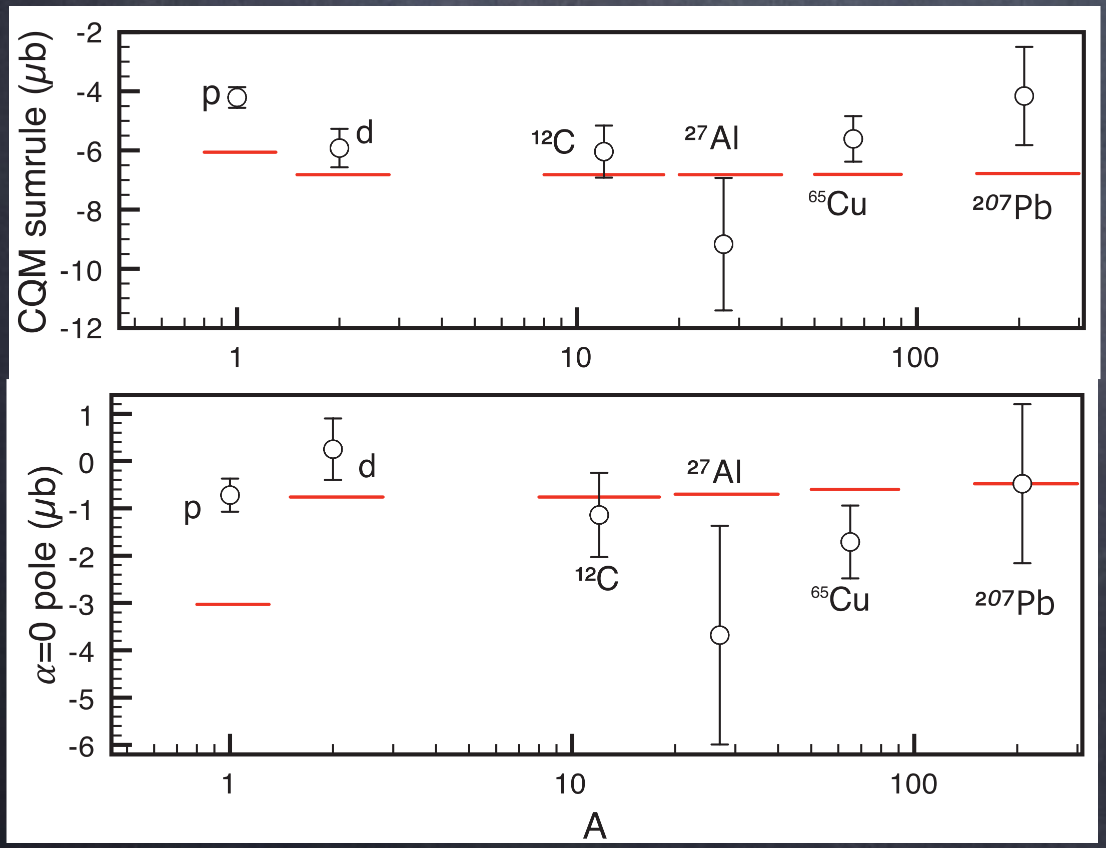
# Fit of photoabsorption data on a few selected nuclei

Resonance + Regge background

MG, Hobbs, Londergan, Szczepaniak 2011




Evaluate sum rules




Message: duality sum rules work;
Can be used for quantitative study;
Precision - can be 10-20%

Generalize Bethe-Levinger SR to finite $Q^2$:

$$\mathrm{Re}\,T_1(\nu_\infty, Q^2) = \mathrm{Re}\,T_1(0, Q^2) - \frac{1}{2\pi M} \int_{\nu_{thr}}^{\nu_{max}} \frac{d\nu}{\nu} F_1(\nu, Q^2)$$

LEX at finite $Q^2$: Dirac (or charge) form factor + magnetic pol.

