# Pion-photon transition form factor in light-cone sum rules

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# Outline:

- Experimental background
- Factorization of Pion-photon Transition Form Factor (FF)
- Light Cone Sum Rule (LC SR)
- Pion Distribution Amplitude (DA) from QCD Sum Rule
- Data Analysis
- Pion DA from Data
- Conclusions

# Pion transition form factor $\gamma^*\gamma \to \pi$ data

One exclusive reaction for which very accurate data exist is the transition FF $F^{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ that has been measured in several experiments $e^+e^- \to e^+e^-\pi^0$ using $q_2^2 \approx 0$.

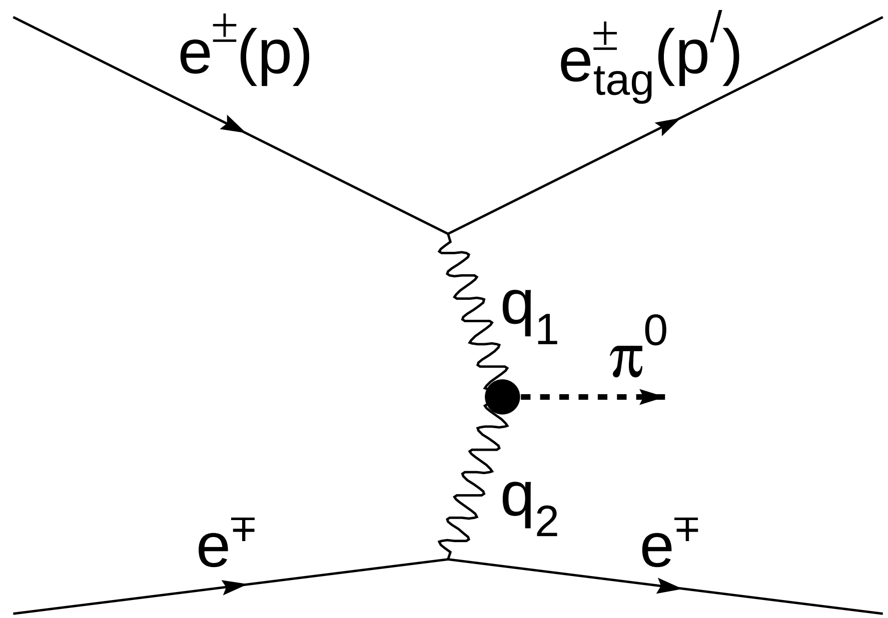
**CELLO** (1991) $0.7 - 2.2$ GeV$^2$ ,

**CLEO** (1998) $1.6 - 8.0$ GeV$^2$ ,

**BaBar** (2009) $4 - 40$ GeV$^2$ ,

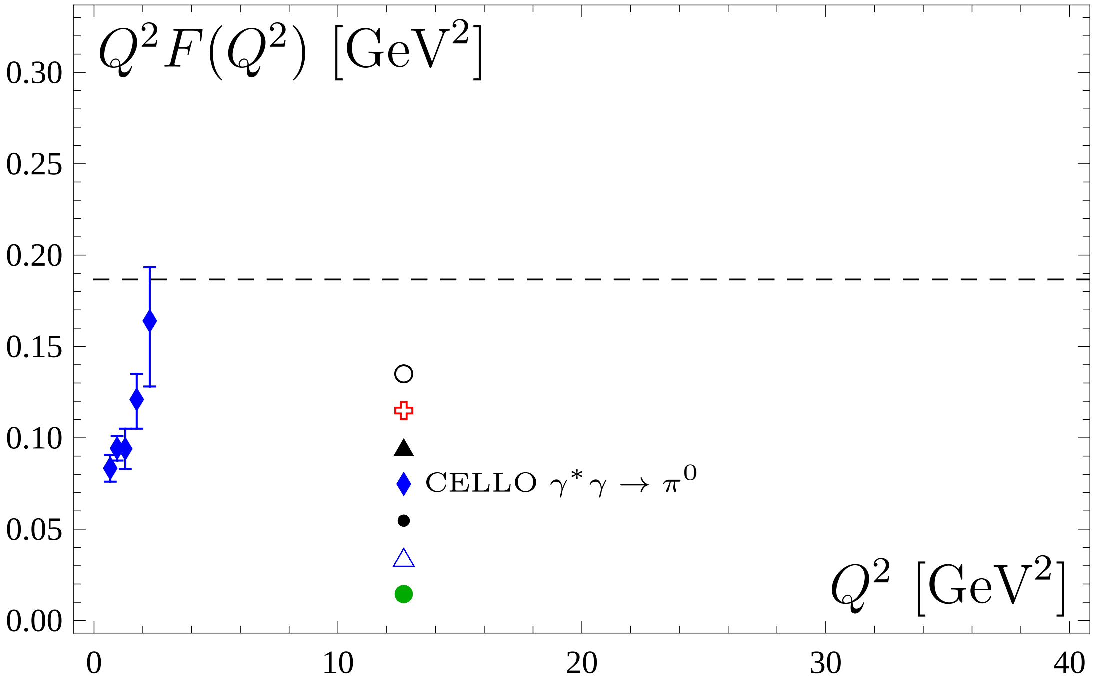
**Belle** (2012) $4 - 40$ GeV$^2$ ,

**BESIII** (????) $< 10$ GeV$^2$ .



# Pion-gamma transition FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle**




| Data | Collab. |
|---|---|
| ◆ | **CELLO (1991)** |
| | |
| dashed line = $\sqrt{2}\, f_\pi$ | |

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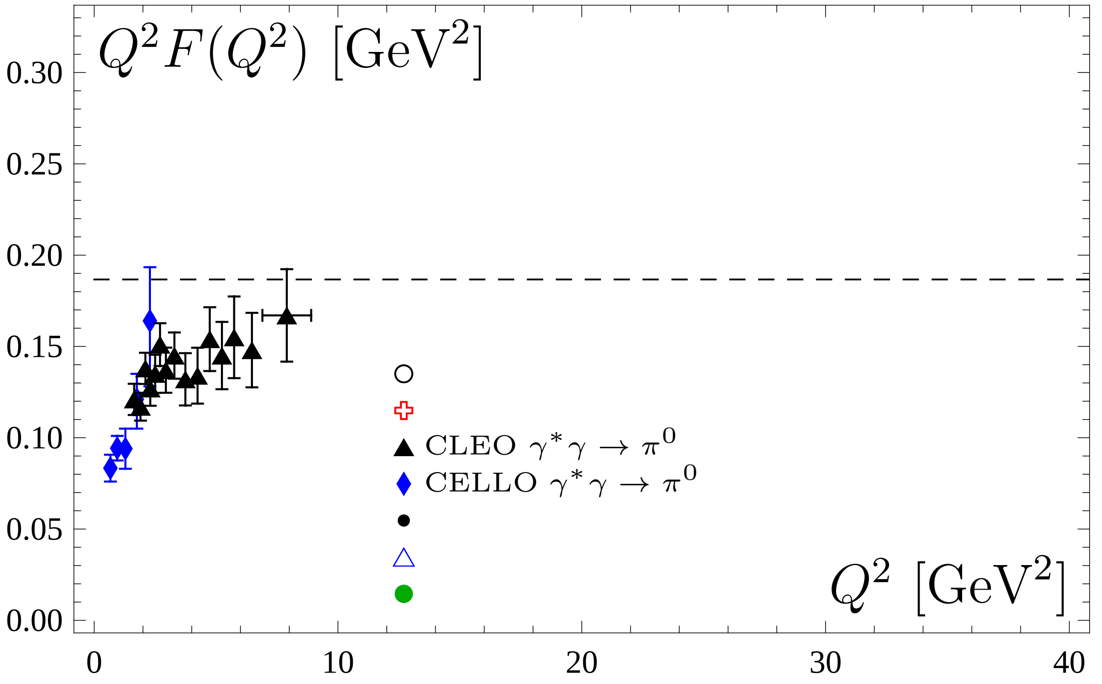


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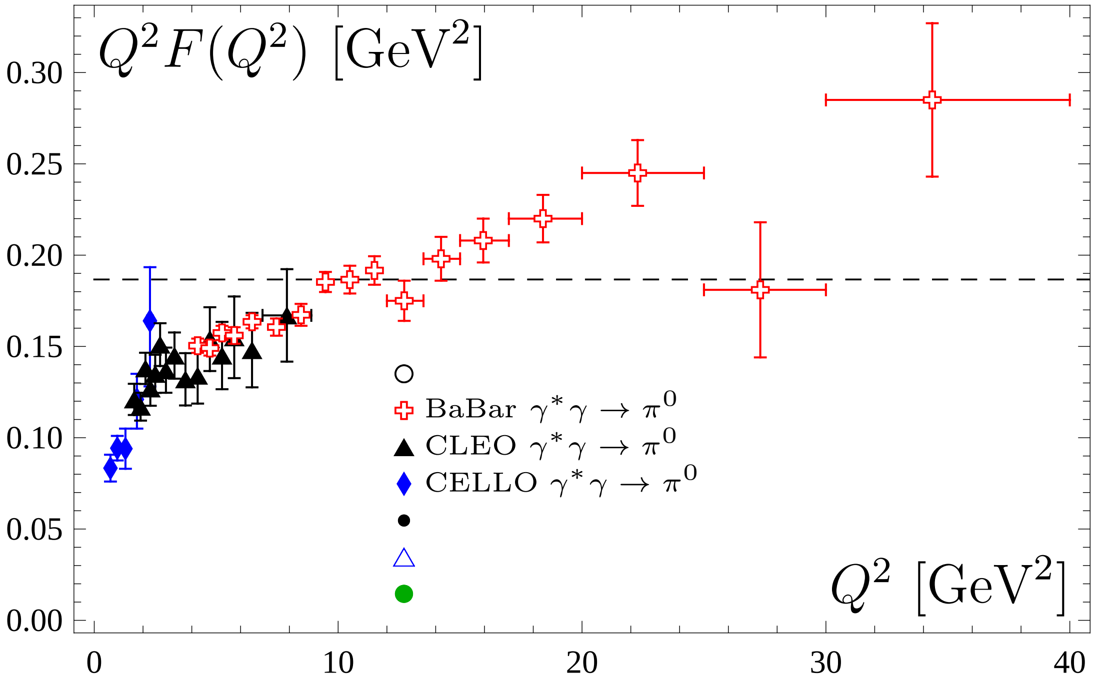


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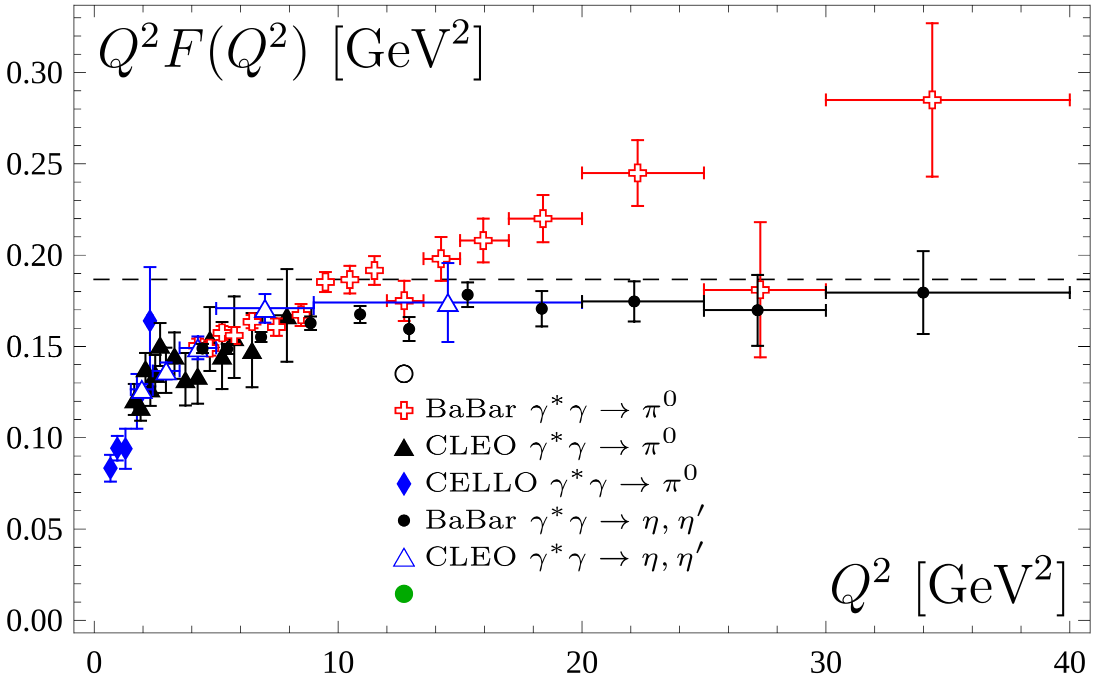


