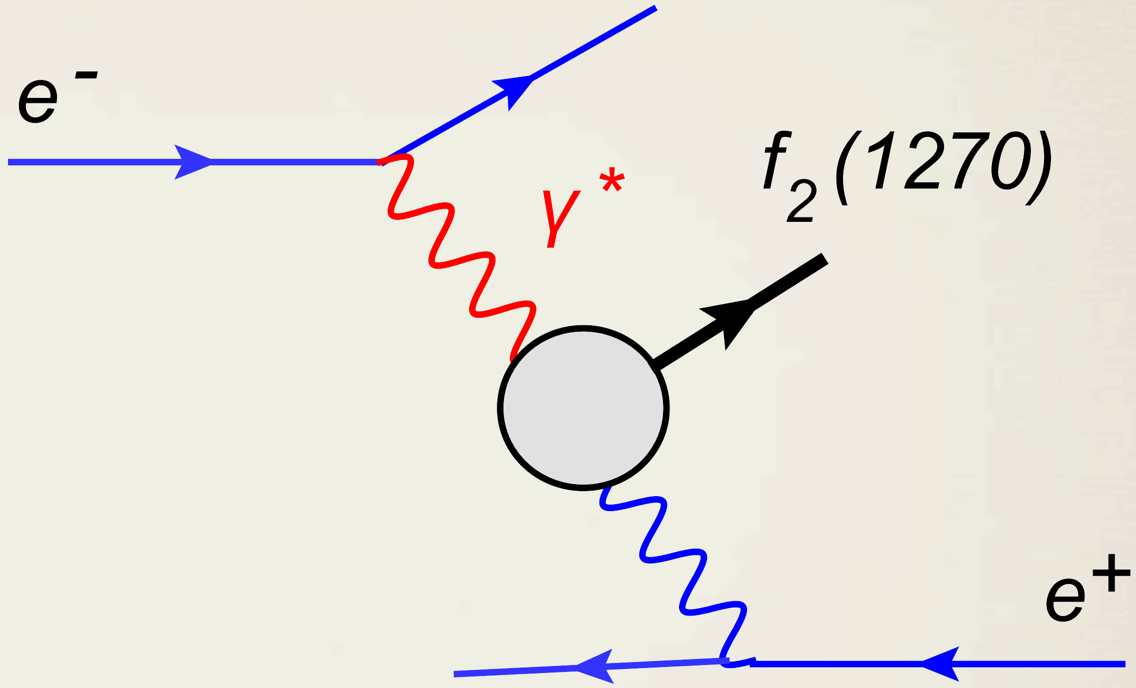
Hard electroproduction of tensor meson $f_2(1270)$ within QCD factorization framework

Nikolay Kivel

HELMHOLTZ Institut Mainz

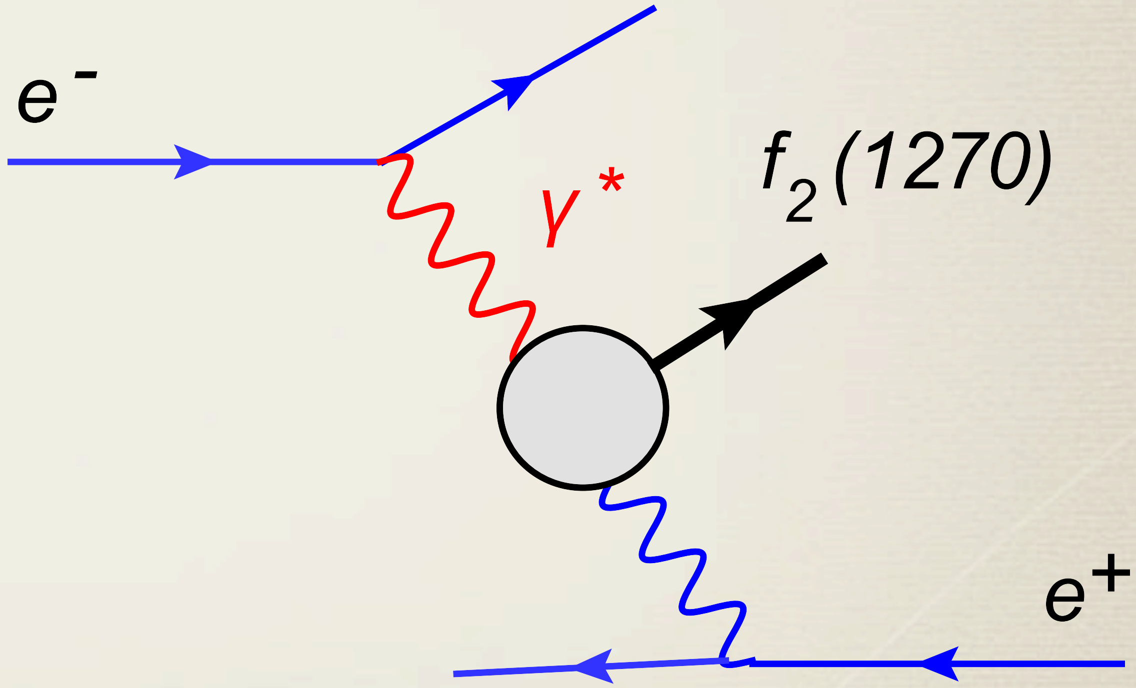


V.Braun, N. Kivel PLB 501, 2001

V.Braun, N. Kivel, M. Strohmaier, A. Vladimirov JHEP, 2016

Outline

- Introduction: theory & existing exp. results
- f2 : theory & data
- Conclusions



Introduction: QCD factorisation

QCD factorization <=> effective field theory approach

Two scales: hard and soft $Q^2 \gg \Lambda^2_{QCD}$

$$A(Q^2) = H(Q^2) * S(\Lambda)$$

Hard part is defined by a hard subprocess $p_h \sim Q \quad p_h^2 \sim Q^2$

$$H(Q^2) = Q^{-2n} H_{\rm LP}(\ln Q^2/\Lambda^2) + \mathcal{O}(Q^{-2n-2})$$

scaling behavior Q^{-2n} is the model independent QCD prediction (!) which can be checked by experiment

$$H_{\rm LP}(\ln Q^2/\Lambda^2) = h_{\rm LO}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{\gamma_{LO}} + h_{\rm NLO}\,\alpha_s(Q^2)\left(\ln\frac{Q^2}{\Lambda^2}\right)^{\gamma_{NLO}} + \mathcal{O}(\alpha_s^2)$$

Log corrections can be computed systematicall in pQCD $\alpha_s(Q^2) \sim \ln^{-1} Q^2/\Lambda^2 \ll 1$