$$T_1(0, Q^2) = -\frac{Z^2}{4\pi M} F_D^2(Q^2) + \frac{Q^2}{e^2} \beta_M F_\beta(Q^2) \qquad F_\beta(0) = 1$$

$$\begin{aligned} T_1(\nu_\infty, Q^2) \quad &= \quad -\frac{Z}{4\pi M_p} F_D^{p\,2}(Q^2) + Z\frac{Q^2}{e^2} \beta_M^p F_\beta^p(Q^2) \\ &\quad -\frac{N}{4\pi M_p} F_D^{n\,2}(Q^2) + N\frac{Q^2}{e^2} \beta_M^n F_\beta^n(Q^2) + O(\nu_\infty^2) \end{aligned}$$

The new sum rule: the $Q^2$ slope of the TRK - BL sum rule

$$\begin{aligned} \beta_M \quad &= \quad \frac{2\alpha}{M} \int_{\nu_{thr}}^{\nu_{max}} \frac{d}{d\nu} \frac{d}{dQ^2} F_1(\nu, Q^2) \Big|_{Q^2 \to 0} \\ &\quad - \quad \frac{Z^2 \alpha R_{Ch}^2}{3M} + \frac{Z\alpha R_p^2 + N\alpha R_n^2}{3M_p} + Z\beta_M^p + N\beta_M^n \end{aligned}$$

Can test the sum rule:

- fit the electrodisintegration data in the nuclear range;
- compare to the value of the nuclear magnetic pol. (if known)

Deuteron: $\beta_M$ known theoretically

EFT (lowest order): $\beta_M^d = 0.068\ \text{fm}^2$ — Chen et al., 2002

Potential models (LO): $\beta_M^d = 0.068\ \text{fm}^2$ — Friar 1997, Khriplovich 1979, ...

Potential models (NLO): $\beta_M^d = 0.078\ \text{fm}^2$ — Friar 1997

Nucleon $\beta_M$: known and generally small (2 o.o.m.)

PDG 2012 $\quad \beta_M^p = 2.5(0.4)\cdot 10^{-4}\,\text{fm}^3, \quad \beta_M^n = 3.7(2.0)\cdot 10^{-4}\,\text{fm}^3$

ChPT $\quad \beta_M^p = 3.9(0.7)\cdot 10^{-4}\,\text{fm}^3, \quad \beta_M^n = 4.6(2.7)\cdot 10^{-4}\,\text{fm}^3$

Hagelstein et al., arXiv:1512.03765

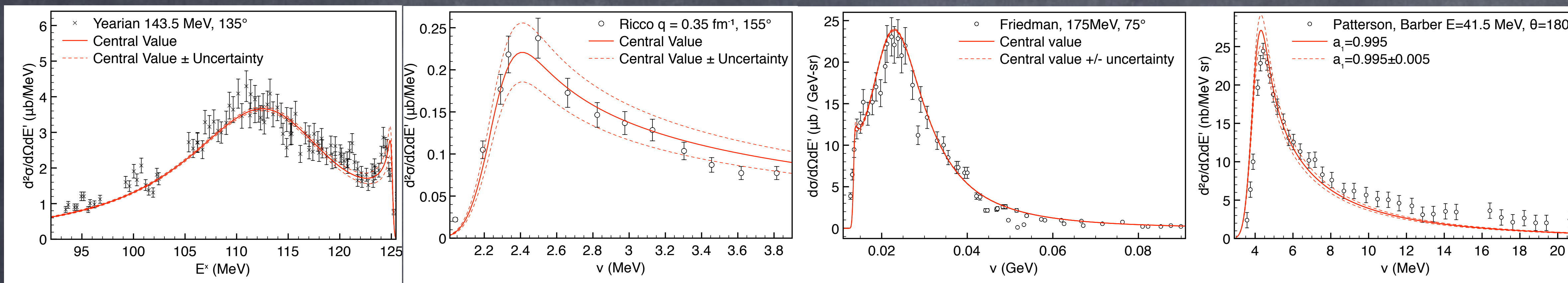
Charge radii known: $R_d = 2.14$ fm, $R_p = 0.840$ fm, $R_n^2 = -0.116\ \text{fm}^2$