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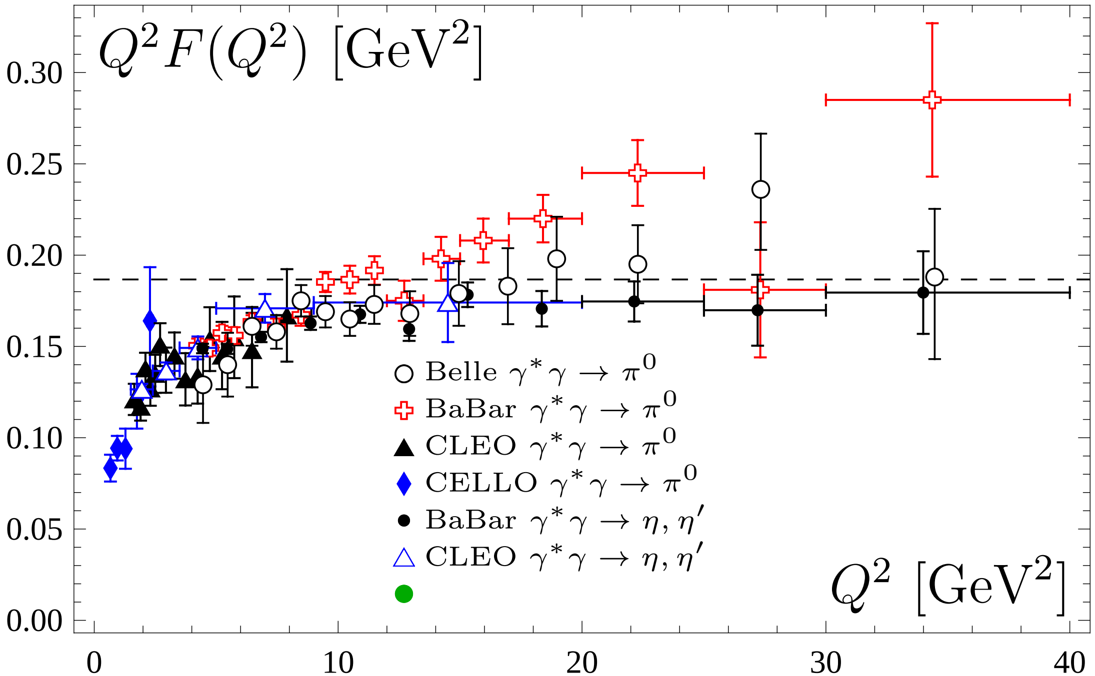
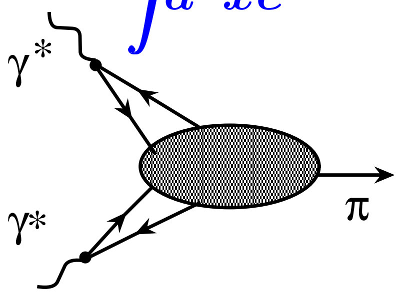



| Data | Collab. |
|---|---|
| ◆ | **CELLO (1991)** |
| ▲ | CLEO (1998) |
| ✚ | **BaBar (2009)** |
| ● | **BaBar$^{\eta}_{\eta'}$ (2011)** |
| ❍ | Belle (2012) |
| dashed line = $\sqrt{2}\, f_\pi$ | |

**Belle data do not confirm auxetic form factor behavior above $10$ GeV$^2$** (except outlier at $Q^2 = 27.33$ **GeV$^2$**).

Factorization of Pion-photon Transition Form Factor

# Factorization $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(P)$ in pQCD



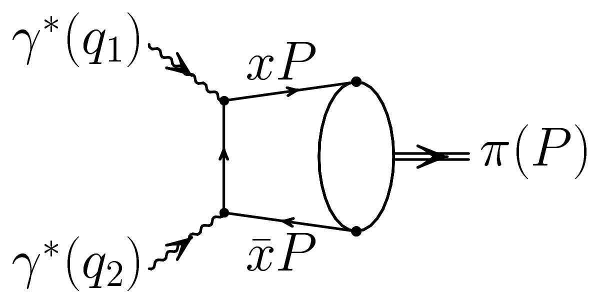

$$\int d^4x e^{-iq_1\cdot z}\langle \pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot F^{\gamma^*\gamma^*\pi}(Q^2,q^2)\,,$$

where $-q_1^2 = Q^2 > 0, \quad -q_2^2 = q^2 \geq 0$

Collinear factorization at $Q^2, q^2 \gg$ **(hadron scale $\sim m_\rho$)$^2$** for the leading twist

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = T(Q^2,q^2,\mu_F^2;x)\otimes\varphi_\pi(x;\mu_F^2) + O\left(\frac{1}{Q^4}\right),$$

$\mu_F^2$ – boundary between large scale $Q^2$ and hadronic one. At the parton level




$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}}{3}f_\pi\int_0^1 dx\,\frac{1}{Q^2x+q^2\bar{x}}\,\varphi_\pi(x).$$

$$Q^2F^{\gamma^*\gamma\pi}(Q^2,q^2\to 0) = \frac{\sqrt{2}}{3}f_\pi\int_0^1\frac{dx}{x}\varphi_\pi(x) \equiv \frac{\sqrt{2}}{3}f_\pi\langle x^{-1}\rangle_\pi$$

# NLO and NNLO amplitudes.

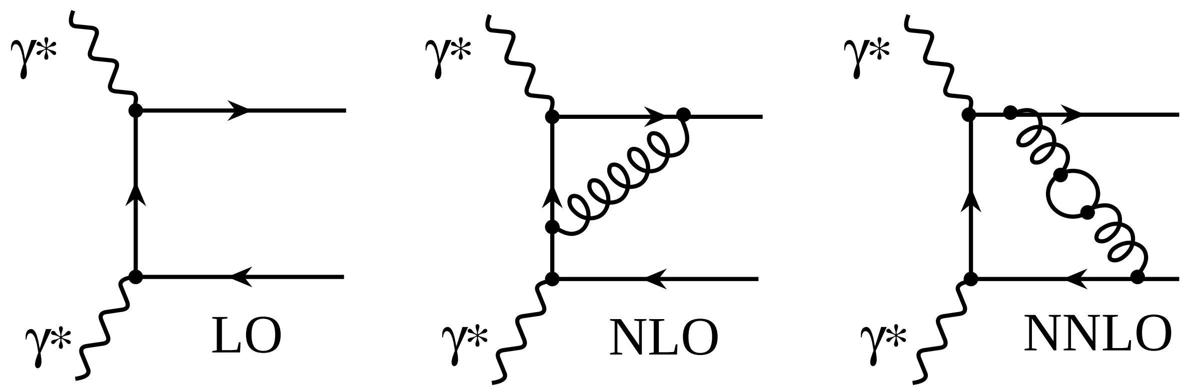
Collinear factorization **[Efremov&Radyushkin 1978]**

$$F^{\gamma^*\gamma^*\pi} \sim \Big(T_0(Q^2,q^2;x) + a_s^1\, T_1(Q^2,q^2;\mu_F^2;x) + a_s^2\, T_2(Q^2,q^2;\mu_{\mathrm{F}}^2;\mu_{\mathrm{R}}^2;x) + \ldots\Big) \otimes \varphi_\pi^{(2)}(x;\mu_{\mathrm{F}}^2) - \delta_{tw4}^2(\mu_F^2)\cdot T_0^2(Q^2,q^2;x) \otimes \varphi_\pi^{(4)}(x)$$

Hard amplitudes $T_i$ — calculable in pQCD, coupling constant — $a_s = \alpha_s(\mu_{\mathrm{R}}^2)/(4\pi)$.

Usual setting $\mu_{\mathrm{R}}^2 = \mu_F^2 = \langle Q^2 \rangle$ to simplify and minimize rad. corrections.

Twist-4 scale parameter – $\delta_{tw4}^2 = (0.19 \pm 0.02)$ GeV$^2$.




LO hard amplitude: $$T_0(Q^2,q^2;x) = \frac{1}{x\,Q^2 + \bar{x}\,q^2}$$

# NLO and NNLO hard amplitudes

NLO: **[Bakulev&Mikhailov&Stefanis(2003),Melić&Müller&Passek(2003)]**

$$T_1\,(x) \otimes \varphi\,(x) = T_0(Q^2, q^2; y) \otimes \left\{C_{\rm F}\mathcal{T}^{(1)}(y,x) + \mathrm{L}(y)\cdot V^{(0)}(y,x)\right\} \otimes \varphi(x;\mu_F^2)$$

$\beta_0$-part of NNLO: **[Melić&Müller&Passek(2003)]**

$T_2 \otimes \varphi = \left(\beta_0 \cdot T_\beta + \ldots\right) \otimes \varphi$, at $\mu_{\rm R}^2 = \mu_{\rm F}^2$

$\beta_0$-term gives the sign and size of NNLO effect following to BLM prescription.

$$a_s^2\beta_0 T_\beta \otimes \varphi = a_s^2\beta_0 T_0 \otimes \Big\{C_{\rm F}\mathcal{T}_\beta^{(2)} - C_{\rm F}\mathrm{L}(y)\cdot\mathcal{T}^{(1)} + \mathrm{L}(y)\cdot\left(V_\beta^{(1)}\right)_+ - \tfrac{1}{2}\mathrm{L}^2(y)\cdot V_+^{(0)}\Big\} \otimes \varphi\,.$$

$V_+^{(0)}$, $V_+^{(1)}$ – 1- and 2-loop ERBL-evolution kernels;

$V_{\beta+}^{(1)}$ – $\beta_0$-part of 2-loop ERBL kernel;

$\mathcal{T}^{(1)}$, $\mathcal{T}_\beta^{(2)}$ – 1-loop part and 2-loop $\beta_0$-part of hard amplitude

# Pion Distribution Amplitude from QCD Sum Rule

# Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

- Pion DA parameterizes this matrix element:

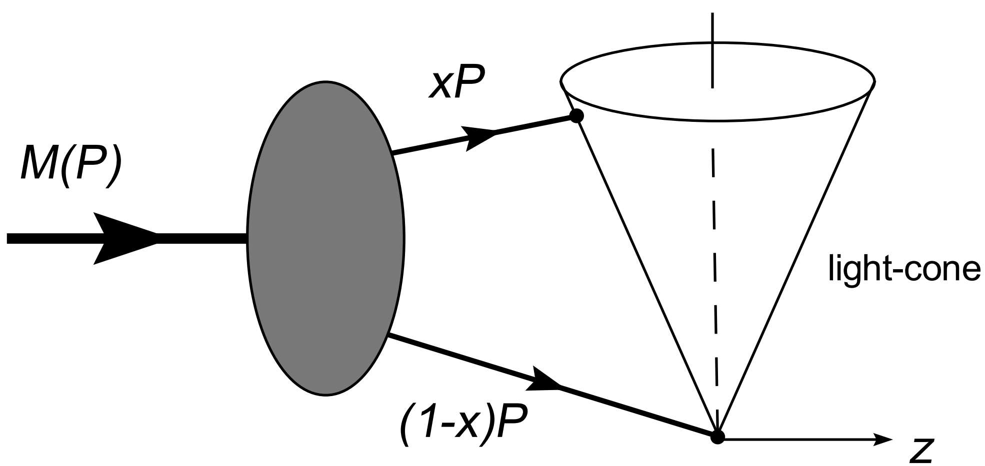
$$\langle 0|\,\bar{d}(z)\gamma_\nu\gamma_5[z,0]u(0)\,|\,\pi(P)\rangle\Big|_{z^2=0} = if_\pi P_\nu \int_0^1 dx\; e^{ix(zP)}\varphi_\pi(x,\mu^2)\,.$$

where path-ordered exponential

$$[z,0] = \mathcal{P}\,exp\left[ig\int_0^z t^a A_\mu^a(y)dy^\mu\right],$$

i.e., light-like gauge link, ensures gauge invariance.