Introduction: QCD factorisation

QCD factorization <=> effective field theory approach

Two scales: hard and soft $Q^2 \gg \Lambda^2_{QCD}$

$$A(Q^2) = H(Q^2) * S(\Lambda)$$

Soft part is

associated with a soft subprocess $p_s^2 \sim \Lambda^2$

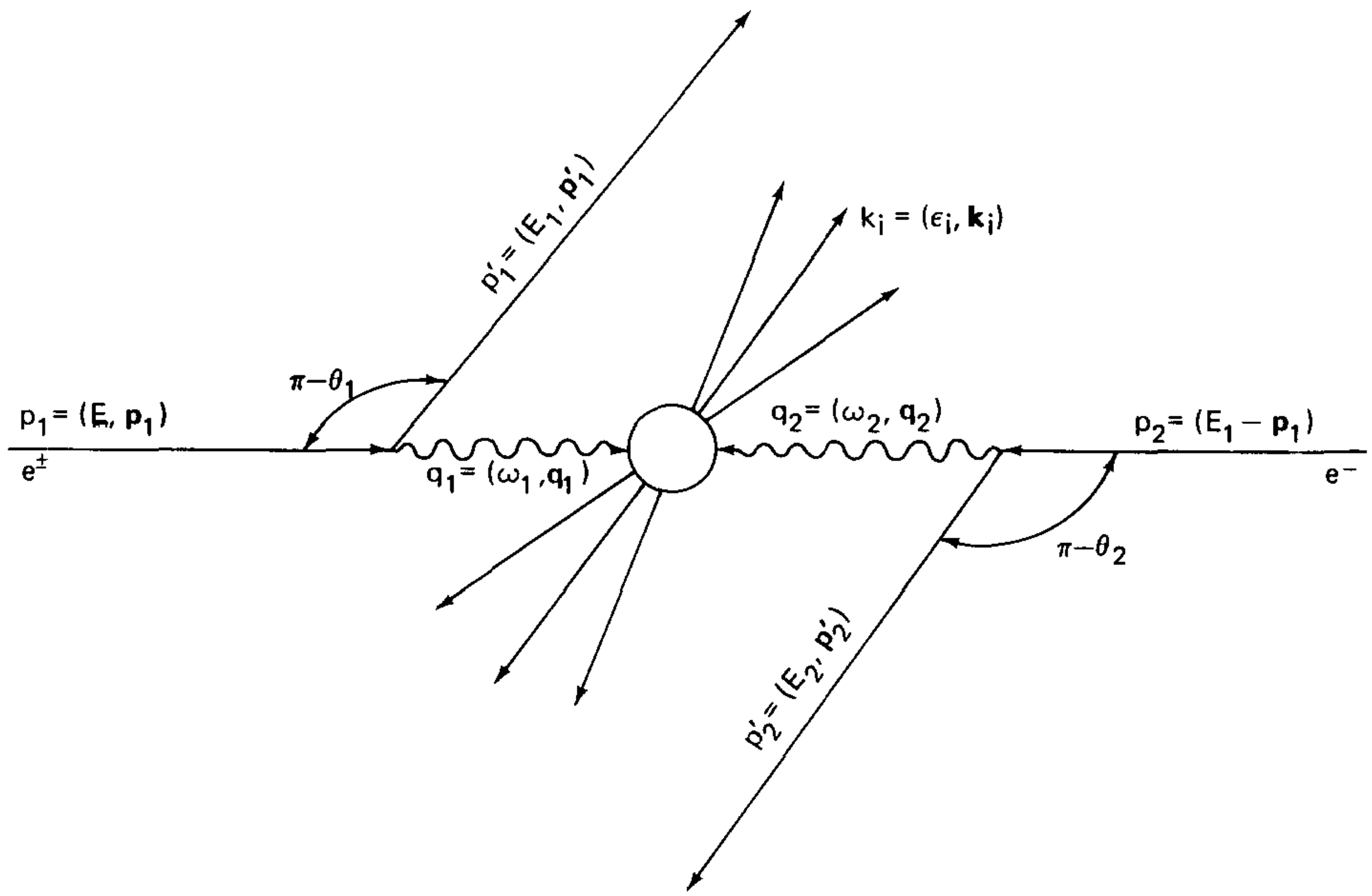
defined as a matrix element in QCD

process independent (universal)

can be estimated only in the framework of some nonperturbative approach or constrained from the experimental data

Introduction: e^+e^- & gamma-gamma fusion

Budnev, Meledin, Ginsburg, Serbo 1974



single tagget experiment cross section

$$\frac{d\sigma}{dE_1 dE_2 d\cos\theta_1} = \frac{\alpha}{4\pi} \frac{(E^2 + E_1^2) N(E_2, \theta_m)}{E^2 (E - E_1)(E - E_2) \sin^2 \frac{1}{2}\theta_1} (\sigma_{TT} + \xi_1 \sigma_{ST});$$

$$N(E_2, \theta_m) = \frac{\alpha}{\pi} \left[\frac{E^2 + E_2^2}{E^2} \ln \frac{E E_2 \theta_m}{m_e (E - E_2)} - \frac{E_2}{E} \right]$$

Introduction: $\gamma^*(q)\gamma(q') \to M(p)$

Process $\gamma^*(q)\gamma(q') \to M(p)$ $J^{PC}=0^{-+}, 0^{++}, 2^{++}, \ldots$

$m_M \ll Q$ $\pi^0, \eta, \pi\pi, f_0, a_0, f_2, \ldots$

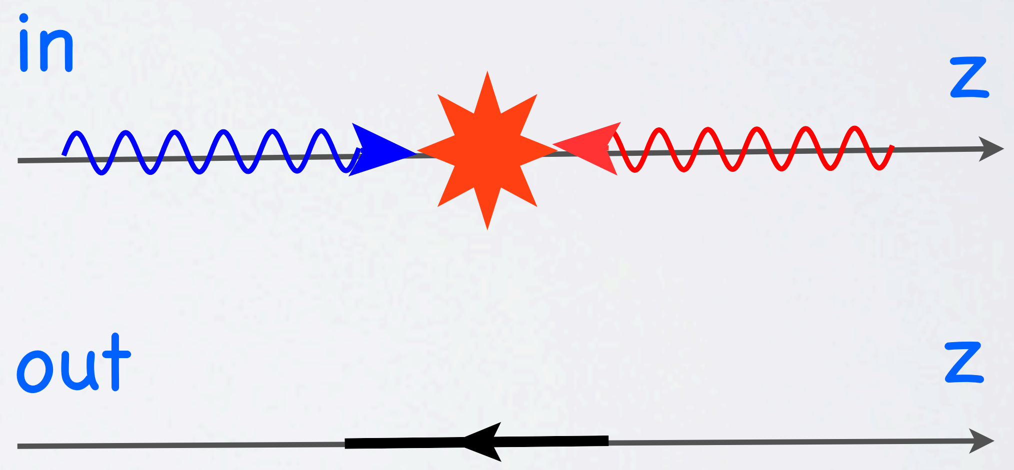
$p^2 = m_M^2$ $p^2 \simeq 0$ $q^2 = -Q^2 < 0$ $q'^2 = 0$

Breit frame

$$q = (0, 0, 0, -Q)$$

$$q' = \frac{1}{2}Q(1, 0, 0, 1)$$

$$p = \frac{1}{2}Q(1, 0, 0, -1)$$



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$\pi^0, \eta, \pi\pi, f_0, a_0, f_2, \ldots$

$m_M \ll Q$

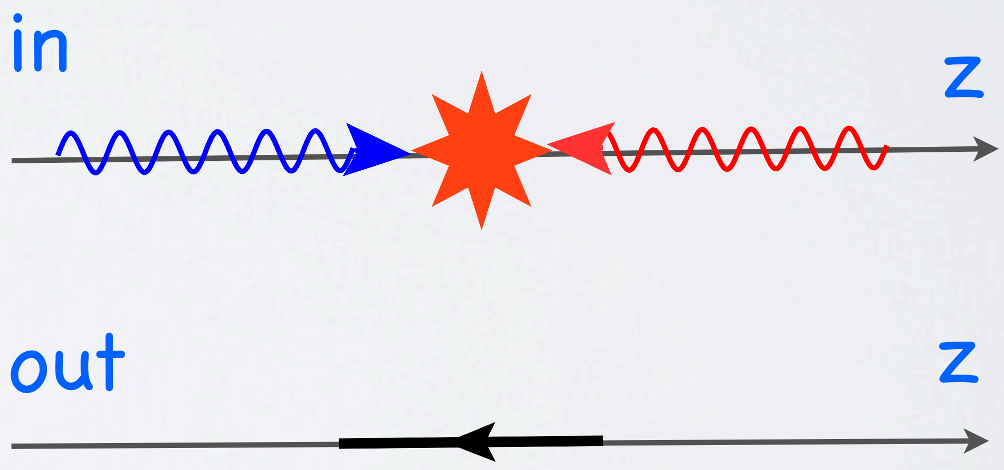
$p^2 = m_M^2$ $p^2 \simeq 0$ $q^2 = -Q^2 < 0$ $q'^2 = 0$

Breit frame

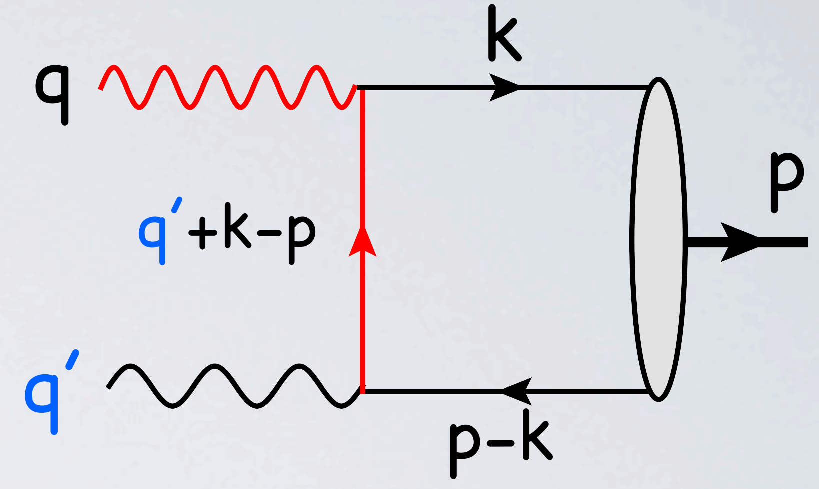
$$q = (0,0,0,-Q)$$

$$q' = \frac{1}{2}Q(1,0,0,1)$$

$$p = \frac{1}{2}Q(1,0,0,-1)$$



Introduction: $\gamma^*(q)\gamma(q') \to M(p)$



Process $\gamma^*(q)\gamma(q') \to M(p)$

$$q' = \frac{1}{2}Q(1,0,0,1) \qquad p = \frac{1}{2}Q(1,0,0,-1)$$

$$A[\gamma^*\gamma \to M] = \int dk \frac{\gamma^\mu(\not{q}' + \not{k} - \not{p})\not{\epsilon}_\gamma}{(q'+k-p)^2} \int dz e^{i(kz)} \langle M(p)|\bar{\psi}(z)\psi(0)|0\rangle$$