Correction term: $$-\frac{\alpha R_d^2}{3M} + \frac{\alpha R_p^2 + \alpha R_n^2}{3M_p} + \beta_M^p + \beta_M^n \approx 1\times 10^{-5}\text{fm}^3$$

# Recent deuteron data fit

Carlson, MG, Vanderhaeghen, PR A89 (2014)

Fit of the form $F^{QE}(\nu,Q^2)\cdot f^{QE}(Q^2) + F^{thr}(\nu,Q^2)\cdot f^{thr}(Q^2)$



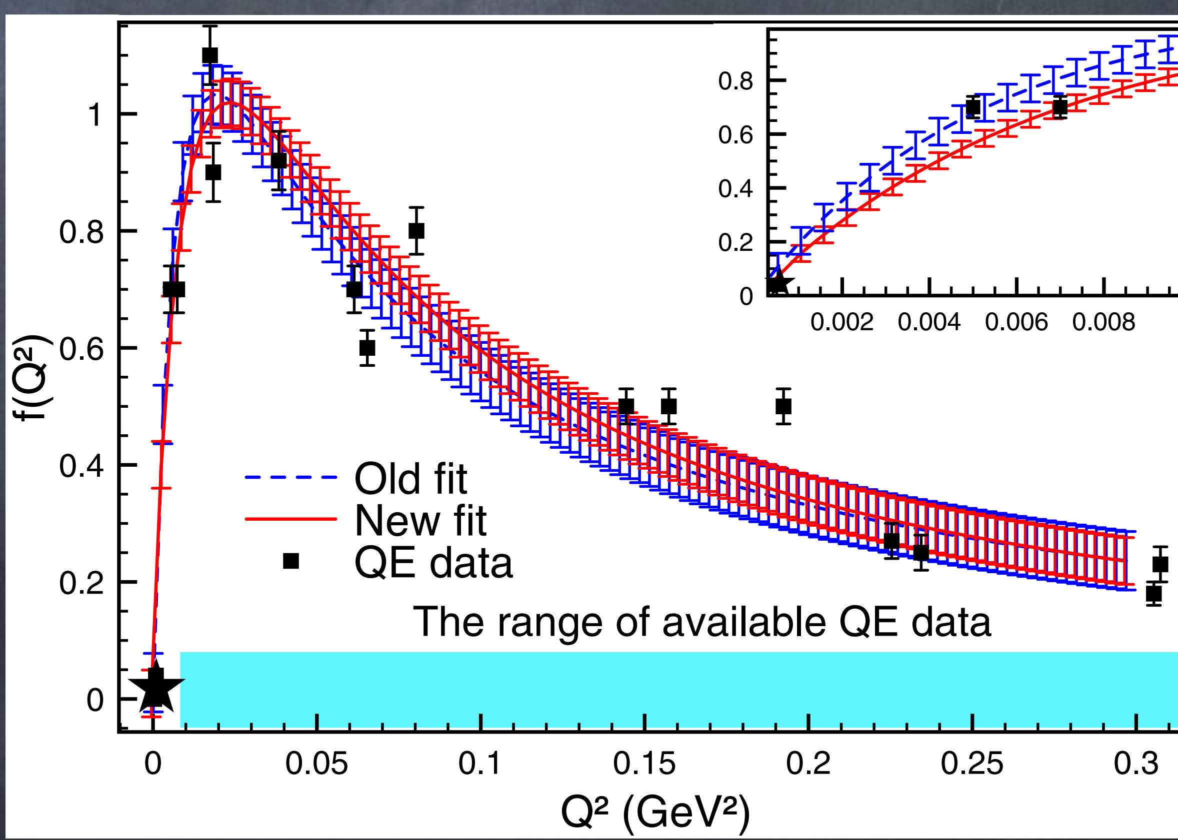

$$\beta_M = \frac{2\alpha}{M}\int_{\nu_{thr}}^{\nu_{max}} \frac{d}{d\nu}\,\frac{d}{dQ^2}F_1(\nu,Q^2)\Big|_{Q^2\to 0}$$