- Pion DA describes transition of physical pion into two valence quarks, separated at light cone.



# Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

- Pion DA parameterizes this matrix element:

$$\langle 0|\,\bar{d}(z)\gamma_\nu\gamma_5[z,0]u(0)\,|\,\pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\nu \int_0^1 dx\; e^{ix(zP)}\varphi_\pi(x,\mu^2)\,.$$

Distribution amplitudes are **nonperturbative** quantities to be derived from

- QCD SR **[CZ 1984]**,
  NLC QCD SR **[M&Radyushkin 1986-91, Bakulev&M&S 1998,2001–04]**
- instanton-vacuum approaches, e.g.
  **[Polyakov *et al.* 1998, 2009; Dorokhov *et al.* 2000]**
- Light-front quark model **[Choi&Ji 2007]**
- Lattice QCD, **[Braun *et al.* 2006; Donnellan *et al.* 2007]**
- from experimental data **[Schmedding&Yakovlev 2000, BMS 2003–2006, Khodjamirian *et al.* 2000, 2002]**

# Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

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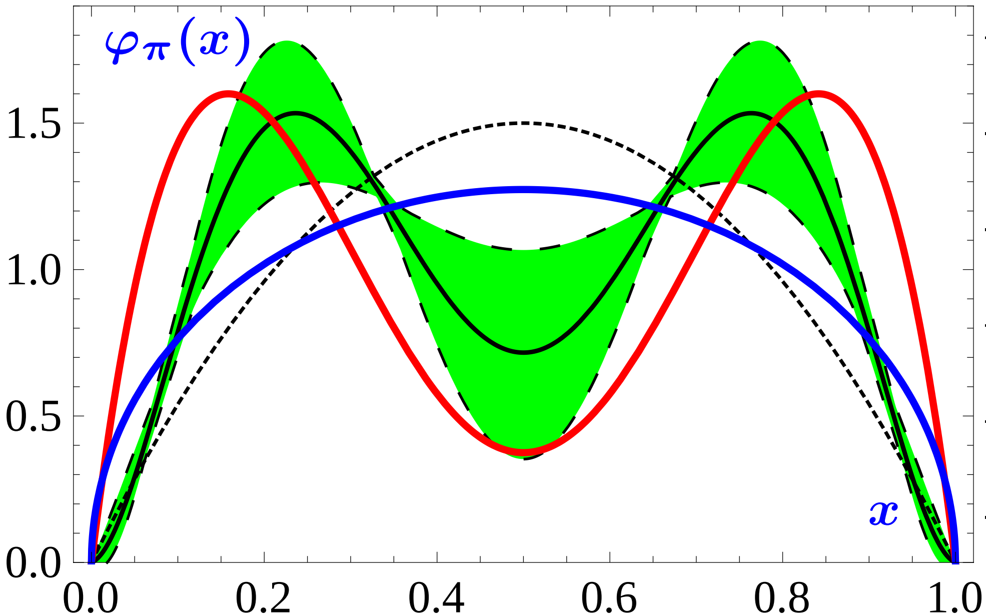
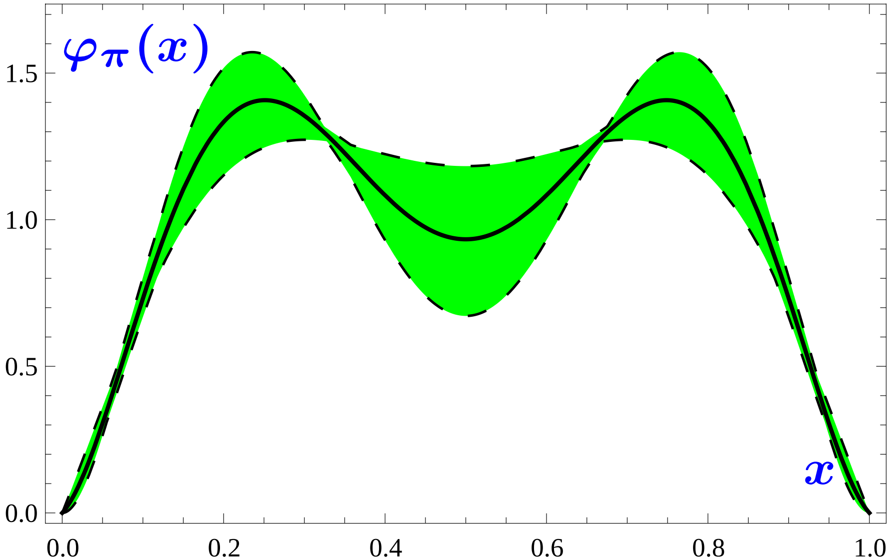


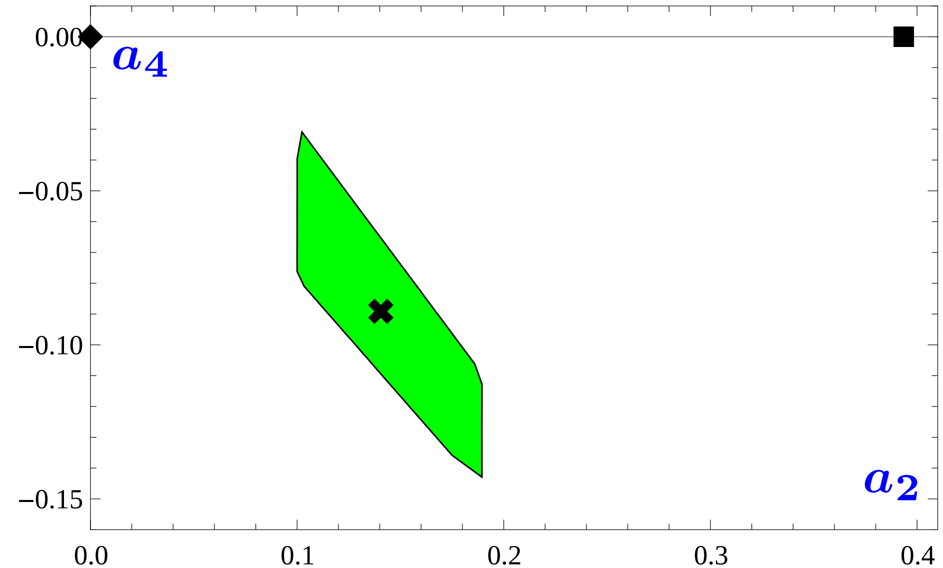

| Curve | Approach |
|---|---|
| dashed black line | Asymptotic |
| green band with black lines | BMS DA, NLC QCD SR |
| red line | CZ from QCD SR |
| blue line | AdS/QCD result |

- DA evolution with $\mu^2$, according to ERBL equation [79-80].
- Gegenbauer expansion of pion DA: $\varphi_\pi(x,\mu^2) \Leftrightarrow a_2, a_4, \ldots, a_n$

$$\varphi_\pi(x,\mu^2) = 6x\bar{x}\left(1 + a_2(\mu^2)C_2^{3/2}(x-\bar{x}) + a_4(\mu^2)C_4^{3/2}(x-\bar{x}) + \ldots\right)$$

# Pion DA from QCD SR with NLC




BMS DA model and bunch were obtained using minimal Gaussian condensate model with single nonlocality parameter $\lambda_q^2 = 0.4\,\mathrm{GeV}^2$.

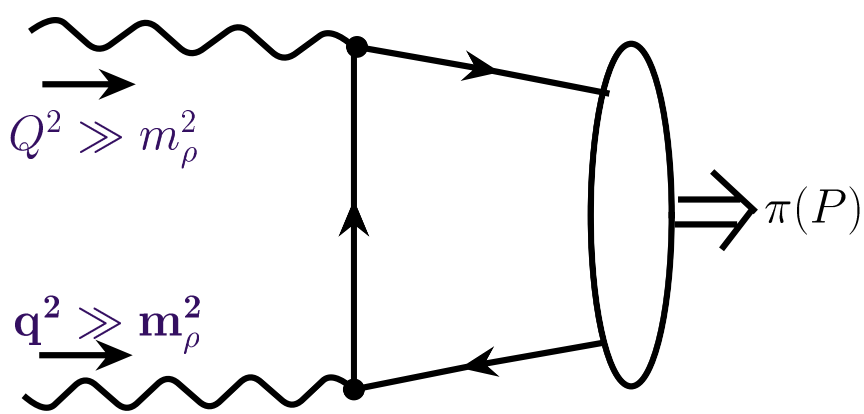
- One can assume higher Gegenbauer coefficients $a_{n\geq 6} = 0$ be equal to 0, that supported by QCD SR.
- QCD SR with NLC provides end-poind suppressed pion DA with slope $\varphi'_\pi(0) \approx 6$ that depends on behavior of quark-condensate at large distances.

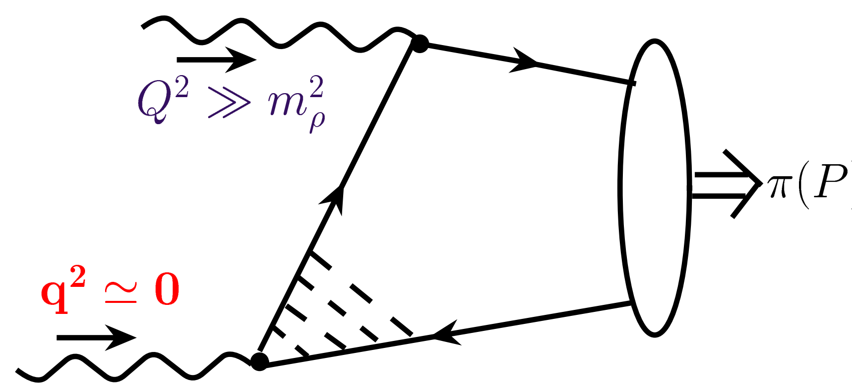
## Light Cone Sum Rule (LC SR)

# $\gamma^*\gamma \to \pi$: Why Light-Cone Sum Rules?

- For $Q^2 \gg m_\rho^2,\ \ q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are important **[Radyushkin–Ruskov, NPB (1996)]**.

- Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances $\sim O(1/\sqrt{q^2})$




**pQCD is OK** **LCSRs should be applied**

# $\gamma^*\gamma \to \pi$: Light-Cone Sum Rules!

LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in $q^2$ (Balitsky et. al.-89, Khodjamirian **[EJPC (1999)]**)

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) \;=\; \int_0^{s_0} \frac{\rho^{\text{PT}}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \int_{s_0}^{\infty} \frac{\rho^{\text{PT}}(Q^2, s)}{s + q^2} ds \,,$$

where $s_0 \simeq 1.5$ **GeV$^2$** – effective threshold in vector channel,
$M^2$ – Borel parameter ($0.7 - 1$ GeV$^2$).
Real-photon limit $q^2 \to 0$ can be easily done.

