

# Light Front Representation of Transverse Densities in the Chiral Periphery of the Nucleon

C. Granados<sup>1</sup>   C. Weiss<sup>2</sup>

<sup>1</sup>Uppsala University

<sup>2</sup>Jefferson Lab

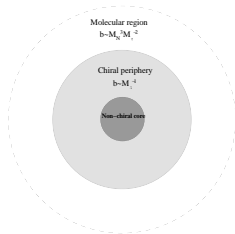
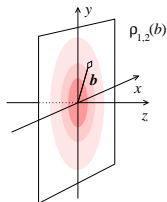
Ruhr-University Bochum

June 25, 2015

# Outline

- ▶ Transverse Densities  
Definitions, properties
- ▶ Light front formulation  
Wave functions and  $\chi$ GPDs
- ▶ Transverse polarization and mechanical interpretation  
CG, C. Weiss, arXiv:1503.02055 [hep-ph]  
CG, C. Weiss, arXiv:1503.04839 [hep-ph]
- ▶ Energy momentum tensor and peripheral OAM

# Transverse Densities



M.Strikman, C.Weiss PRC82(2010)  
CG, C.Weiss JHEP 1401 (2014)

Connect Form Factors and GPDs to nucleon intrinsic spacial structure

$$F_{1(2)}(-\Delta_T^2) = \int d^2 b e^{i\Delta_T \cdot b} \rho(b)_{1(2)}$$

$$\rho_1(b) = \int dx \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i\Delta_T \cdot b} H(x, -\Delta_T^2)$$

G.A. Miller, ARNPS 60 (2010)

M.Burkardt, PRD62(2000)

Parametrize the light front electromagnetic current density

$$\frac{J^+(b)}{2p^+} = \underbrace{\rho_1(b)}_{spin-ind.} + 2 \underbrace{\mathbf{S}_T \cdot (\mathbf{e}_z \times \mathbf{e}_b)}_{spin-dep.} \tilde{\rho}_2(b)$$

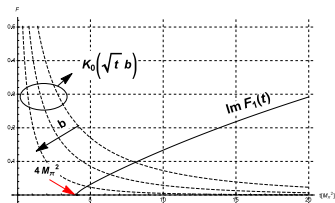
with

$$\tilde{\rho}_2(b) = \frac{1}{2M_N} \frac{\partial \rho_2(b)}{\partial b}$$

Boost invariants; structure of the nucleon as a relativistic multiparticle systems.

Nucleon transverse profile; Define dynamical regions in impact parameter space; e.g., non chiral, chiral  $[M_\pi^{-1}]$ , and molecular  $[M_N^2 M_\pi^{-3}]$

# TD from spectral functions in $\chi$ PT

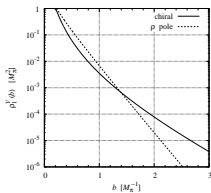
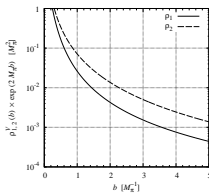
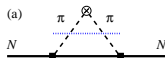


Filter high momentum contributions

$$\rho_{1(2)} = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\text{Im} F_{1(2)}(t+i0)}{\pi}$$

FF from leading order  $\chi$ PT

$$\begin{aligned} \mathcal{L}_{\pi N} = & -\frac{g_A}{F_\pi} \bar{\psi} \gamma^\mu \gamma_5 \tau \psi \cdot \partial_\mu \pi \\ & -\frac{1}{4F_\pi^2} \bar{\psi} \gamma^\mu \tau \psi \cdot \pi \times \partial_\mu \pi \end{aligned}$$

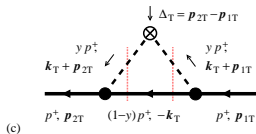
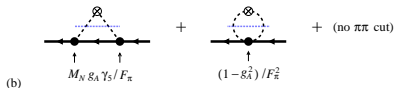
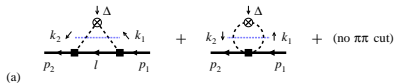


M.Strikman, C.Weiss PRC82(2010)