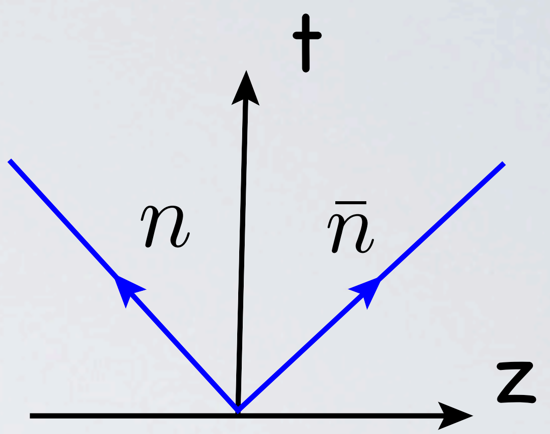
the blob is soft if $\quad k \sim p \qquad k^2 \sim p^2 \sim (kp) \sim \Lambda^2$

$$(q'+k-p)^2 = (k-p)^2 + 2q'(k-p) \simeq 2q'(k-p) \sim Q^2$$

Light-cone coordinates

$$q' = \frac{1}{2}Q(1,0,0,1) \qquad \bar{n} = (1,0,0,+1)$$

$$p = \frac{1}{2}Q(1,0,0,-1) \qquad n = (1,0,0,-1)$$



Light-cone expansion

$$V = (V\bar{n})\frac{n}{2} + (Vn)\frac{\bar{n}}{2} + V_\perp$$

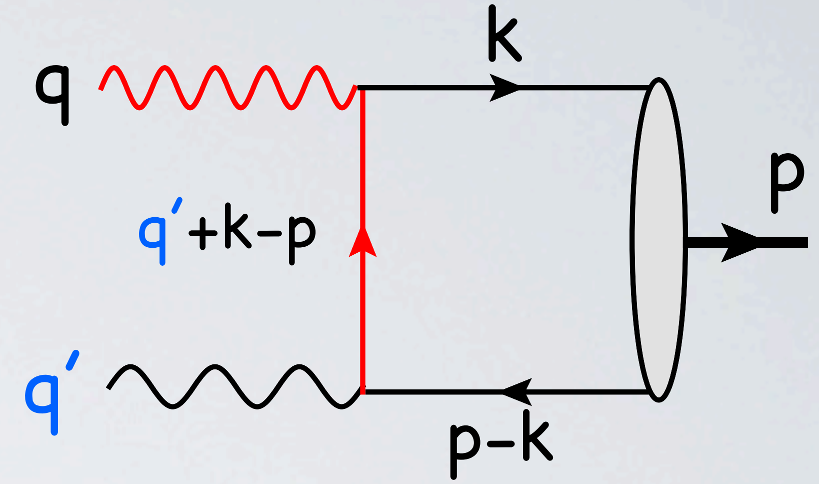
$$(Vn) \equiv V_+ = V_0 + V_z$$

$$(V\bar{n}) \equiv V_- = V_0 - V_z$$

$$(q'n) = q'_+ = Q \qquad (p\bar{n}) = p_- = Q$$

$$\int d^4k\, F[k_+, k_-, k_\perp] = \frac{1}{2}\int dk_+\, dk_-\, d^2k_\perp F[k_+, k_-, k_\perp]$$

Introduction: $\gamma^*(q)\gamma(q') \to M(p)$



Process $\gamma^*(q)\gamma(q') \to M(p)$

$$q' = Q\frac{\bar{n}}{2} \qquad p = Q\frac{n}{2}$$

$$A[\gamma^*\gamma \to M] = \int dk \frac{\gamma^\mu(\not{q}' + \not{k} - \not{p})\not{\epsilon}_\gamma}{(q'+k-p)^2} \int dz e^{i(kz)} \langle M(p)|\bar{\psi}(z)\psi(0)|0\rangle$$

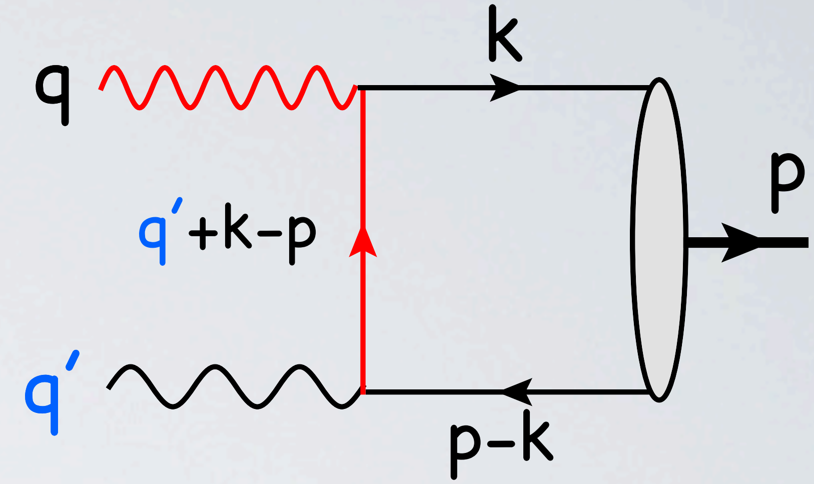
the blob is soft if $\quad k \sim p \qquad k^2 \sim p^2 \sim (kp) \sim \Lambda^2$

$$(q'+k-p)^2 = (k-p)^2 + 2q'(k-p) \simeq 2q'(k-p) \sim Q^2$$

$$\frac{\gamma^\mu(\not{q}' + \not{k} - \not{p})\not{\epsilon}_\gamma}{(q'+k-p)^2} \simeq \frac{\gamma^\mu(\not{q}' + \frac{1}{2}\not{n}(k_- - p_-))\not{\epsilon}_\gamma}{q'_+(k_- - p_-)} + \mathcal{O}(\Lambda/Q)$$

Introduction: $\gamma^*(q)\gamma(q') \to M(p)$

Process $\gamma^*(q)\gamma(q') \to M(p)$



$$q' = \frac{1}{2}Q(1,0,0,1) \qquad p = \frac{1}{2}Q(1,0,0,-1)$$

$$A[\gamma^*\gamma \to M] = \frac{1}{2}\int dk_- \frac{\mathrm{Tr}[\hat{S}(k_-/p_-)\gamma^\mu(\not{q}' + \frac{1}{2}\not{n}\,(k_- - p_-))\not{\epsilon}_\gamma]}{q'_+\,(k_- - p_-)}$$

The soft part is defined as light-cone matrix element

$$\hat{S}(k_-/p_-) = \int dz_+ e^{ik_- z_+}\langle M(p)|\bar{\psi}(z_+\bar{n})\psi(0)|0\rangle$$

Introduction: light-cone distribution amplitude

$$\hat{S}(k_-/p_-) = \int dz_+ e^{ik_- z_+} \langle M(p)|\bar{\psi}(z_+\bar{n})\psi(0)|0\rangle$$

Example: $k_-/p_- = x \qquad 0 < x < 1$

$$-ip^\mu f_\pi \phi_\pi(x) = p_- \int dz_+ e^{ixp_- z_+} \langle \pi(p)|\bar{\psi}(z_+\bar{n})\gamma^\mu\gamma_5\psi(0)|0\rangle$$

$$\langle \pi(p)|\bar{\psi}(z_+\bar{n})\gamma^\mu\gamma_5\psi(0)|0\rangle = -if_\pi p^\mu \int_0^1 dx e^{-ixp_- z_+} \phi_\pi(x)$$

$$\int_0^1 dx \phi_\pi(x) = 1 \qquad \phi_\pi(1-x) = \phi_\pi(x) \qquad |\bar{q}q(^1S_0)\rangle$$

$$\langle \pi(p)|\bar{\psi}(0)(\overleftarrow{\partial}\cdot\bar{n})^n\gamma^\mu\gamma_5\psi(0)|0\rangle = -if_\pi p^\mu(-ip_-)^n \int_0^1 dx x^n \phi_\pi(x)$$

simplest model $\qquad \phi_\pi(x,\mu) \simeq 6x(1-x) + a_2(\mu) C_2^{3/2}(2x-1)$