Spectral density was calculated in QCD:

$$\rho^{\text{PT}}(Q^2, s) = \frac{1}{\pi} \textbf{Im} F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2, -s - \imath\varepsilon) = \text{Tw-2} + \text{Tw-4} + \text{Tw-6} + \ldots \,,$$

where twists contributions given in a form of convolution with pion DA:

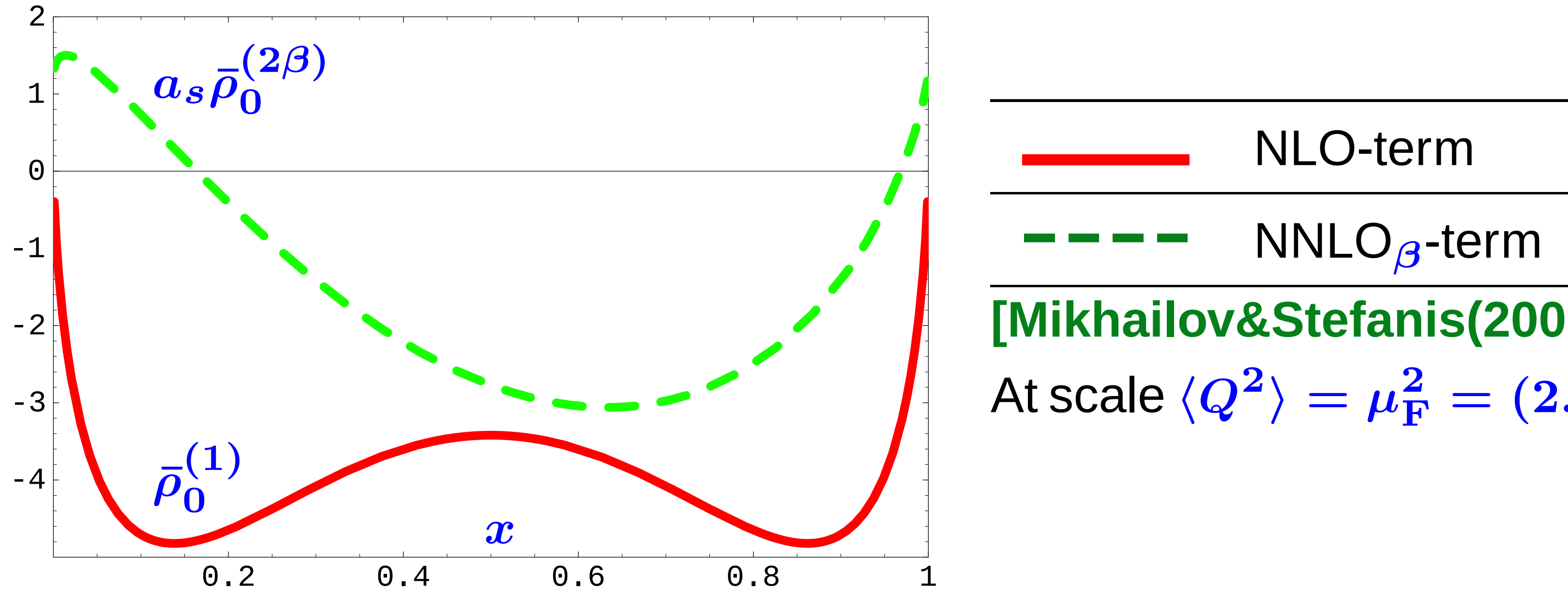
$$\text{Tw-2} \sim \frac{1}{\pi} \textbf{Im}\left(T_{\text{LO}} + T_{\text{NLO}} + T_{\text{NNLO}_{\beta_0}} + \ldots\right) \otimes \varphi_\pi^{\text{Tw2}}(x, \mu) \,.$$

# $\text{NNLO}_{\beta_0}$ and NLO spectral density

$$\rho^{\text{Tw-2}}(Q^2,s) = \sum_{n=0} a_n(Q^2)\left(\bar{\rho}_n^{(0)}(Q^2,x) + a_s\bar{\rho}_n^{(1)}(Q^2,x) + a_s^2\bar{\rho}_n^{(2\beta)}(Q^2,x)\right)$$

where $x = Q^2/(s+Q^2)$.

$x$-dependence for 0-harmonic (asymptotic DA) of NLO and $\text{NNLO}_\beta$ spectral densities.




**[Mikhailov&Stefanis(2009)]**

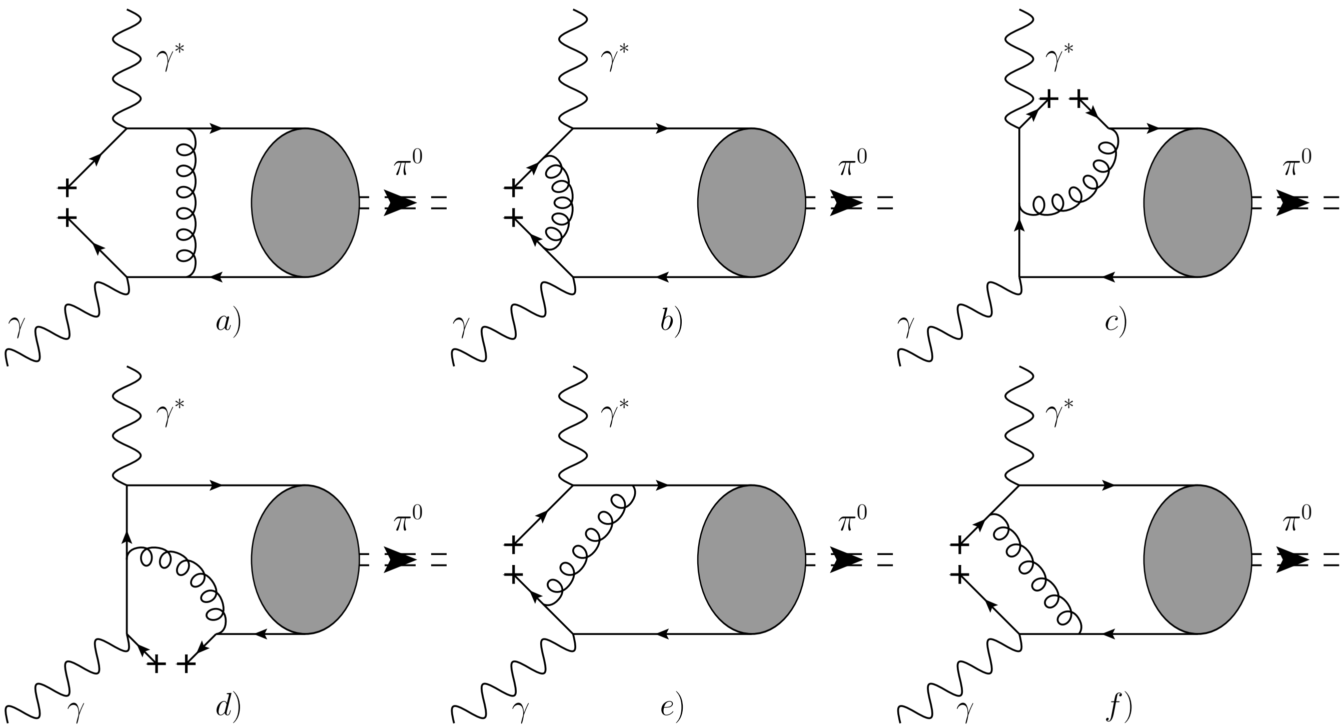
At scale $\langle Q^2\rangle = \mu_F^2 = (2.4\ \text{GeV})^2$.

The $\text{NNLO}_\beta$ spectral density and $F^{\gamma\gamma^*\pi}$ are obtained for 6 numbers of Gegenbauer harmonics

# "Twist-6" contribution

"Twist-6"-term expresses inverse power correction to coefficient function rather than geometric twist-6

$$\rho^{\rm tw6}(Q^2,x) = 8\pi C_F \alpha_s(\mu) \frac{\langle \bar{q}q\rangle^2}{N_c f_\pi^2} \frac{x^2}{Q^6} \left[ 2x\log x + 2x\log\bar{x} - x + 2\delta(\bar{x}) - \left[\frac{1}{1-x}\right]_+ \right].$$

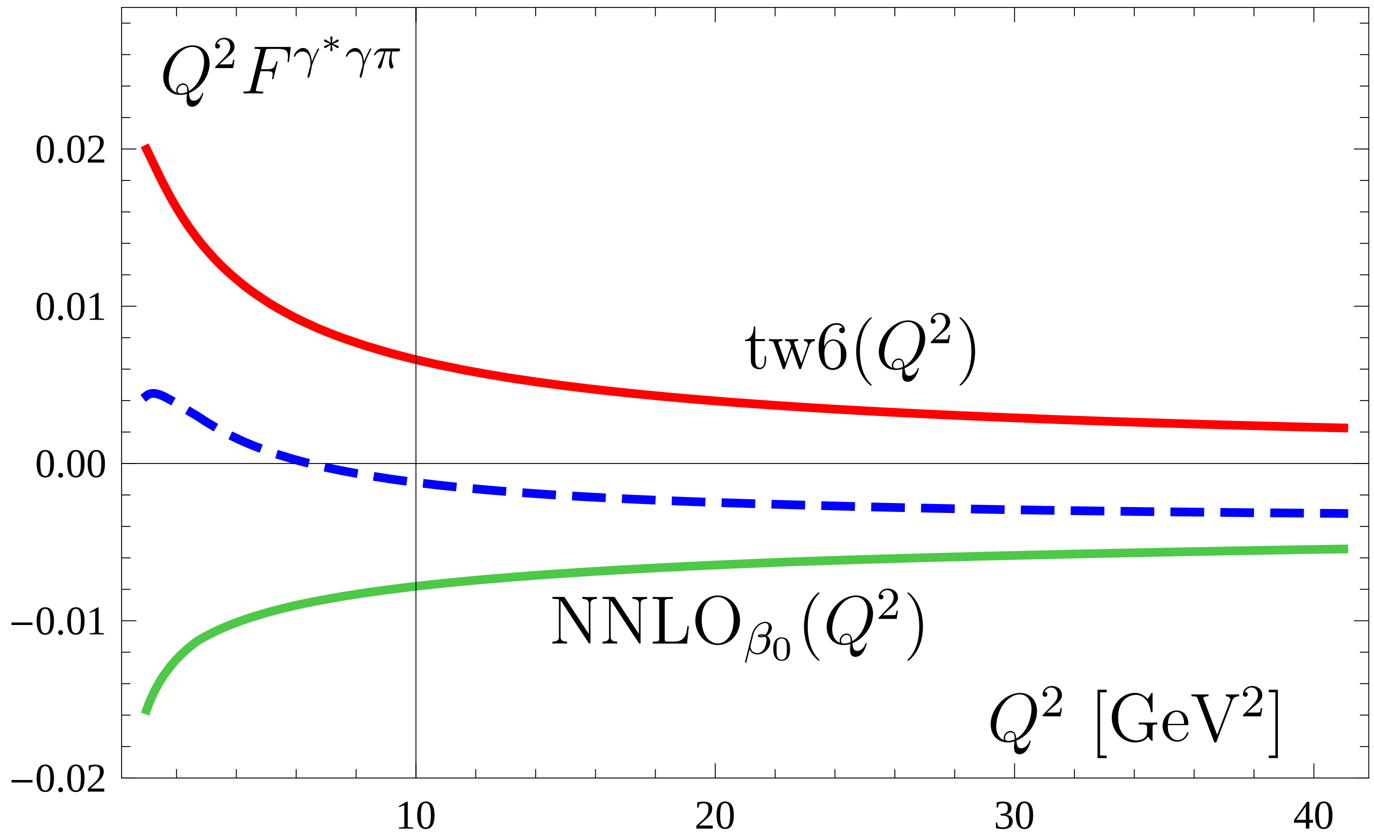



**[Agaev et al, PRD83,0540020(2011)]**

# High order corrections result

**Twist-6** and **NNLO**$_{\beta_0}$ contributions to the $\boldsymbol{Q^2 F^{\gamma^*\gamma\pi}(Q^2)}$ with **BMS**-like **Pion DA**

They practically cancel out each other **[BMPS(2011)]**




We use this residual as **theoretical uncertainty** of our prediction, that provides us with an additional **$3\%$-uncertainty**.

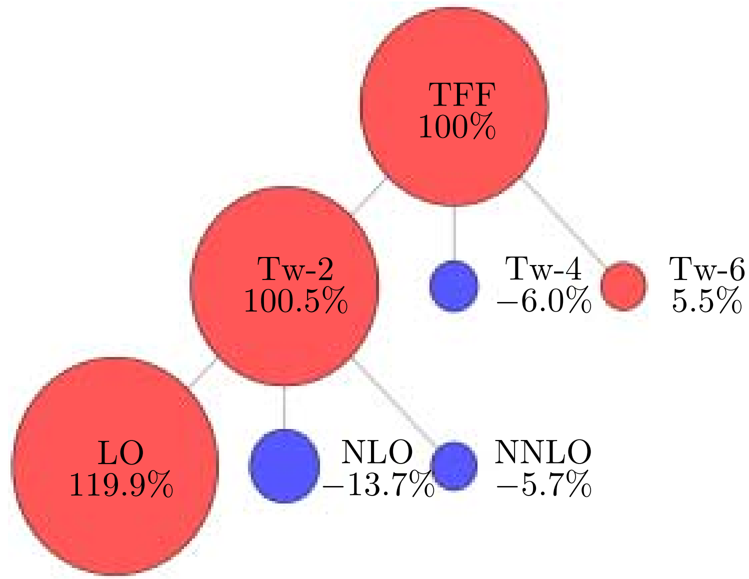
# Main Ingredients of Spectral Density

- **LO** Spectral Density, **Tw-4** term — **Khodjamirian[EJPC (1999)]**
- **NLO** Spectral Density — in **[Mikhailov&Stefanis(2009)]**
- **NNLO**$_{\beta_0}$ Spectral Density — in **[M&S(2009)]**
- **Tw-6** contribution — in **[Agaev et.al.–PRD83(2011)0540020]**

- **NLO** evolution of pion DA **[Samadi, Dittes, Radyushkin&Mikhailov]**

Terms of Pion-Photon FF at $Q^2 = 8\,\text{GeV}^2$

- Result is dominated by Hard Part of Twist-2 LO contribution.
- Twist-6 contribution is taken into account together with **NNLO**$_{\beta_0}$ one — they has close absolute values and opposite signs.