CG, C.Weiss JHEP 1401 (2014)

- ▶ subthreshold singularity (on-shell intermediate nucleon)Molecular region
- ▶ use of HB expansions in chiral region
- ▶ chiral component becomes dominant for  $b > 2\text{fm}$

# Pion loops in Light-front PT



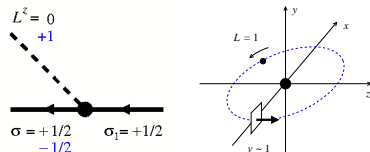
Chiral dynamics in the nucleon's periphery;  
 Defines structure at  $b \sim \frac{1}{M_\pi}$ . Dominated by the behavior of spectral functions near the  $2\pi$  threshold ( $k_\pi \sim M_\pi$ , already relativistic pions).

Pion current contributions: from Chiral Lagrangian with axial vector  $\pi N$  coupling ( $\gamma_5 \not{k}_\pi$ ) + contact term to pseudoscalar coupling + modified contact term ( $\sim 10\%$  contribution)

In LF variables ( $v^\pm = v^0 \pm v^z$ ),

$$\frac{J^+}{2p^+} = \frac{1}{2\pi} \int \frac{dy}{y(\bar{y} = 1 - y)} \frac{d^2 k_T}{(2\pi)^2} \Psi^\dagger(y, k_T, \mathbf{p}_{2T}) \Psi(y, k_T, \mathbf{p}_{1T}) + \underbrace{(1 - g_A)\delta(y)[\dots]}_{\text{contact term}}$$

# Light front wave functions



$$\Psi(y, \tilde{\mathbf{k}}_T = \mathbf{k}_T + y\mathbf{p}_{1T}) \equiv \frac{\Gamma(y, \tilde{\mathbf{k}}_T)}{\underbrace{\Delta\mathcal{M}^2(y, \tilde{\mathbf{k}}_T)}_{\text{Inv. Mass difference}}}$$

while in transverse coordinate space,

$$\begin{aligned} \Phi(y, r_T) &= \int \frac{d^2\tilde{\mathbf{k}}_T}{(2\pi)^2} e^{i\tilde{\mathbf{k}}_T \cdot \mathbf{r}_T} \Psi(y, \tilde{\mathbf{k}}_T) \\ &= -2i \left[ U_0(y, r_T) S^z + i \frac{U_1(y, r_T) \mathbf{r}_T \cdot \mathbf{S}_T}{r_T} \right] \end{aligned}$$

Eigenfunctions of LF Hamiltonian.

Allow quantum mechanical description of peripheral dynamics.

Computable at leading order in chiral periphery

CG, C. Weiss, arXiv:1503.04839 [hep-ph]

$$\begin{aligned} \Gamma(y, \tilde{\mathbf{k}}_T) &\approx \frac{g_A M_N}{F_\pi} \bar{u}(y, \mathbf{k}_T) i\gamma_5 u(p_{1T}) \\ &= \frac{2ig_A M_N^2}{F_\pi \sqrt{y}} \left[ y \mathbf{S}_z + \frac{\tilde{\mathbf{k}}_T \cdot \mathbf{S}_T}{M_N} \right] \end{aligned}$$

with radial functions,

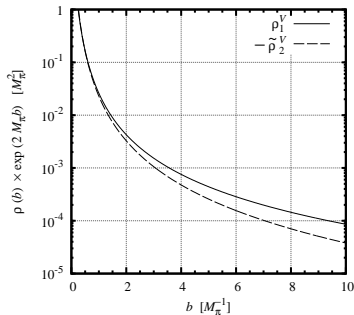
$$\left. \begin{aligned} U_0(y, r_T) \\ U_1(y, r_T) \end{aligned} \right\} = \frac{g_A M_N y \sqrt{y}}{2\pi F_\pi} \begin{cases} y M_N K_0(M_T r_T) \\ M_T K_1(M_T r_T) \end{cases},$$