$0.073(5)\ \text{fm}^3$ <-> $0.096(16)\ \text{fm}^3$

$1.5\sigma$ off, but in the ballpark

The problem: need to extrapolate down to $Q^2$=0 from finite $Q^2$; Nuclear slopes are large

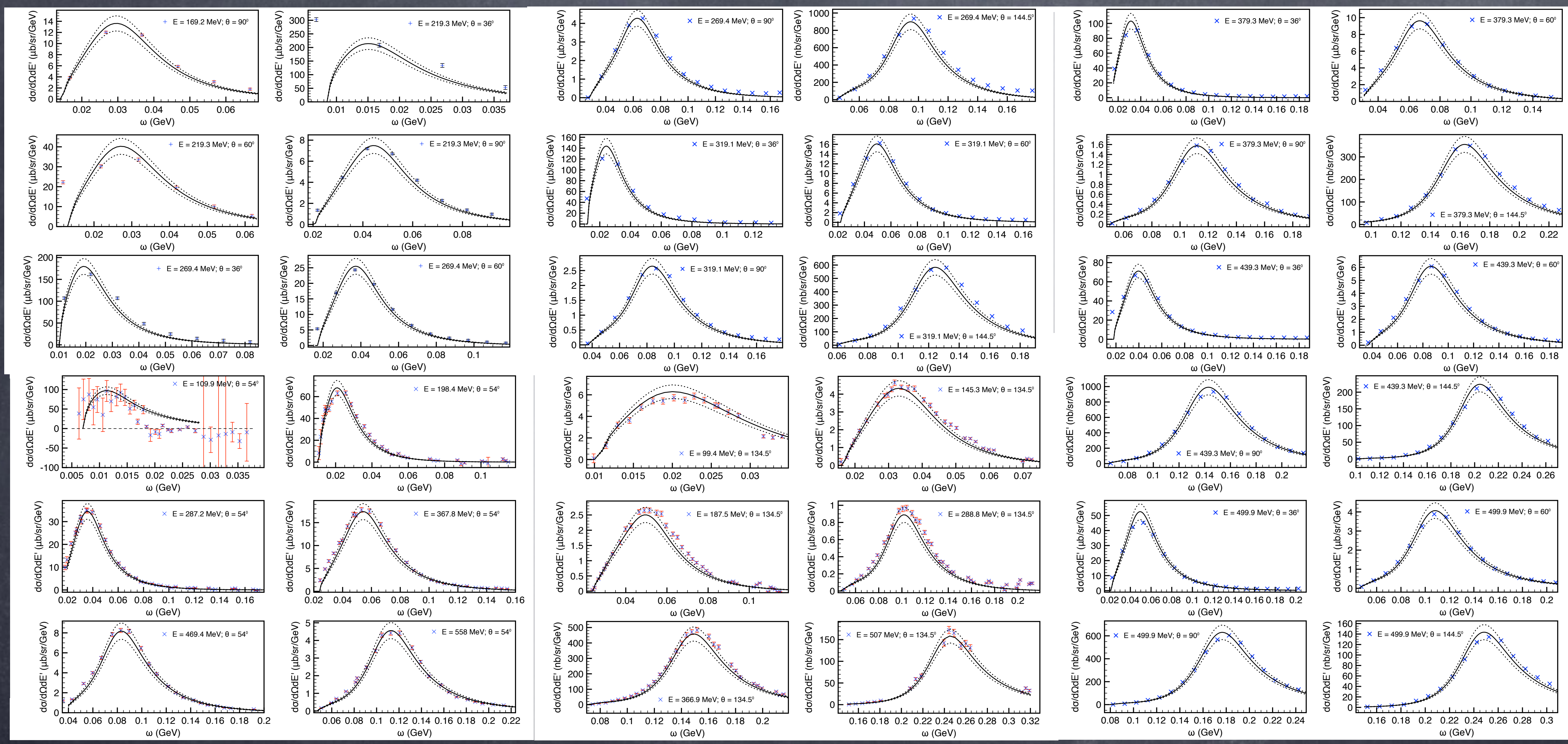
Fit done not using the SR




Can impose the value of $\beta_d$ – new fit

# New application: He-3 Carlson, MG, Vanderhaeghen, in progress

## Fit of the form $F^{QE}(\nu,Q^2) \cdot f^{QE}(Q^2) + F^{thr}(\nu,Q^2) \cdot f^{thr}(Q^2)$




Sum rule prediction for the magnetic polarizability

$$\beta_M^{He-3} = [4.20 - 2.44 + 0.67 + 1.24] \cdot 10^{-3}\,\mathrm{fm}^3 = 3.9 \cdot 10^{-3}\,\mathrm{fm}^3$$

Uncertainty? 10% from $\beta_p$ and $\beta_n$; 10% from the fit; systematics?

Further generalization: the full $Q^2$ dependence of $\beta(Q^2)$

$$\beta_M(Q^2) = \frac{2\alpha}{M} \int\limits_{\nu_{thr}}^{\nu_{max}} \frac{d}{d\nu} \frac{F_1(\nu, Q^2) - F_1(\nu, 0)}{Q^2}$$

Confront to the simple-minded FF-like model of our PR A89

Effect on the Lamb shift calculation

Estimate w/o sum rule

$$\Delta E_{2S}^{DR} = -1.75(74)\,\mathrm{meV}$$

Estimate with sum rule

$$\Delta E_{2S}^{SR} = -1.94(74)\,\mathrm{meV}$$

Uncertainty dominated by the dispersion integral;
once more precise data allow to reduce the uncertainty -
may lead to a shift in the extracted value of $R_d$ !

Effect due to different $R_p$ ~ 0.38 meV in $\mu$D; here - 0.19 meV

# Summary

- Proton radius puzzle - inconsistency between the e-scattering and eH on one hand, and $\mu$H data on the other hand.
- Each part has subtleties but no clear solution found - the puzzle persists
- Scattering experiments: extrapolation issue
- Electronic hydrogen: sensitivity issue