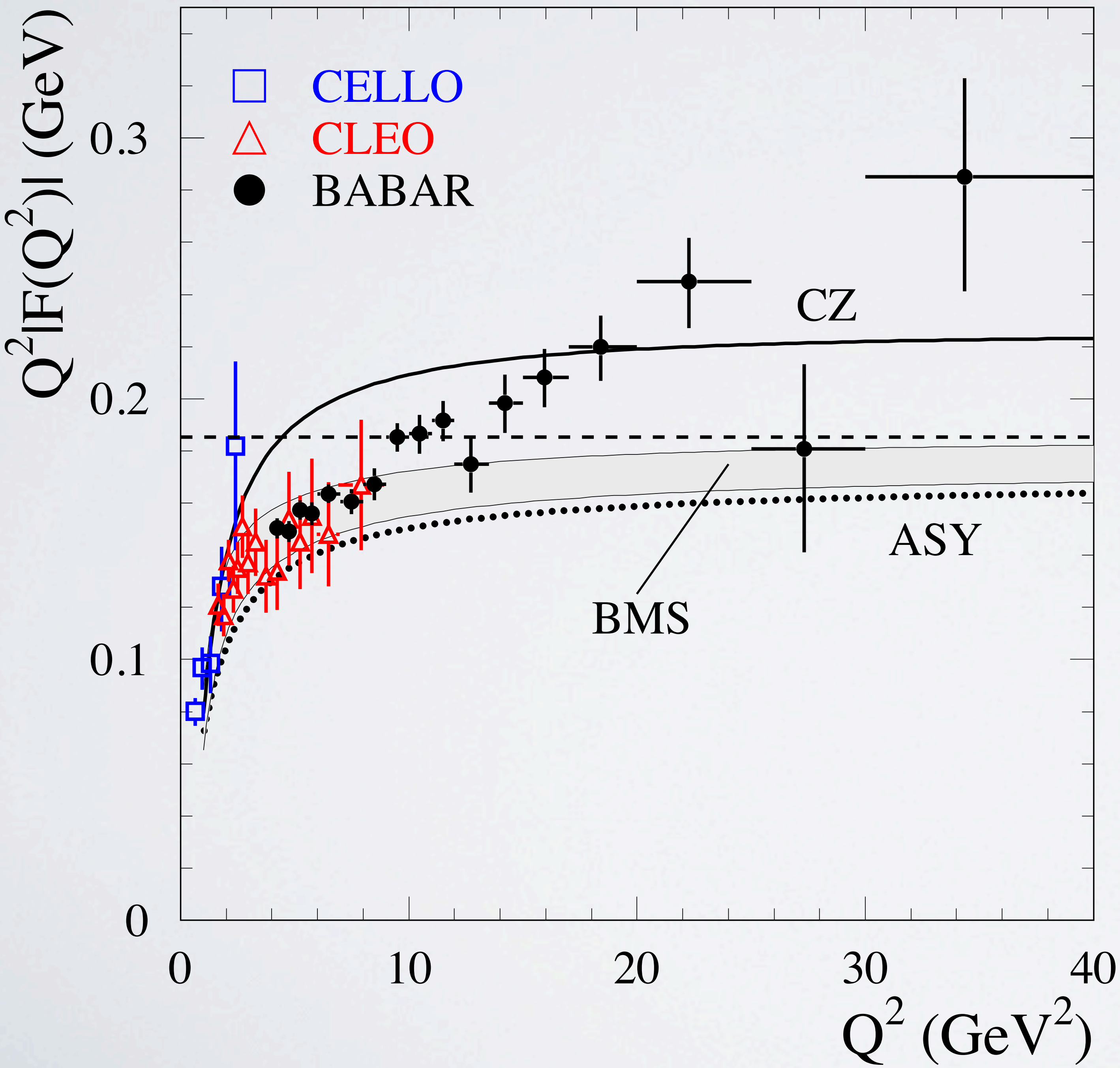
Introduction: data v. theory gamma-pion FF

$$Q^2 F_{\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int dx \frac{\phi_\pi(x)}{x}$$

$$Q^2 \to \infty$$

$$Q^2 F_{\gamma\pi}(Q^2) \to \sqrt{2} f_\pi = 0.185 \text{GeV}$$

Babar 2009



Theor. curves

Bakulev, Mikhailov, Stephanis
2001,2003,2004

Problems with understanding of DA shape and power corrections

Introduction: data v. theory gamma-pion FF

$$Q^2 F_{\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int dx \frac{\phi_\pi(x)}{x}$$

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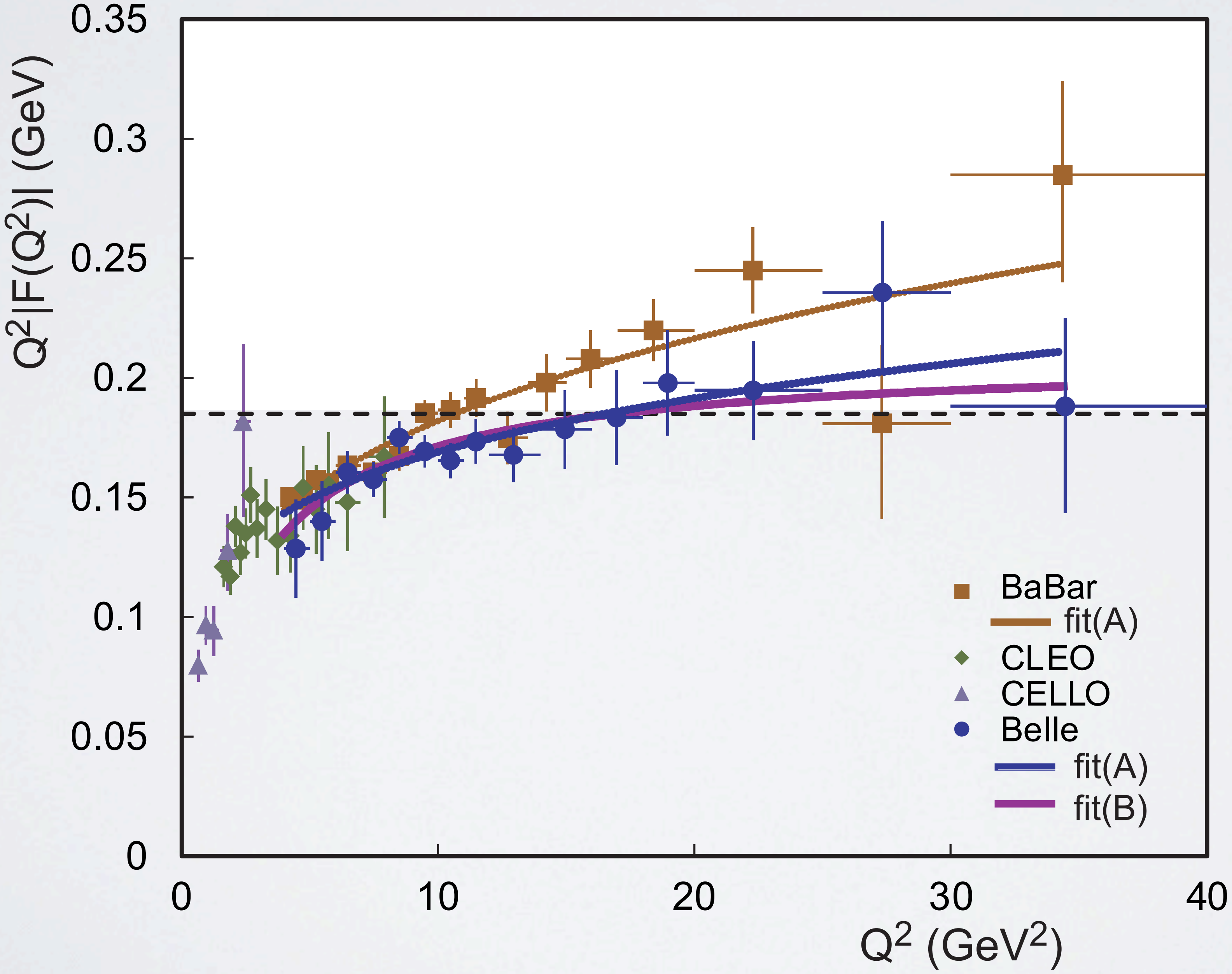
BELLE 2012