Blue - negative terms

Red - positive terms

# Parameters of LC SR

From PDG:

- $\alpha_s(m_Z^2)$
- Masses $m_\rho$, $m_\omega$
- Decay Widths $\Gamma_\rho$, $\Gamma_\omega$

From QCD SR:

- Borel parameter
  $M^2_{\text{LCSR}} \in [0.7, 1]$ GeV$^2$
- Vector Chan. Threshold $s_0$
- Twist-4 $\delta^2 = \lambda_q^2/2 \pm 20\%$
- Twist-6 ($\alpha_S\langle\bar{q}q\rangle$)

Light-Cone Sum Rules:

$$\text{FF} = (\text{LO} + \text{NLO}) \otimes (\pi\text{-DA}_{\text{NLO}}) + \text{Tw-4} \pm \Delta\text{FF}$$

$$\Delta\text{FF} = \pi\text{-}\Delta\text{DA} + \Delta\text{Tw-4} + (\text{NNLO}_{\beta_0} \otimes (\pi\text{-DA}) + \text{Tw-6})$$

$\pi$-DA model

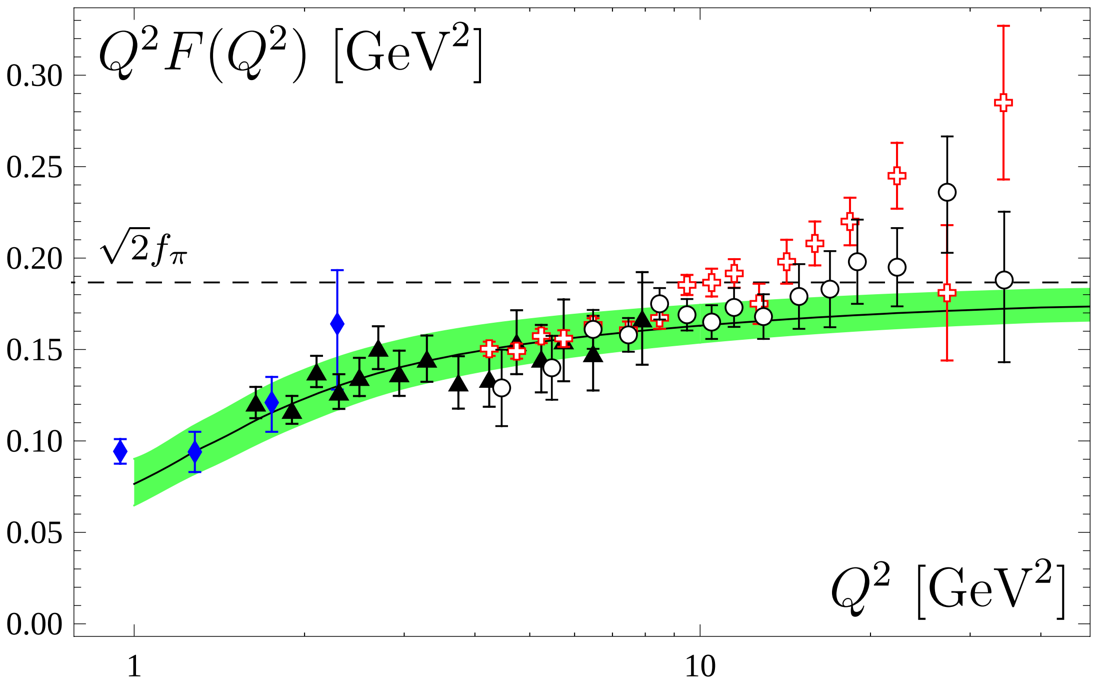
Data on FF

FF Prediction

Fitting $\pi$-DA ($a_n$)

# Pion-gamma transition FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle [1205.3249[hep-ex]]**

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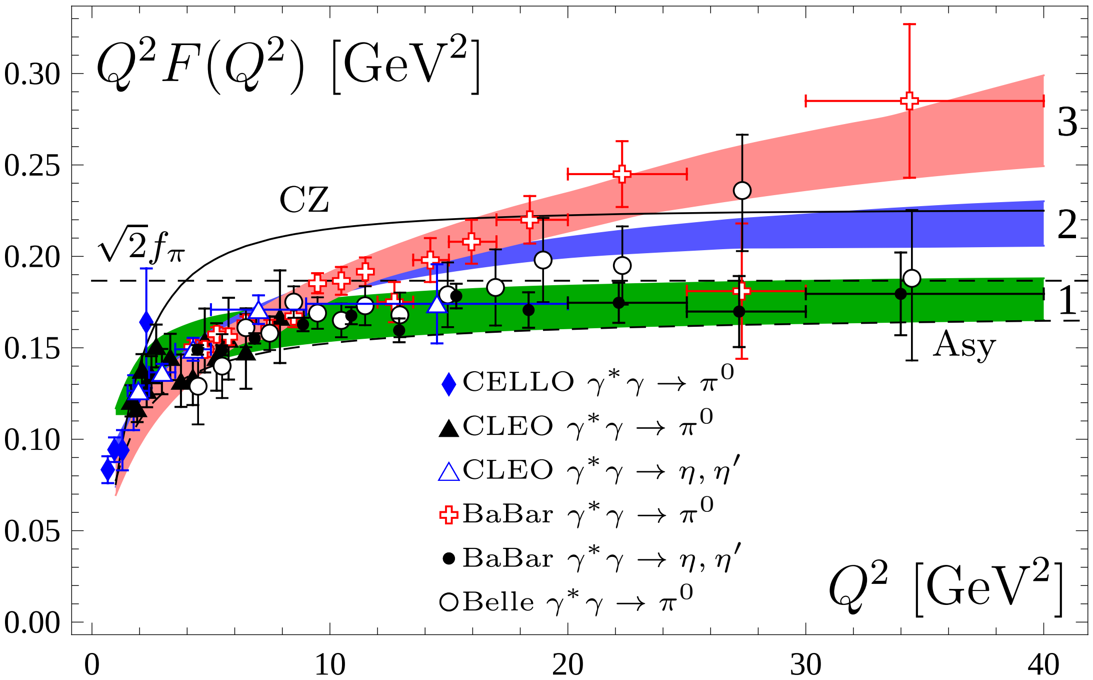
| | our result **BMPS** |

**Belle data do not confirm auxetic form factor behavior above $10$ GeV$^2$** (except outlier at $Q^2 = 27.33$ **GeV$^2$**).

**Predicted FF agrees well** with CELLO, CLEO, BaBar$_{Q^2<9\ \text{GeV}^2}$ (2009), BaBar$^{\eta}_{\eta'}$ (2011), and most Belle (2012).

Data Analysis

# Pion FF Data and Models



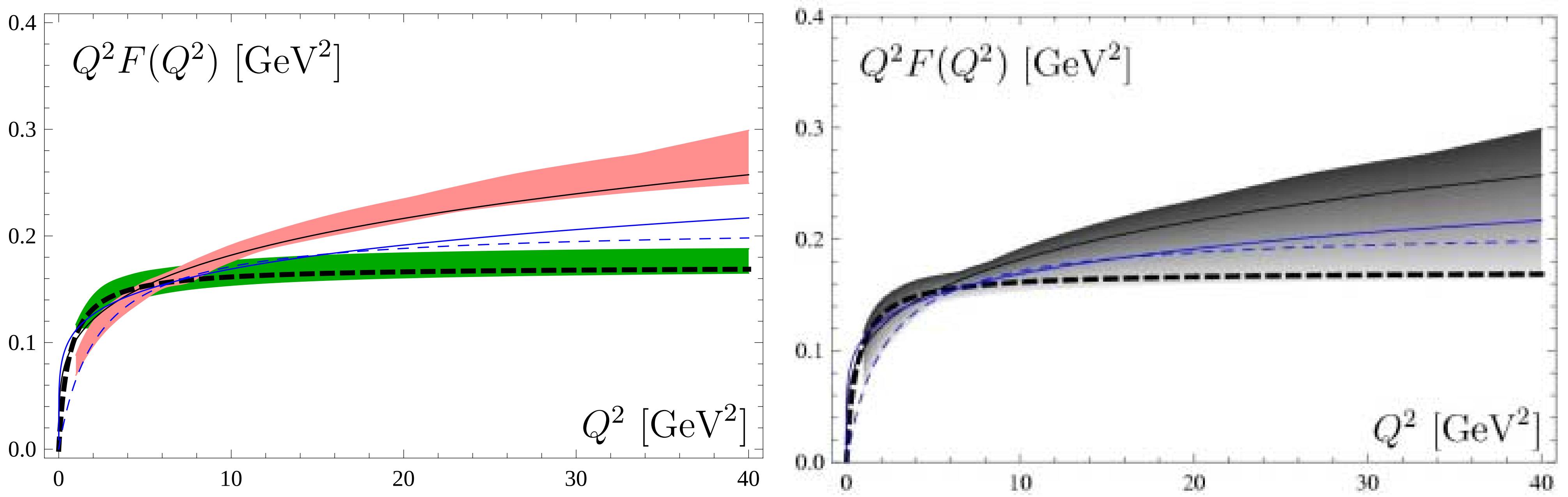

| Data | Collab. |
|---|---|
| ■■ | **BaBar, 8 models** |
| ■■ | **Intermediate, 5 models** |
| ■■ | **BMPS & Holography, 5 models** |

**PRD86(2012)031501(R)**

- Most data points either inside **green “Belle” strip** (**scaling**) or within **red “BaBar” strip** (**auxesis**).
- BaBar $\eta, \eta'$ data are **within green strip**
- **Blue strip** mostly theoretical.

# Alternatives for Pion-Gamma FF Analysis

Alternative: Consider data as forming **two independent data strips** (left) or **one single data strip** (right) **PRD86(2012)031501(R)**




We suggest to explore the first **Alternative**:

To consider all data as forming

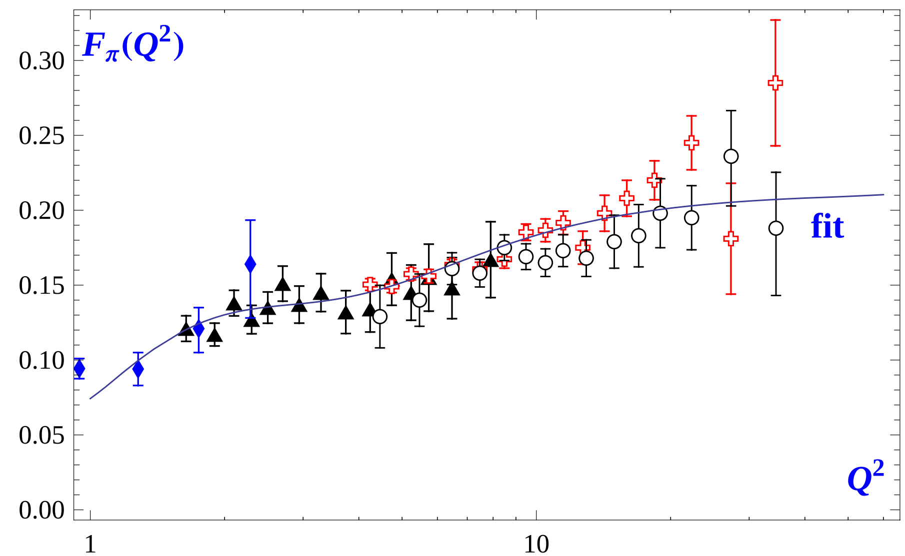
**two independent data strips**,

namely, **CELLO&CLEO&Belle** and **CELLO&CLEO&BaBar**

# Fitting pion DA under LC SR

- We fit experimental data on transition pion FF by varying the Gegenbauer coefficients of pion DA model.
- Two sets of experimental data CELLO&CLEO&BaBar and CELLO&CLEO&Belle were analyzed to show the discrepancy Babar and Belle data in terms of properties of pion DA in LCSR.