- Muonic hydrogen: no experimental issues found to date further muonic atoms consistent with $\mu$H (preliminary)
- BSM explanation possible but requires both lepton non-universality and fine tuning to evade known constraints from other observables

# Proton Radius Puzzle: what's next?

- More precise eH experiments coming (2S-2P, 1S-3S, 2S-4S);
- e-p scattering: $Q^2$ down to $2 \times 10^{-4}$ GeV$^2$ @ Mainz, JLab
- Deuteron radius from e-D scattering: new data at Mainz under analysis $Q^2 > 0.002$ GeV$^2$, radius under 0.25%
- To push $Q^2$ down and get the radius under 1%:
  improved radiative corrections (TPE) necessary.
  Recent works: MG '14; Tomalak, Vanderhaeghen '14, '15(2)
- Study lepton non-universality with $\mu$-p scattering:
  MUSE @ PSI - elastic $\mu$-p scattering at $Q^2 > 0.002$ GeV$^2$ (2017/18);
  $\gamma p \to \mu^+\mu^- p / \gamma p \to e^+e^- p$ measurement may be more sensitive
  Pauk, Vanderhaeghen '15 - proposal under consideration in Mainz

# Proton Radius Puzzle: what's next?

- Further muonic atoms: $\mu$D, $\mu$He-3, $\mu$He-4 - data taken at PSI, now analyzed or finalized

- $\mu$D - more precise DR calculation needed:
  new QE data on deuteron analyzed at Mainz
  - to reduce the uncertainty of dispersion integrals by factor 2-4

  sum rule for the nuclear magnetic polarizability derived (MG, '15)
  - to reduce model dependence of the subtraction contribution

  DR treatment of hyperfine splitting in $\mu$D underway
  - with Carlson and Vanderhaeghen

- $\mu$He-3,4 - DR analysis underway (with Carlson and Vanderhaeghen)
  potential model calculation by Bacca and Co arXiv: 1512.05773