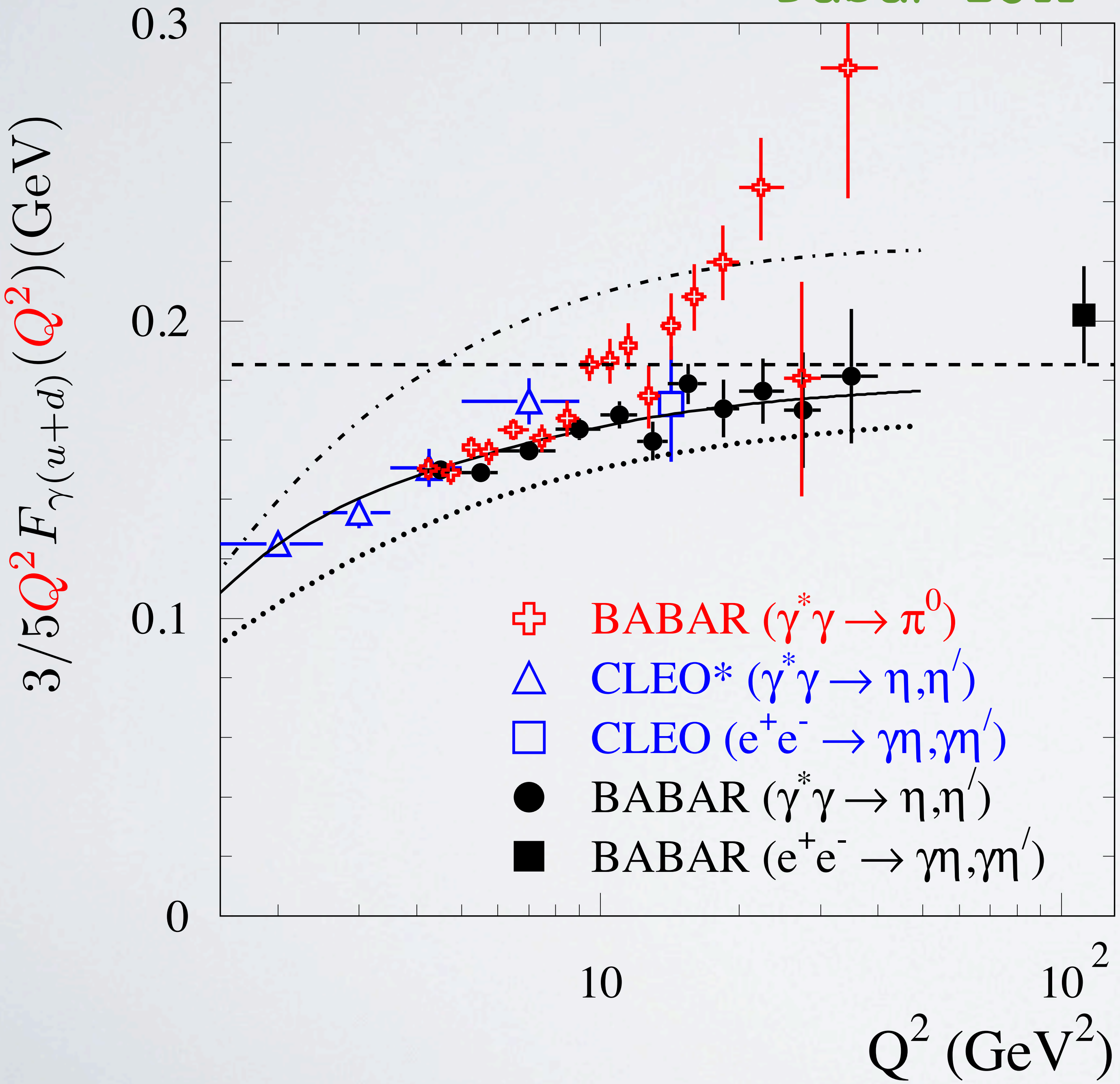


Introduction: data v. theory gamma-eta FFs

$$|\eta\rangle = \cos\varphi \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle - \sin\varphi|s\bar{s}\rangle \qquad |\eta'\rangle = \sin\varphi \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle + \cos\varphi|s\bar{s}\rangle$$

$$F_{\gamma\eta}(Q^2) = \cos\varphi F_{\gamma(u+d)}(Q^2) - \sin\varphi F_{\gamma s}(Q^2) \qquad F_{\gamma\eta'}(Q^2) = \sin\varphi F_{\gamma(u+d)}(Q^2) + \cos\varphi F_{\gamma s}(Q^2)$$

Babar 2011



$\varphi = 41^o$ Thomas, 2007

$Q^2 \to \infty$

$$Q^2 F_{\gamma(u+d)}(Q^2) \to \frac{5}{3}\sqrt{2} f_\pi$$

$$Q^2 F_{\gamma\pi}(Q^2) \to \sqrt{2} f_\pi$$

Theor. curves

Bakulev, Mikhailov, Stephanis 2003,2004

gamma-f_2 FFs

$\gamma^*(q)\gamma(q') \to f_2(p)$ $\Gamma[f_2] = 185\text{MeV}$ $Br[f_2 \to \pi\pi] \sim 85\%$

Three amplitudes (FFs)

$\gamma^*(\pm)\gamma(\pm) \to f_2(0)$ $\gamma^*(0)\gamma(\pm) \to f_2(\mp)$ $\gamma^*(\mp)\gamma(\pm) \to f_2(\mp 2)$

$T_0(Q^2)$ $T_1(Q^2)$ $T_2(Q^2)$

Q^2=0

$$\Gamma[f_2 \to \gamma\gamma] = \frac{\pi\alpha^2}{5m}\left(\frac{2}{3}|T_0(0)|^2 + |T_2(0)|^2\right) = 3.03(40)\,\text{keV}$$

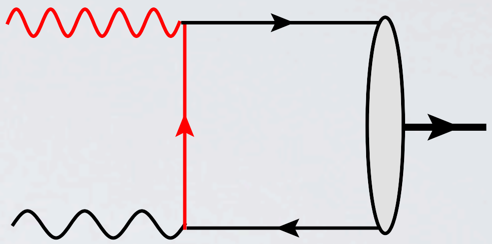
$$\frac{\Gamma^{\Lambda=0}_{\gamma\gamma}}{\Gamma^{\Lambda=2}_{\gamma\gamma}} \simeq (3.7 \pm 0.3) \times 10^{-2}$$

Belle 2008

$$|T_2(0)| \simeq \sqrt{\frac{5m}{\pi\alpha^2}\Gamma[f_2 \to \gamma\gamma]} = 339(22)\,\text{MeV}$$

gamma-f_2 T_0 FF

$T_0(Q^2)$ $\gamma^*(\pm)\gamma(\pm) \to f_2(0)$



$|\bar{q}q(^1S_0)\rangle$

$$T_0(Q^2) \simeq \langle f_q \rangle \int dx \frac{\phi_2(x)}{x}$$

$$\langle f_q \rangle = \frac{4}{9} f_u(\mu) + \frac{1}{9} f_d(\mu) + \frac{1}{9} f_s(\mu)$$

DA properties: $\phi_2(1-x) = -\phi_2(x)$ $\int_0^1 dx(2x-1)\phi_2(x) = 1$

simplest model $\phi_2(x) = 30x(1-x)(2x-1)$

normalization constant $\frac{1}{2}\langle f_2(P,\lambda)|\bar{q}\left[\gamma_\mu i \overleftrightarrow{D}_\nu + \gamma_\nu i \overleftrightarrow{D}_\mu\right] q|0\rangle = f_q m^2 e^{(\lambda)*}_{\mu\nu}$

$f_q(1\text{GeV}) = 101(10)\text{MeV}$

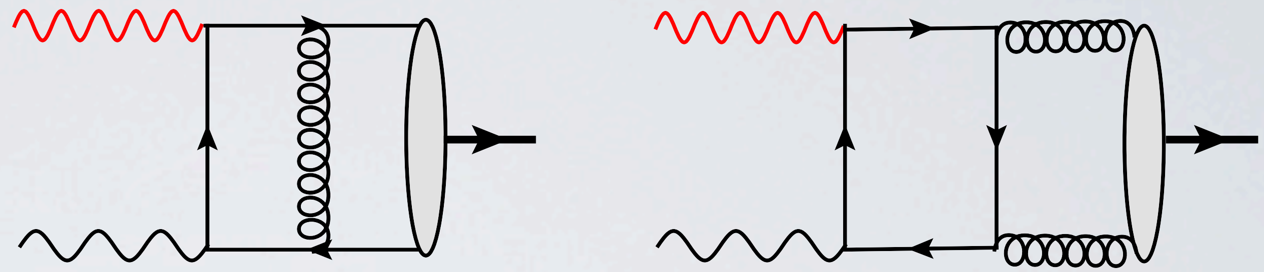
Aliev, Shifman 1982 (QCD SR, TM dom.)

Cheng, Koike, Yang 2010 (QCD SR, TM dom.)