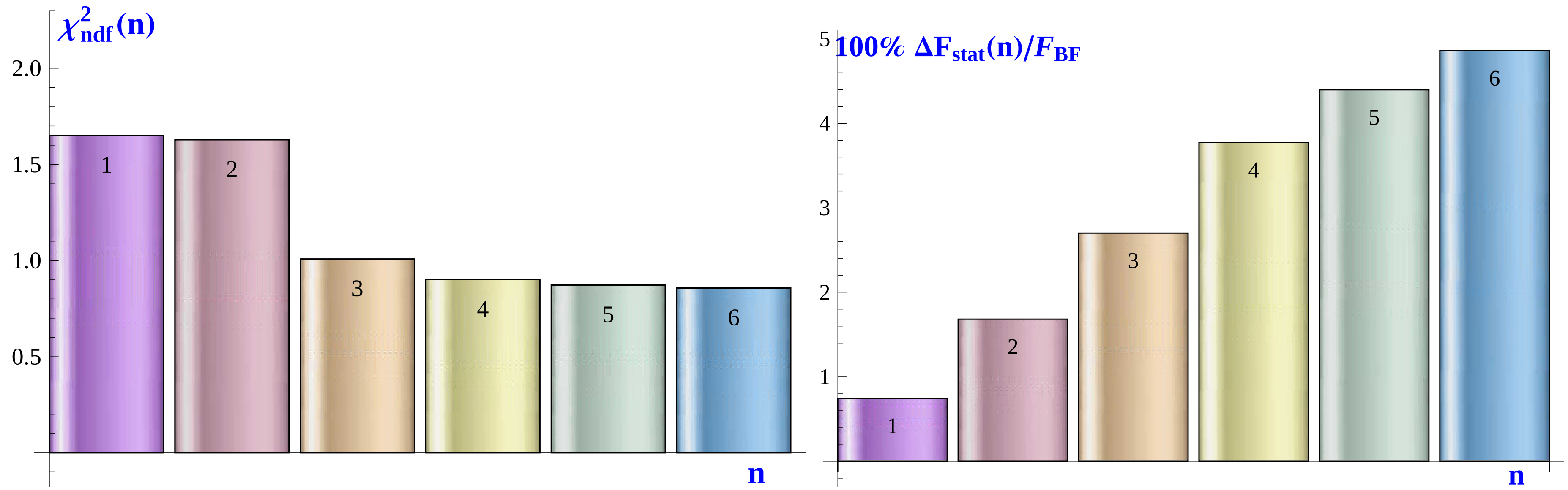
  - How many harmonics take into account?
  - Confidence regions on Gegenbauer coef. $a_2, a_4, \ldots, a_?$
  - Confidence bunches of pion DA $\varphi_\pi(x)$
  - Moments
  - Derivative, “integral” derivative




CELLO&CLEO&BaBar data fitting by FF based on LC SR with NLO$\oplus$Tw4$\oplus$3 Gegen. harmonics

# How many harmonics take into account?

The goodness-of-fit $\chi^2_{\text{ndf}}$-criterion vs conventional error ($68.3\%$ CL) as a function on number $n$ of fit parameters

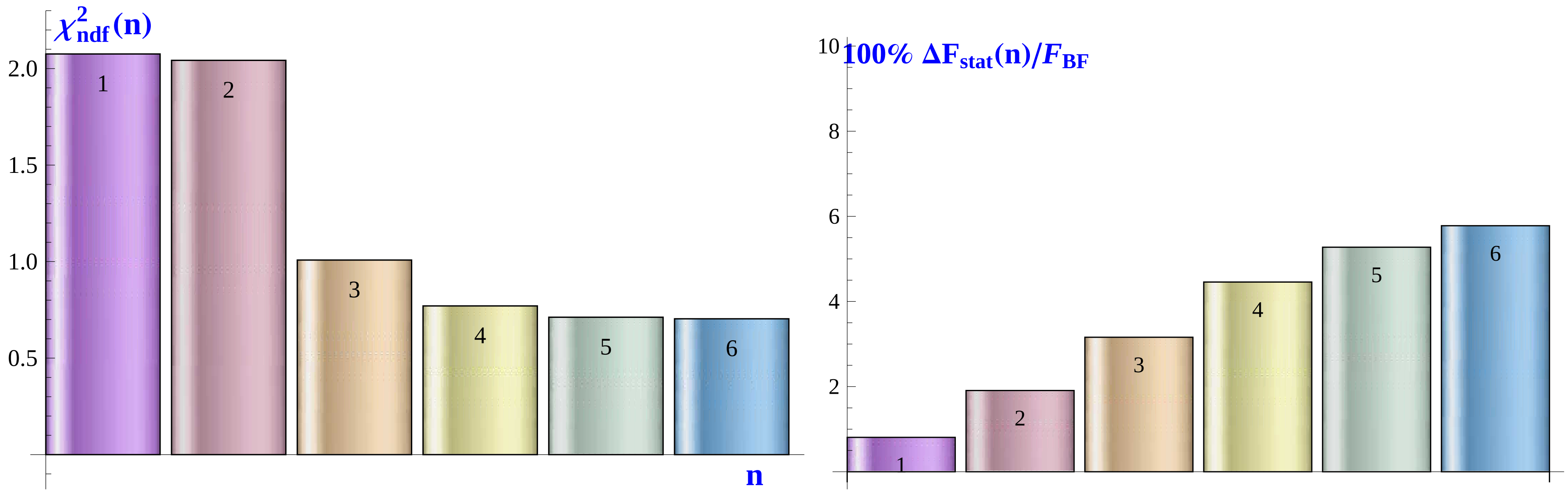



All data: CELLO&CLEO&BaBar&Belle

- Goodness - stable, while the error grows with $n$
- The compromise at $\chi^2_{\text{ndf}} \approx 0.5 - 1$ and $n = 2, 3$ is enough.
- For fitting BaBar data one should take $n \geq 3$ parameters.

# How many harmonics take into account?

The goodness-of-fit $\chi^2_{\text{ndf}}$-criterion vs conventional error ($\mathbf{68.3\%}$ CL) as a function on number $n$ of fit parameters




CELLO&CLEO&BaBar data set

- Goodness - stable, while the error grows with $n$
- The compromise at $\chi^2_{\text{ndf}} \approx \mathbf{0.5 - 1}$ and $n = \mathbf{2, 3}$ is enough.
- For fitting BaBar data one should take $n \geq \mathbf{3}$ parameters.

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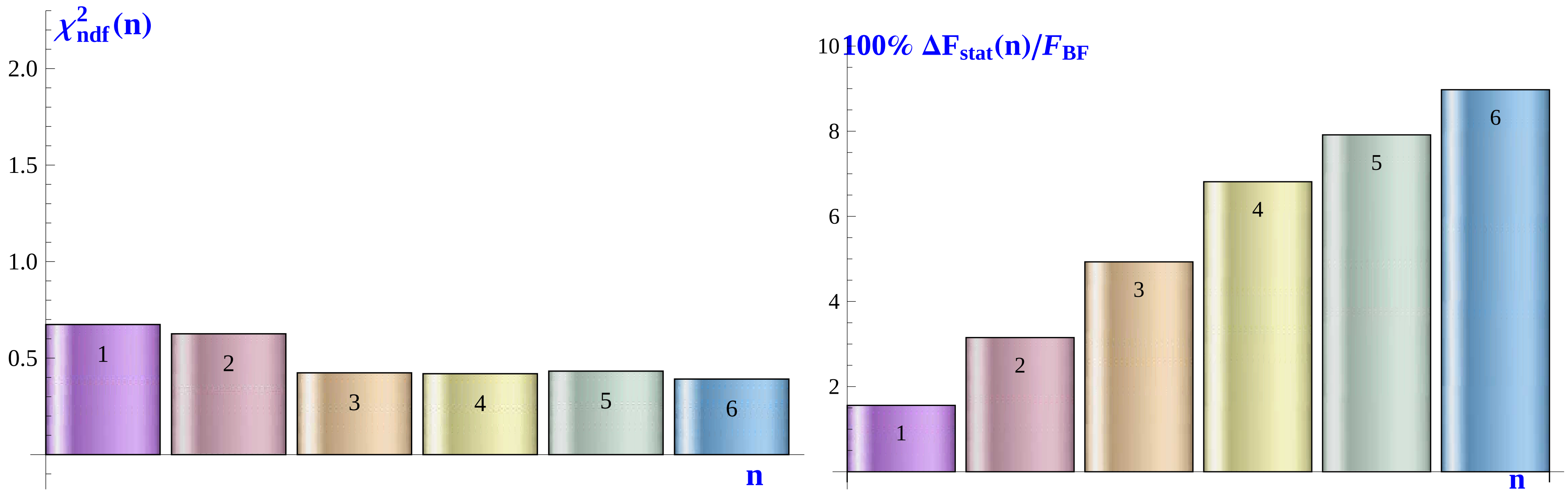


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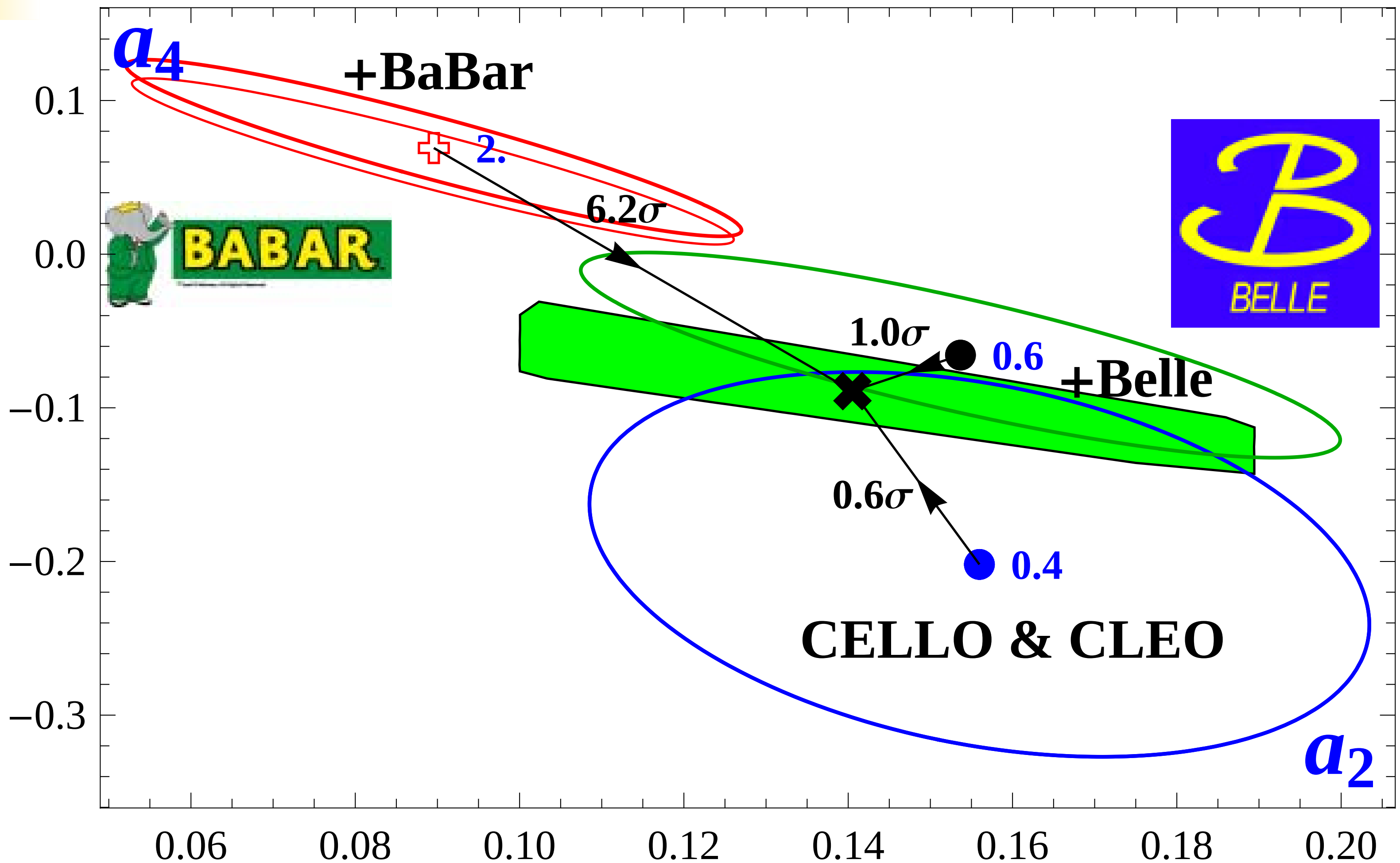
# Pion DA from Data