and transverse mass

$$M_T^2 = \tilde{y}^2 M_\pi + y^2 M_N^2$$

# Charge and magnetization densities

$$\rho_1^V = \begin{array}{cc} \begin{array}{c} 0 \\ 1 \\ \nearrow \\ \leftarrow \bullet \leftarrow \\ + \quad \quad \quad \pm \end{array} & \begin{array}{c} 0 \\ 1 \\ \nwarrow \\ \leftarrow \bullet \leftarrow \\ \pm \quad \quad \quad + \end{array} \\ \rho_2^V = \begin{array}{cc} \begin{array}{c} 1 \\ 0 \\ \nearrow \\ \leftarrow \bullet \leftarrow \\ - \quad \quad \quad \pm \end{array} & \begin{array}{c} 0 \\ 1 \\ \nwarrow \\ \leftarrow \bullet \leftarrow \\ \pm \quad \quad \quad + \end{array} \end{array}$$



Light front current matrix as wavefunction overlap,

$$\frac{J(b)}{2\rho^+} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}} \Phi^\dagger \left( y, \frac{b}{\bar{y}} \right) \Phi \left( y, \frac{b}{\bar{y}} \right),$$

then in terms of radial functions

$$\left. \begin{array}{l} \rho_1^V(b) \\ \tilde{\rho}_2^V(b) \end{array} \right\} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^3} \left\{ \begin{array}{l} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2 \\ -2 U_0(y, b/\bar{y}) U_1(y, b/\bar{y}) \end{array} \right\}.$$

Inequality and positive definiteness of light front current,

$$|\rho_1(b)| > \tilde{\rho}_2(b) \Rightarrow J^+(b) > 0$$

weakly bound pions.

Near parametric equality,

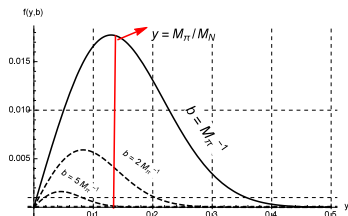
$$\rho_1(b) \approx -\tilde{\rho}_2(b) \Rightarrow \text{relativistic pion-nucleon system}$$

Explain through left right asymmetry in Transverse densities (See transverse polarization.)

# Charge magnetization densities and $\chi$ GPDs

$$\rho_1^V = \begin{array}{cc} & \begin{array}{c} 0 \\ 1 \end{array} \\ \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} & \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} \\ \begin{array}{c} + \\ \underline{\quad} \\ - \end{array} & \begin{array}{c} + \\ \underline{\quad} \\ - \end{array} \end{array} \quad \begin{array}{cc} & \begin{array}{c} 0 \\ 1 \end{array} \\ \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} & \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} \\ \begin{array}{c} - \\ \underline{\quad} \\ + \end{array} & \begin{array}{c} - \\ \underline{\quad} \\ + \end{array} \end{array}$$

$$\tilde{\rho}_2^V = \begin{array}{cc} & \begin{array}{c} 1 \\ 0 \end{array} \\ \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} & \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} \\ \begin{array}{c} - \\ \underline{\quad} \\ + \end{array} & \begin{array}{c} - \\ \underline{\quad} \\ + \end{array} \end{array} \quad \begin{array}{cc} & \begin{array}{c} 0 \\ 1 \end{array} \\ \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} & \begin{array}{c} \leftarrow \\ \bullet \\ \rightarrow \end{array} \\ \begin{array}{c} + \\ \underline{\quad} \\ - \end{array} & \begin{array}{c} + \\ \underline{\quad} \\ - \end{array} \end{array}$$



Light front current matrix as wavefunction overlap,

$$\frac{J(b)}{2\rho^+} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}} \Phi^\dagger \left( y, \frac{b}{\bar{y}} \right) \Phi \left( y, \frac{b}{\bar{y}} \right),$$

then in terms of radial functions

$$\left. \begin{array}{l} \rho_1^V(b) \\ \tilde{\rho}_2^V(b) \end{array} \right\} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^3} \left\{ \begin{array}{l} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2 \\ -2 U_0(y, b/\bar{y}) U_1(y, b/\bar{y}) \end{array} \right\}.$$