Terazawa, 1990/ Suzuki 1993 (TM dom.)

gamma-f_2 To FF

NLO & LLog resummation
involves gluons



Gluon DA:

$$\langle f_2(P,\lambda)|G^a_{-\mu}(z_+\bar{n})G^a_{-\mu}(0)|0\rangle = -2f_g^S m^2 e^{(\lambda)}_{--}\int_0^1 dx\, e^{ixp_- z_+}\phi_g^S(x)$$

$$(Vn) \equiv V_+$$

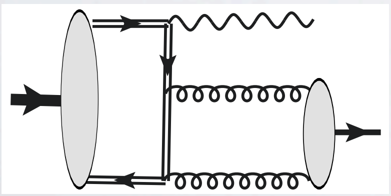
$$(V\bar{n}) \equiv V_-$$

properties $\quad \phi_g^S(1-x) = \phi_g^S(x) \qquad \int_0^1 dx\phi_g^S(x) = 1$

simplest model $\quad \phi_g^S(x) = 30x^2(1-x)^2$

normalization constant $\quad f_g^S(1\,\mathrm{GeV}) \approx 0 \qquad f_g^S(\mu^2 = 4m_b^2) = (14.9 \pm 0.8)\,\mathrm{MeV}$

$\Upsilon(1S) \to \gamma f_2$



$$\frac{Br[\Upsilon(1S)\to\gamma\ f_2]}{Br[\Upsilon(1S)\to e^+e^-]} = \frac{64\pi}{3}\frac{\alpha_s^2(4m_b^2)}{\alpha}\left(1-\frac{m^2}{M_\Upsilon^2}\right)\frac{[5f_g^S/4]^2}{m_b^2}$$

gamma-f_2 T_0 FF

$$T_0 = 5\langle f_q \rangle - \frac{\alpha_s(Q^2)}{\pi}\left(\frac{5}{27}\langle f_q \rangle + \frac{215}{27} f_g^S\right) - 5\frac{m^2}{Q^2}\langle f_q \rangle + T_0^{\rm PC}(Q^2)$$

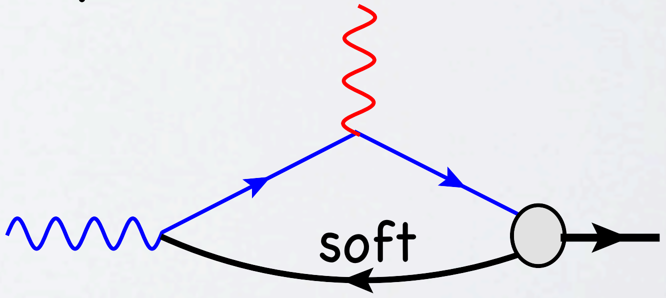
LO NLO kin. p. cor. soft overlap

DAs $\phi_2(x) = 30x(1-x)(2x-1)$ $\phi_g^S(x) = 30x^2(1-x)^2$

NLO Kivel, Mankiewicz, Polyakov 1999

kin. p. cor. a part of the target mass correction

soft overlap power corrections due to the overlap of the in-out wave functions (LCSR method)

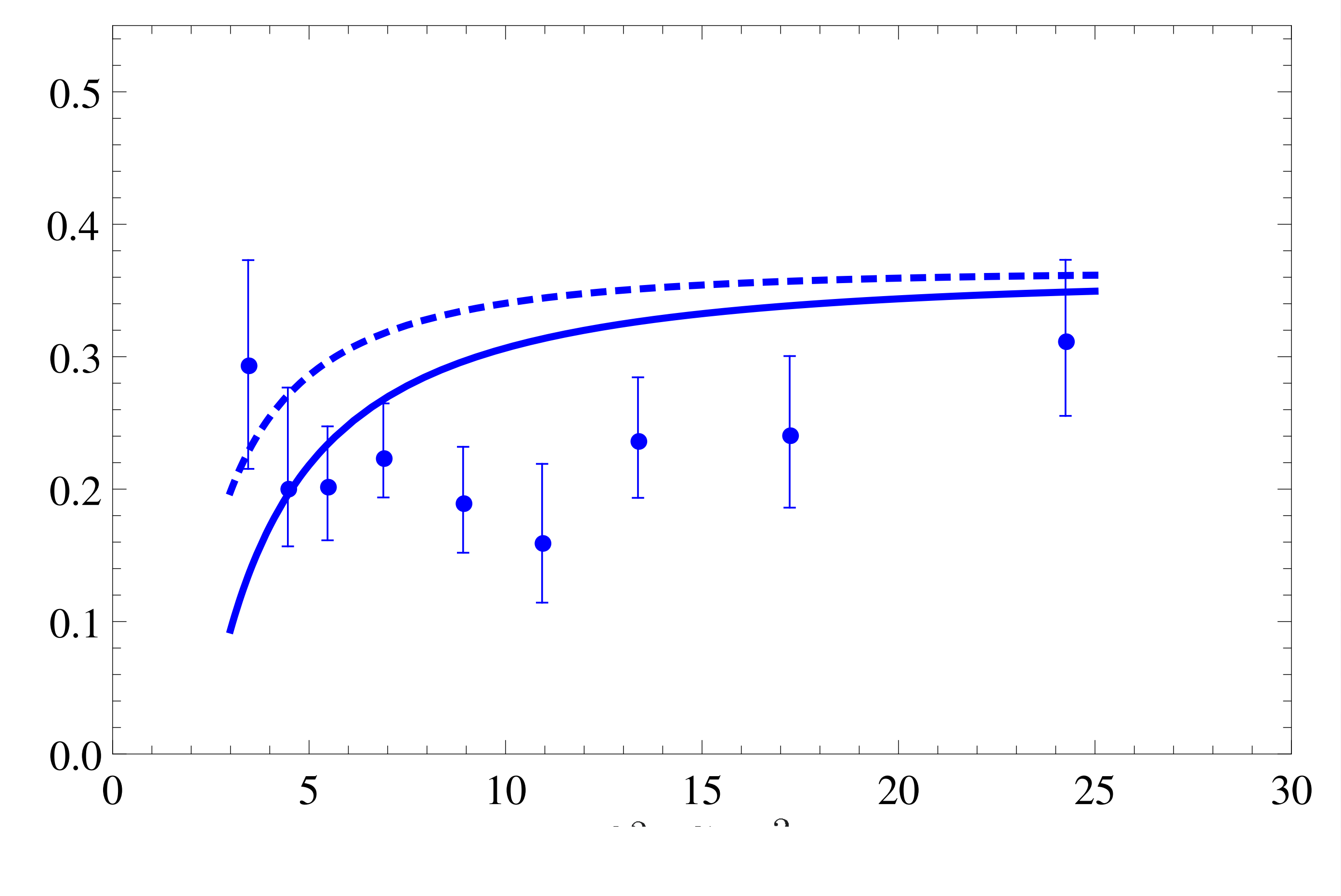


$\sim \Lambda^2/Q^2$

gamma-f_2 T_0 FF

$$T_0 = 5\langle f_q\rangle - \frac{\alpha_s(Q^2)}{\pi}\left(\frac{5}{27}\langle f_q\rangle + \frac{215}{27} f_g^S\right) - 5\frac{m^2}{Q^2}\langle f_q\rangle + T_0^{\rm PC}(Q^2)$$

soft overlap



$\frac{T_0(Q^2)}{T_2(0)}$

Q^2, GeV^2

$T_0(0)/T_2(0) \simeq 4\%$

data BELLE 2015

$3.5\mathrm{GeV}^2 \leq Q^2 \leq 24\mathrm{GeV}^2$

it seems that the value of f_q is overrated ...

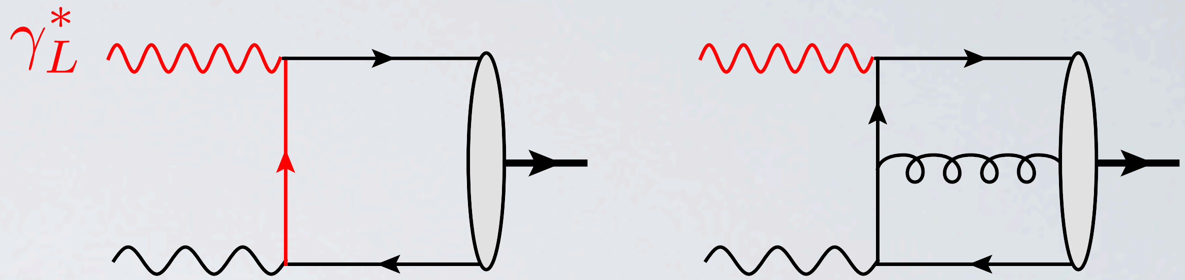
good idea: to extract normalization from

$e^+e^- \to \gamma^* \to f_2 + \gamma$

$q^2 \simeq 100\mathrm{GeV}^2$

gamma-f_2 T_1 FF

$T_1(Q^2)$ $\gamma^*(0)\gamma(\pm) \to f_2(\mp)$



DAs of twist-3:

$$\langle f_2(P,\lambda)|\bar\psi(z_+\bar n)\overleftrightarrow{D}_\perp\gamma_-\psi(0)|0\rangle \qquad |\bar q q(^1P_1)\rangle$$

$$\langle f_2(P,\lambda)|\bar\psi(z_+\bar n)G_{-\perp}(vz_+\bar n)\gamma_-(1,\gamma_5)\psi(0)|0\rangle \quad |\bar q q(^1S_0)g\rangle$$