# Confidence regions in 2D $(a_2, a_4)$




- In vertexes of a triangle – $\chi^2/ndf$, all estimates at $\mu_{\text{SY}} = 2.4$ GeV.
- On sides of triangle: discrepancy in terms of stand. deviation ($1\sigma \approx 68\%$)

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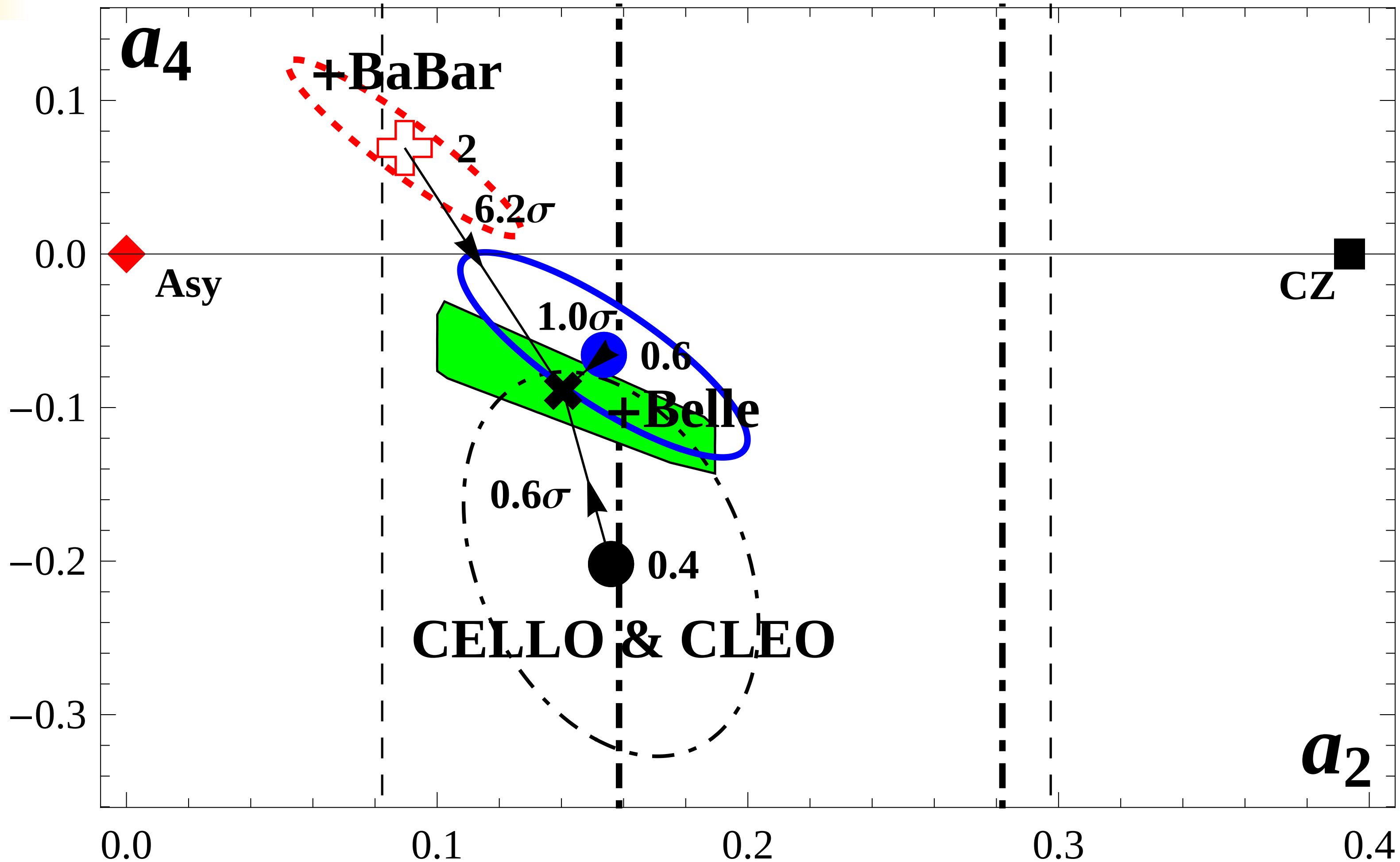
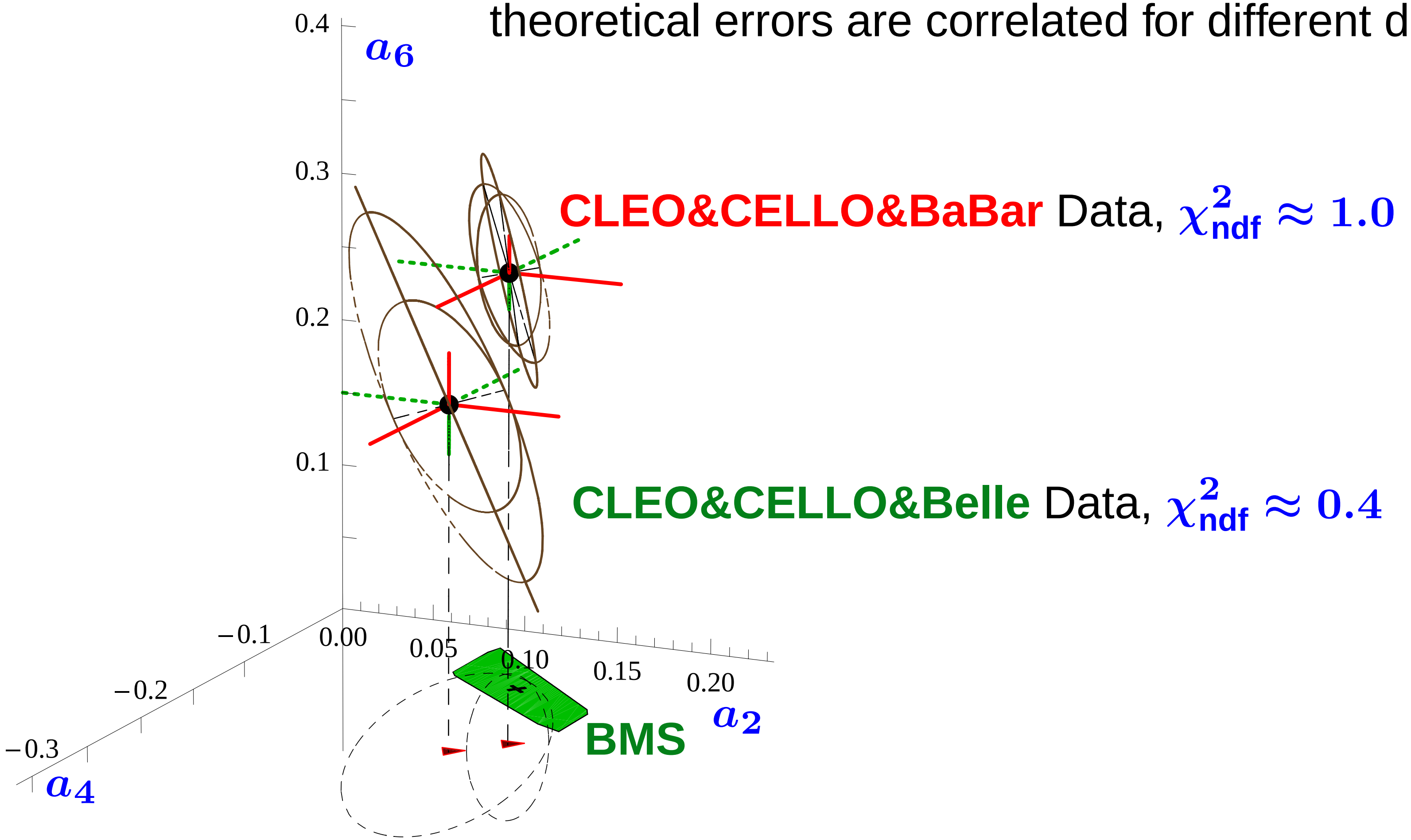



- **BMS DA (✖ &green bunch)** from QCD SR with nonlocal condensates: $\lesssim 1\sigma$.
- ◆ **Asy**mptotic DA, CZ DA: $> 6\sigma$.
- Lattice constraints (vert. lines): dash-dotted–**Braun et al.[2006]**, dashed–**[2011]**.

# NLC SR Results vs 3D Constraints $(a_2, a_4, a_6)$

3D $1\sigma$-error ellipsoid for $(a_2, a_4, a_6)$ at $\mu_{SY} = 2.4$ GeV scale
with theoretical $\mp\Delta\delta^2_{\text{tw4}}$-error shown by **green($-$)** and **red($+$)** length.

theoretical errors are correlated for different data sets.



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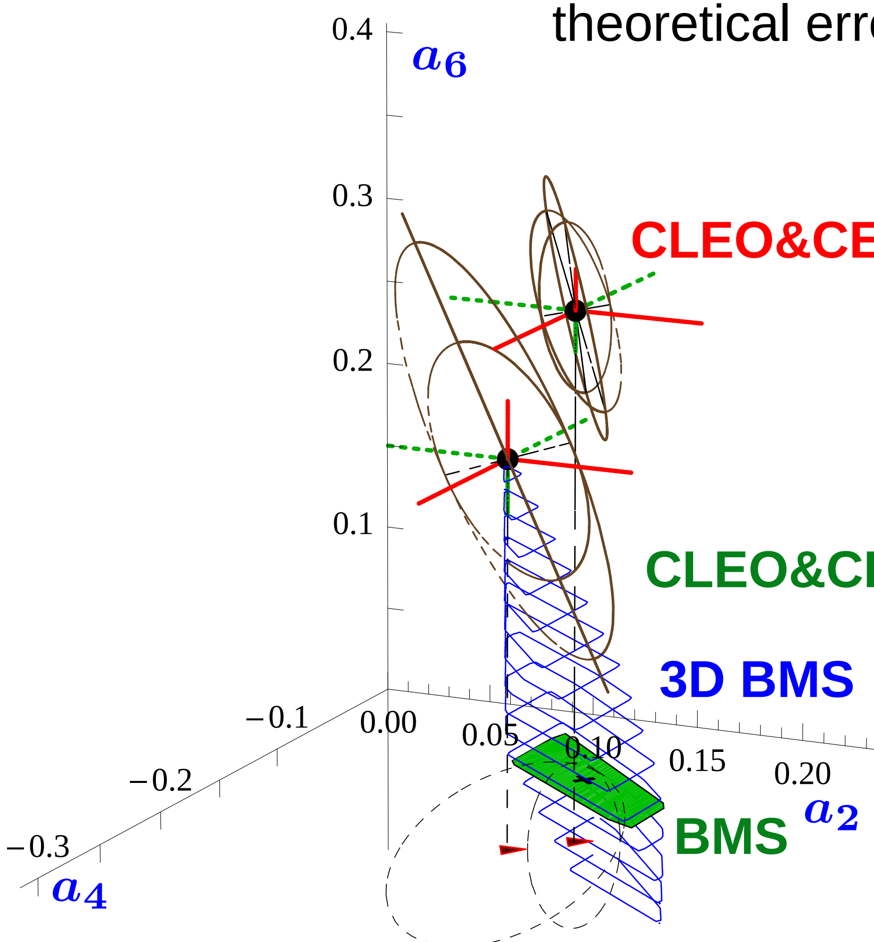


- QCD SR with Nonlocal Condensate result: $a_6 \approx 0 \pm 0.2$.
- $a_{n\geq 6} \to 0$ assumption gives **BMS** pion DA model.
- $a_{n\geq 8} \to 0$ assumption leads to **3D BMS** pion DA model.