Integrands correspond to Fourier trans. of GPDs,  $H(y, t)$  and  $E(y, t)$ , i.e.,  $f_1(y, b)$ ,  $f_2(y, b)$  respectively,

$$\left. \begin{array}{l} f_1^V(y, b) \\ \tilde{f}_2^V(y, b) \end{array} \right\} = \frac{1}{2\pi} \frac{1}{y\bar{y}^3} \left\{ \begin{array}{l} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2 \\ -2 U_0(y, b/\bar{y}) U_1(y, b/\bar{y}) \end{array} \right\}.$$



# Transverse polarization and asymmetry

CG, C. Weiss, arXiv:1503.04839 [hep-ph]

LFWF components of a transversely polarized nucleon,

$$\begin{aligned}\Phi_{\text{tr}}(+, +) &= \sin \alpha U_1, \\ \Phi_{\text{tr}}(-, -) &= -\sin \alpha U_1, \\ \Phi_{\text{tr}}(+, -) &= U_0 + \cos \alpha U_1, \\ \Phi_{\text{tr}}(-, +) &= -U_0 + \cos \alpha U_1,\end{aligned}$$

Define Left and Right transverse densities from LFWF at  $\alpha = 0$ ,

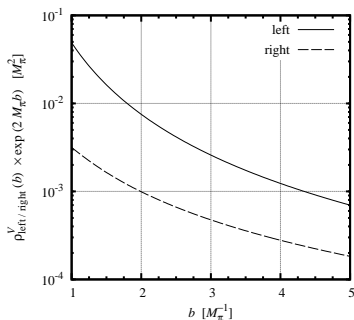
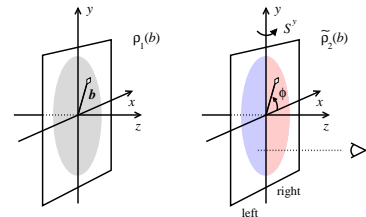
$$\left. \begin{aligned}\rho_{\text{left}}^{\text{V}}(b) \\ \rho_{\text{right}}^{\text{V}}(b)\end{aligned} \right\} = \int_0^1 dy \frac{|\Phi_{\text{tr}}(y, \mp r_T \mathbf{e}_x; -, +)|^2}{2\pi y \bar{y}^3} \quad [r_T = b/\bar{y}]$$

to find for the charge and magnetization densities that

$$\left. \begin{aligned}\rho_1^{\text{V}}(b) \\ \tilde{\rho}_2^{\text{V}}(b)\end{aligned} \right\} = \frac{1}{2} [\pm \rho_{\text{left}}^{\text{V}}(b) + \rho_{\text{right}}^{\text{V}}(b)].$$

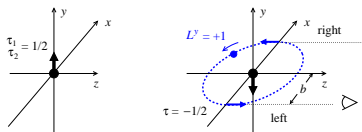
$-\tilde{\rho}_2$  measures Left-Right asymmetry of LF currents in the nucleon.

Strikingly large in the chiral periphery, generates the near equality  $\rho_1 \approx -\tilde{\rho}_2$ .



# Quantum Mechanical Picture

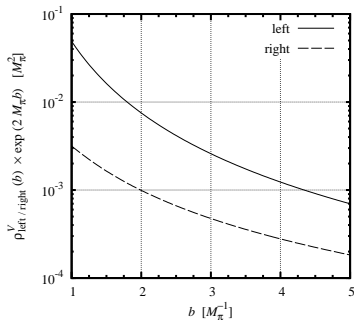
CG, C. Weiss, arXiv:1503.02055 [hep-ph]



initial/final state



intermediate state



For a nucleon to pion-nucleon transition with  $J_y = +1/2$ ,

$$\Phi_{\text{tr}}(-, +)$$

is the only contribution in x-z plane and classically represents its  $L^y = +1$  state. Then,

$$\langle J^+(\pm b \mathbf{e}_x) \rangle = \rho_{r(l)}(b)$$

Peripheral pions are weakly bound,

$$\begin{aligned} J^+ &= 2k^+ \\ &\approx 2(\sqrt{k^2 + M_\pi^2} + k_z) \end{aligned}$$

Then

$$J^+ > 0$$

as shown by the inequality  $|\rho_1| > |\tilde{\rho}_2|$ .