In the cross section contribution with $T_1(Q^2)$ is suppressed by factor $\frac{\Lambda^2}{Q^2}$

At LO there are only 2 new DAs

models

$$\Phi_3(\alpha) = 360\alpha_1\alpha_2^2\alpha_3\left[\zeta_3 + \frac{1}{2}\omega_3(7\alpha_2 - 3) + \dots\right]$$

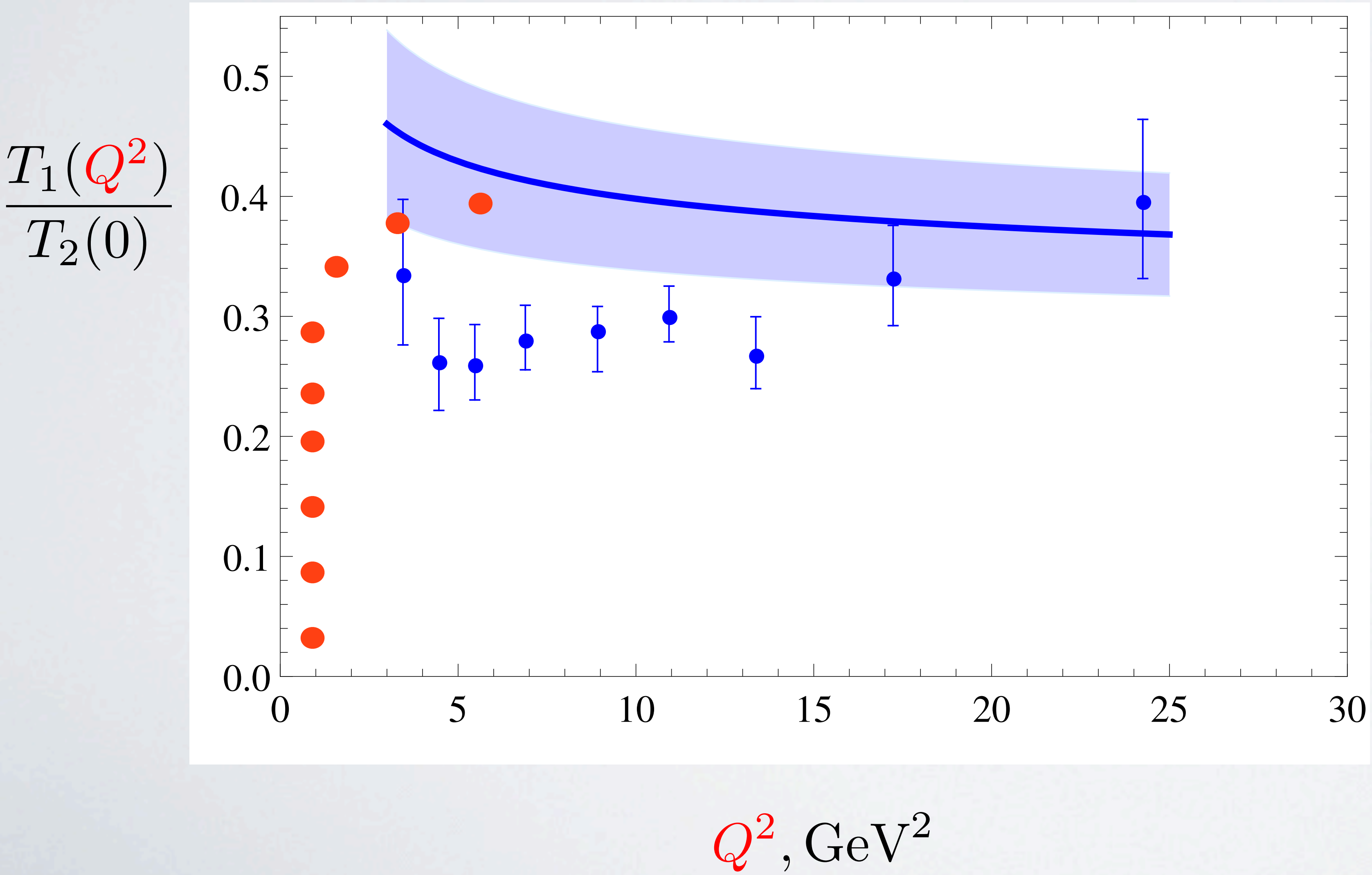
$$\widetilde\Phi_3(\alpha) = 360\alpha_1\alpha_2^2\alpha_3\left[0 + \frac{1}{2}\widetilde\omega_3(\alpha_1 - \alpha_3) + \dots\right]$$

$\mu = 1\text{GeV}$ $\zeta_3 = 0.15(8)$ $\omega_3 = -0.2(3)$ $\widetilde\omega_3 = 0.06(1)$ QCD sum rule estimates

gamma-f₂ T₀ FF

$$T_1(Q^2) = \frac{10}{3}\langle f_q\rangle\left[1 + 4\zeta_3 + \frac{9}{16}(\omega_3 - \widetilde{\omega}_3)\right]$$

$\mu = 1\text{GeV}$ $\zeta_3 = 0.15(8)$ $\omega_3 = -0.2(3)$ $\widetilde{\omega}_3 = 0.06(1)$ QCD SR



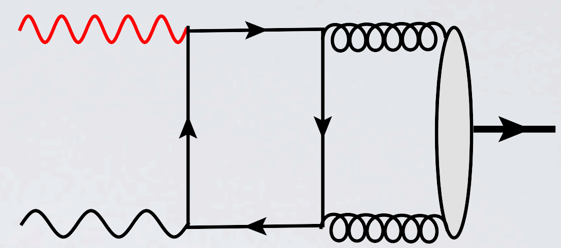
data BELLE 2015

$3.5\text{GeV}^2 \leq Q^2 \leq 24\text{GeV}^2$

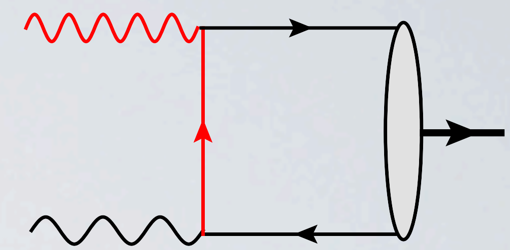
it seems that the value of f_q is overrated ...

gamma-f$_2$ T$_2$ FF

$T_2(Q^2)$ $\gamma^*(\mp)\gamma(\pm) \to f_2(\mp 2)$



$|gg(^5S_2)\rangle$



$|\bar{q}q(^1D_2)\rangle$

gluon DA of twist-2: do not mix with quarks!

$|gg(^5S_2)\rangle$ $$\langle f_2(P,\lambda)|G^a_{-\{\mu}(z_+\bar{n})G^a_{-\nu\}}(0)|0\rangle = f_g^T[e^\perp_{\mu\nu} - \frac{1}{2}g^\perp_{\mu\nu}m^2 e^{(\lambda)}_{--}]\int_0^1 dx\, e^{ixp_-z_+}\phi_g^T(x)$$

$|\bar{q}q(^1D_2)\rangle$ $$\langle f_2(P,\lambda)|\bar{\psi}(z_+\bar{n})\overleftrightarrow{D}_{\{\perp\mu}\overleftrightarrow{D}_{\perp\nu\}}\gamma_-\psi(0)|0\rangle \sim \frac{\Lambda^2}{Q^2}$$ QCD EOM $\to \phi_q(x) + \ldots$