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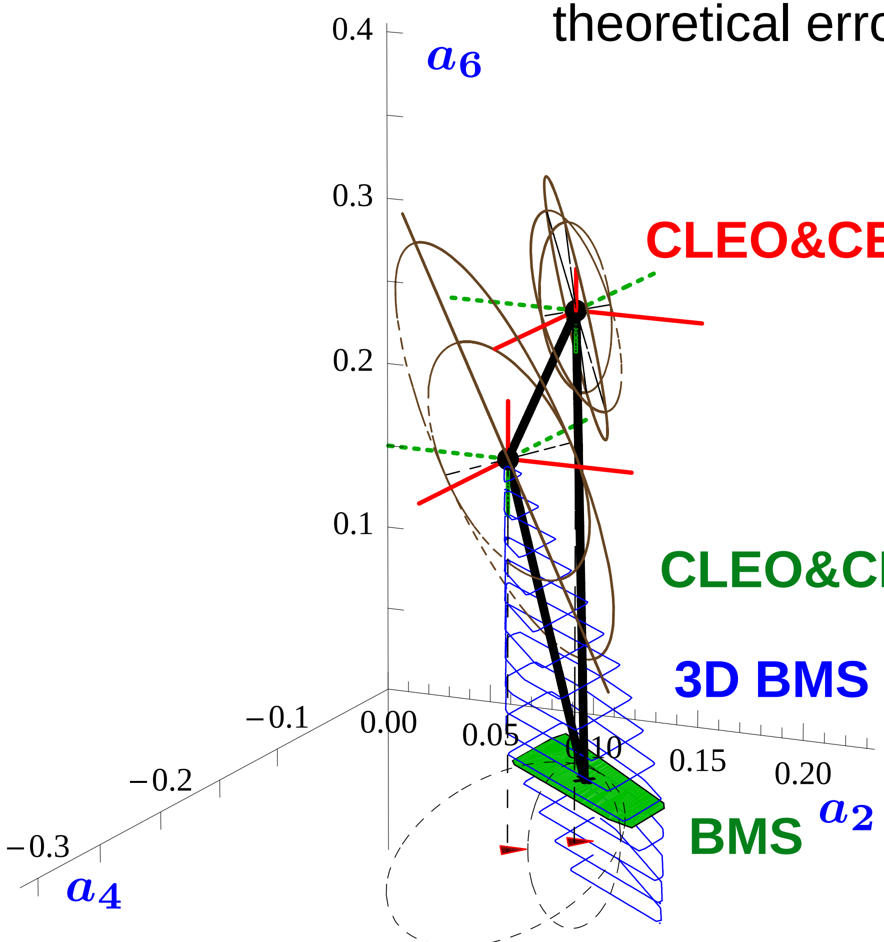


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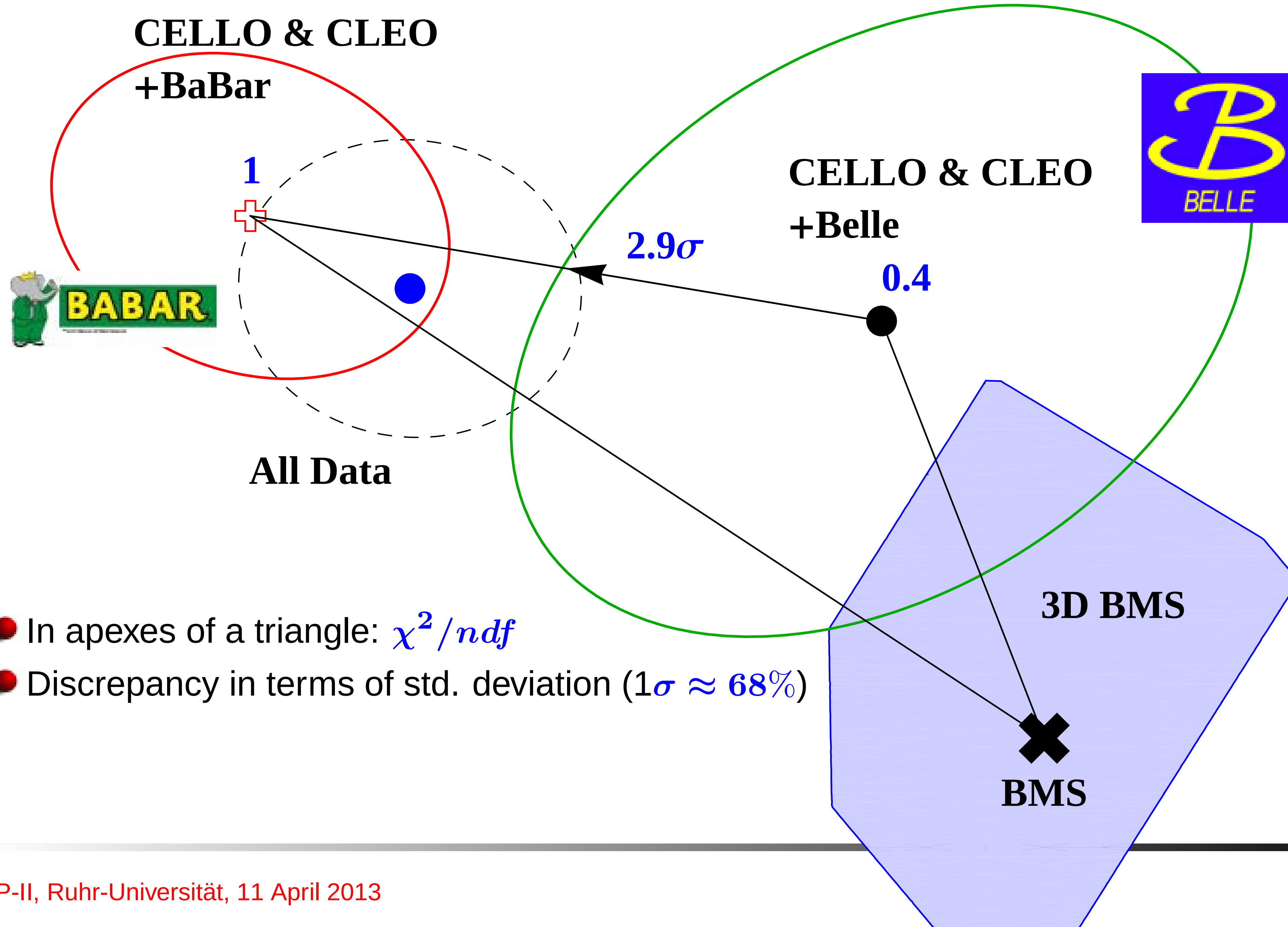
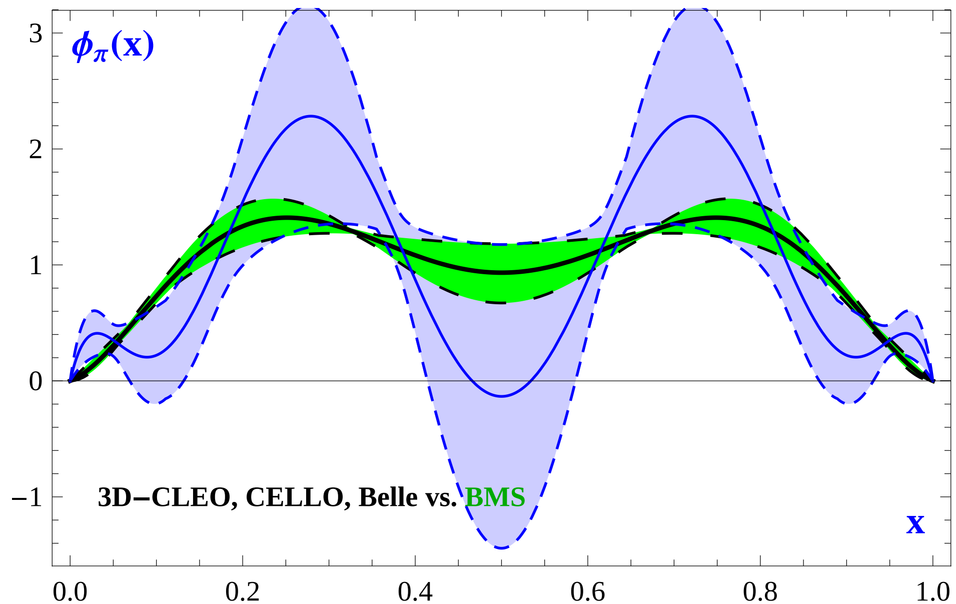
# 2D cut of 3D confidence regions $(a_2, a_4, a_6)$

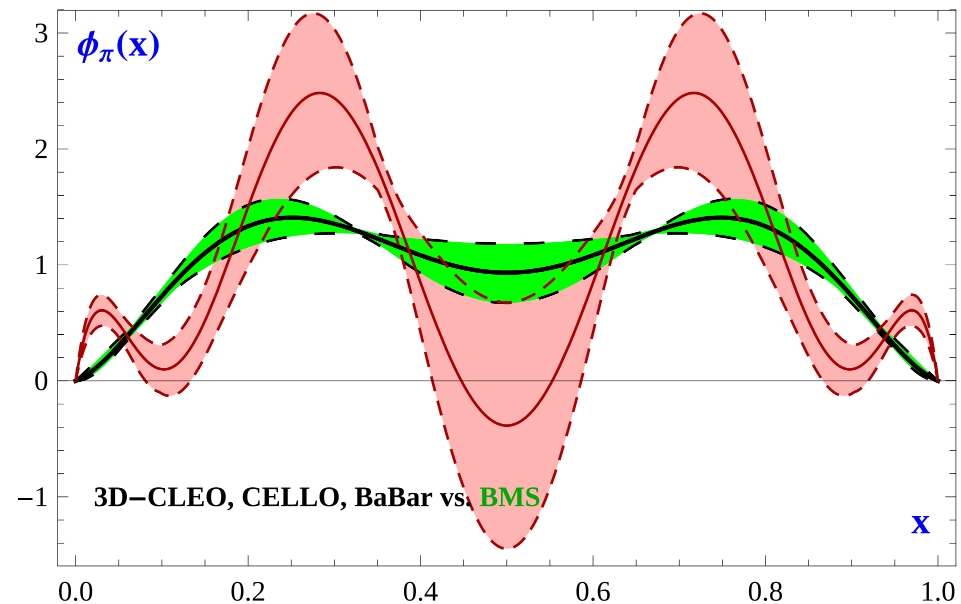



- In apexes of a triangle: $\chi^2/ndf$
- Discrepancy in terms of std. deviation ($1\sigma \approx 68\%$)

# Data fit of pion DA vs NLC SR profiles

→ +Belle, → **BMS**, $\mathbf{1-40}$ GeV$^2$ at $\mu_{\text{SY}}=\mathbf{2.4}$ GeV, → **+BaBar**




**average incline:** $\mathbf{17.2\pm 8.5}$; $\mathbf{25.6\pm 5.25}$

- The main difference – **a sharper behavior** of **BaBar** near endpoints.
- **CELLO, CLEO, Belle** data agrees with **BMS bunch** based on NLC QCD SR.
- **BaBar** data above $\mathbf{10}$ GeV$^2$ does not support BMS bunch.

# Conclusions

- We extract confidence region for different characteristics of pion DA from experimental data on pion transition FF in framework of LC SR at NLO level and “twist-4” term taking into account [NNLO$_\beta$ + “twist-6”] as uncertainties.
- Performed 2D and 3D analysis of CELLO, CLEO, BaBar, Belle data.
  - We showed in 2D analysis that data from CELLO, CLEO, BaBar, and Belle in $[\mathbf{1} \div \mathbf{9}]$ GeV$^{\mathbf{2}}$ favor pion DA with endpoint suppression, like BMS.
  - Beyond $\mathbf{9}$ GeV$^{\mathbf{2}}$, best fit to data, including higher BaBar points, requires sizeable coefficient $\boldsymbol{a_6}$, while $\boldsymbol{a_2}$ and $\boldsymbol{a_4}$ remain the same.
  - Both, the 2D and 3D analysis of CLEO&CELLO&Belle data comply with BMS “bunch” but disagree with CLEO&CELLO&BaBar data.
- Belle data are compatible with collinear factorization and QCD scaling, whereas BaBar data demand some enhancement mechanism for the transition FF. The promising fine accuracy of future BESIII experiment can clarify current ambiguity of data.