For non relativistic pion

$$\frac{\rho_{\text{left}}}{\rho_{\text{right}}} = \frac{E_\pi + k^z}{E_\pi - k^z} = 1 + O(v)$$

while for peripheral pions  $k_z \sim M_\pi$ , then

$$\frac{\rho_{\text{left}}}{\rho_{\text{right}}} \gg 1$$

# Energy Momentum Tensor

- ▶ EMT decomposition and form factors

$$\langle N' | \Theta^{\mu\nu} | N \rangle = \hat{u}(p') \left[ P^{(\mu} \gamma^{\nu)} A(\Delta^2) + \frac{P^{(\mu} \sigma^{\nu)\alpha} q_\alpha}{2M_N} B(\Delta^2) + \dots \right] u(p).$$

- ▶ FF associated to angular momentum

$$J(t) \equiv \left( \frac{1}{2} + t \frac{1}{3} \frac{\partial}{\partial t} \right) (A(t) + B(t))$$

e.g., M. Polyakov PLB555(2003)

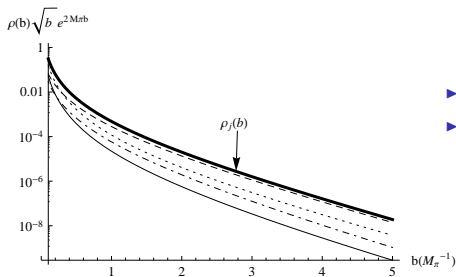
satisfies

$$J_N = \frac{1}{2}(A(0) + B(0)),$$

- ▶ Transverse density of Angular momentum

$$\rho_J(b) = \frac{1}{3} \left[ (\rho_A(b) + \rho_B(b)) - b \frac{\partial}{\partial b} (\rho_A(b) + \rho_B(b)) \right]$$

# OAM in Chiral Periphery



- ▶ Transverse densities from spectral functions.

$$\rho_{A(B)} = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\text{Im}A(B)(t+i0)}{\pi}$$

- ▶ No contact term contributions to  $A$  and  $B$ .
- ▶ Light Front representation as overlap integral

$$\frac{\Theta^{++}}{2p^+} = \frac{1}{2\pi} \frac{3}{2} \int \frac{dy}{y\bar{y}} \frac{d^2k_T}{(2\pi)^2} y \Psi^\dagger(y, k_T, \mathbf{p}_{2T}) \Psi(y, k_T, \mathbf{p}_{1T})$$

while for transverse densities,

$$\left. \begin{array}{l} \rho_A^V(b) \\ \tilde{\rho}_B^V(b) \end{array} \right\} = \frac{1}{2\pi} \frac{3}{2} \int \frac{dy}{y\bar{y}^3} \mathbf{y} \left\{ \begin{array}{l} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2 \\ -2 U_0(y, b/\bar{y}) U_1(y, b/\bar{y}) \end{array} \right\}.$$

## Summary and Outlook

Transverse densities computed in Chiral EFT were used in a model independent approach to describe features of the long distance structure of the nucleon.

The light front formulation reveals how general properties of chiral transverse densities are connected to a mechanical picture of the nucleon's periphery.

- ▶ Inequality and thus positive definiteness of  $J^+$  tied to the onset of quasi-free pions.
- ▶ Near equality  $\rho_1 \approx \tilde{\rho}_2$  originated by large asymmetry evidencing the relativistic dynamics dominating the periphery. Potentially observable in form factor extractions at low momentum transfer.

This framework opens the possibility of fully exploring the role that chiral dynamics plays constraining the nucleons internal structure.

- ▶ Transverse densities associated with form factors of the energy momentum tensor. Distributions of matter and OAM in impact parameter space.
- ▶ Expand on different intermediate baryons. (Ongoing) work on intermediate  $\Delta$  probes large  $N_c$  limit properties of LCWF and allows the study of higher orbital modes