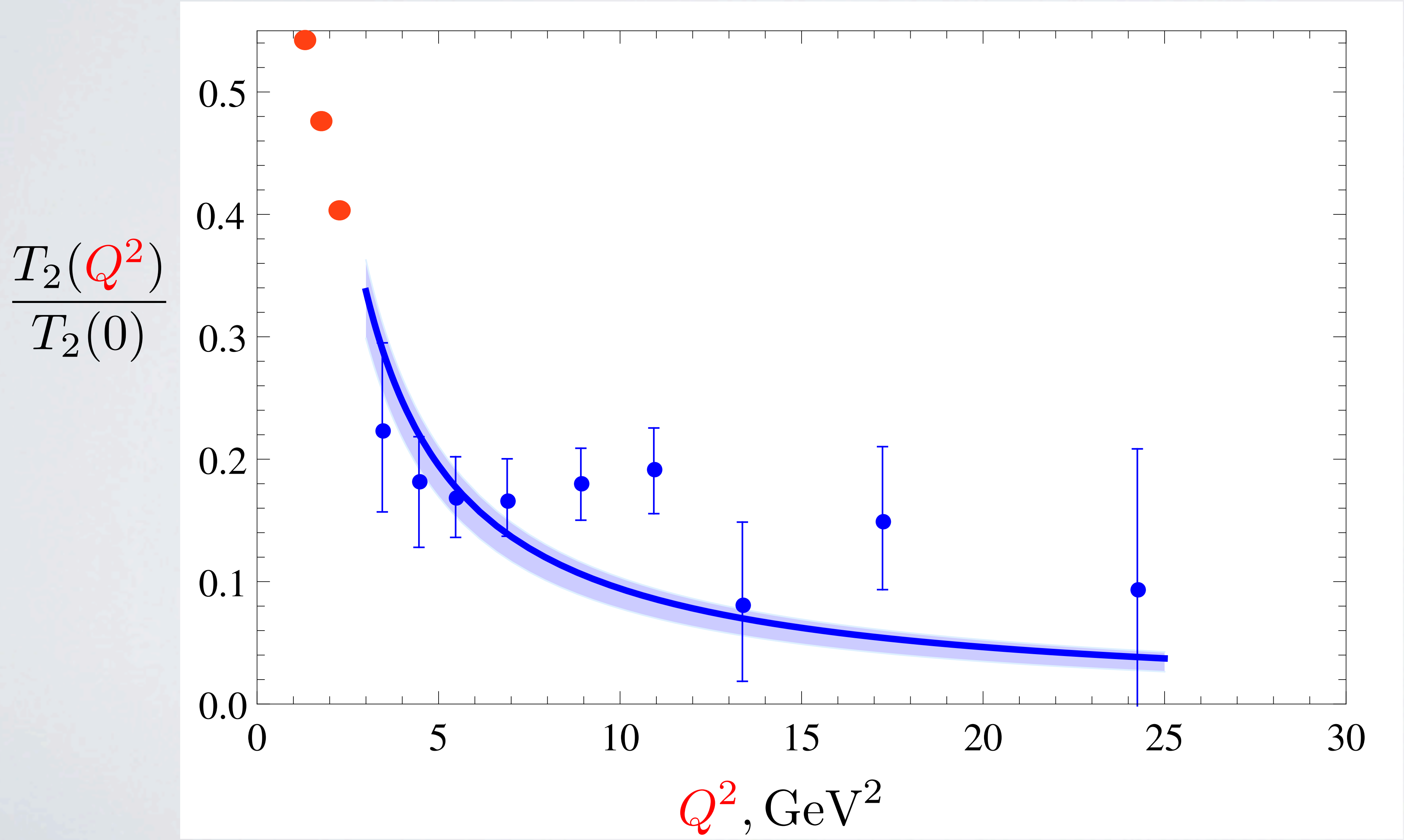
properties $\phi_g^T(1-x) = \phi_g^T(x)$ $\int_0^1 dx\phi_g^T(x) = 1$

simplest model $\phi_g^T(x) = 30x^2(1-x)^2$

gamma-f_2 T_2 FF

$$T_2(Q^2) = \frac{20}{3}\frac{m^2}{Q^2}\langle f_q\rangle + \frac{5}{3}\frac{\alpha_s(Q^2)}{\pi} f_g^T \left[1 + \frac{8}{3}\lambda(m_c^2/Q^2)\right]$$

$$f_g^T = 10 \pm 50\text{MeV}$$



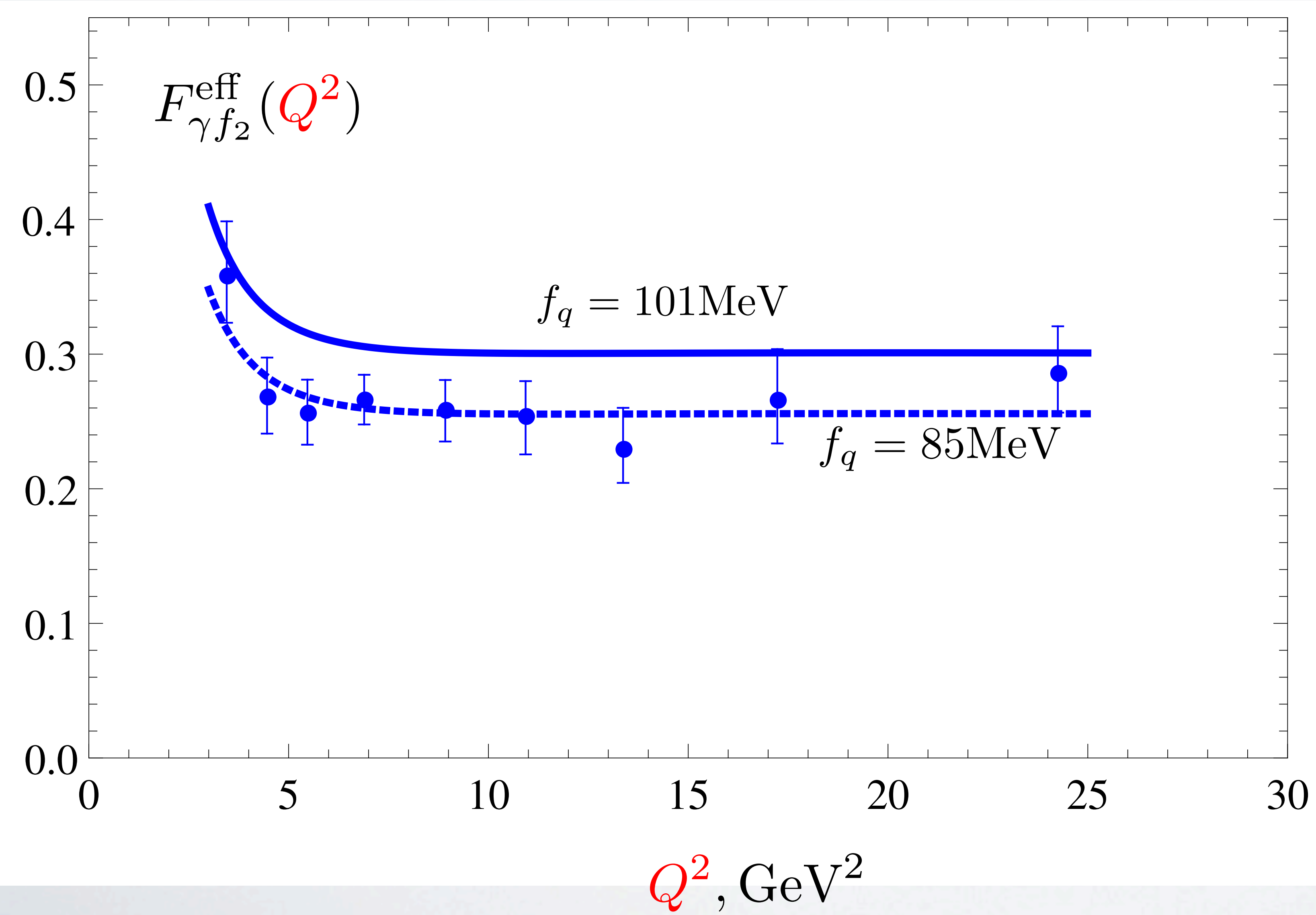
data BELLE 2015

$3.5\text{GeV}^2 \le Q^2 \le 24\text{GeV}^2$

gamma-f_2 F_{eff} FF

$$F^{\rm eff}_{\gamma f_2}(Q^2) = \sqrt{\frac{2}{3}\left|\frac{T_0(Q^2)}{T_2(0)}\right|^2 + \frac{Q^2 m^2}{(m^2+Q^2)^2}\left|\frac{T_1(Q^2)}{T_2(0)}\right|^2 + \left|\frac{T_2(Q^2)}{T_2(0)}\right|^2}$$

good scaling behavior for Q^2>5 GeV^2



data BELLE 2015

$$3.5{\rm GeV}^2 \le Q^2 \le 24{\rm GeV}^2$$

$$T_0 \simeq 5\langle f_q\rangle - \frac{\alpha_s(Q^2)}{\pi}\left(\frac{5}{27}\langle f_q\rangle + \frac{215}{27}f_g^S\right)$$

$$T_1 \simeq \frac{10}{3}\langle f_q\rangle\left[1 + 4\zeta_3 + \frac{9}{16}(\omega_3 - \tilde{\omega}_3)\right]$$

$$T_2 \simeq \frac{5}{3}\frac{\alpha_s(Q^2)}{\pi}f_g^T\left[1 + \frac{8}{3}\lambda(m_c^2/Q^2)\right]$$

Conclusions

- Theory: there is no problem with factorization as expected
- One gets a direct possibility for analysis of the subleading FFs which sensitive to the higher Fock states and specific gluonic components
- The data allows to conclude that QCD scaling is observed
- It seems that there is problem with the normalization estimate in T_0 which is about 20% overrated, but within the theor. uncertanties. This must be clarified ($e^+e^- \to \gamma^* \to f_2 + \gamma$)
- The data for the individual helicity FFs must be improved in order to perform a more quantitative theoretical analysis

Thank you!