



Departamento de Física Teórica II.

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# Light Scalar mesons: No ordinary hadrons

J. R. Peláez

1) Scalar Mesons: motivation & perspective

2) The  $\sigma$  or  $f_0(500)$

3) The  $f_0(980)$

4) The  $\kappa$  or  $K(800)$  and  $a_0(980)$

5) Nature and classification.

Regge sigma trajectory

6) Summary

I will focus on progress  
after PDG2010

Following two points of view:

i) PDG

Consensual, conservative

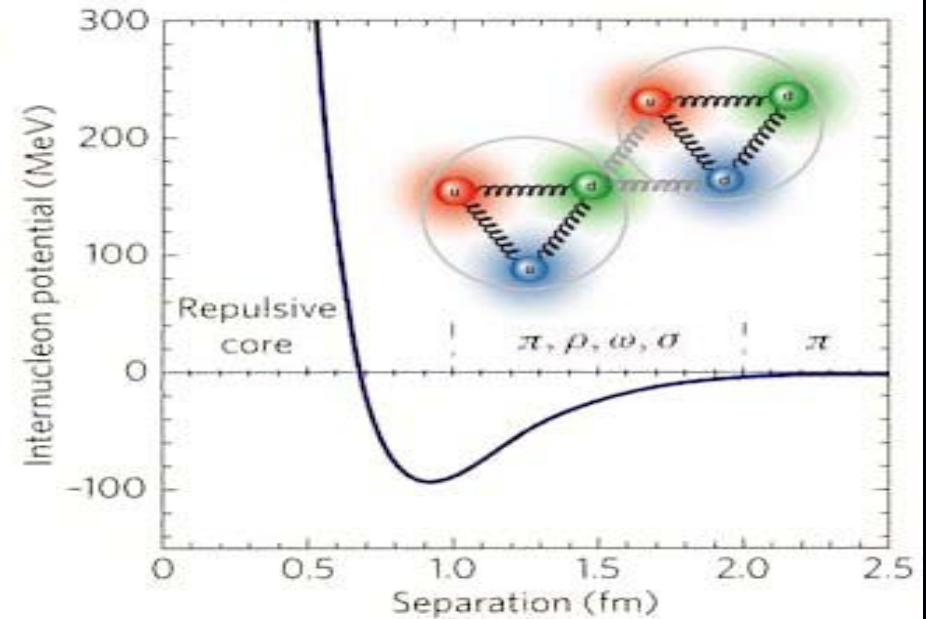
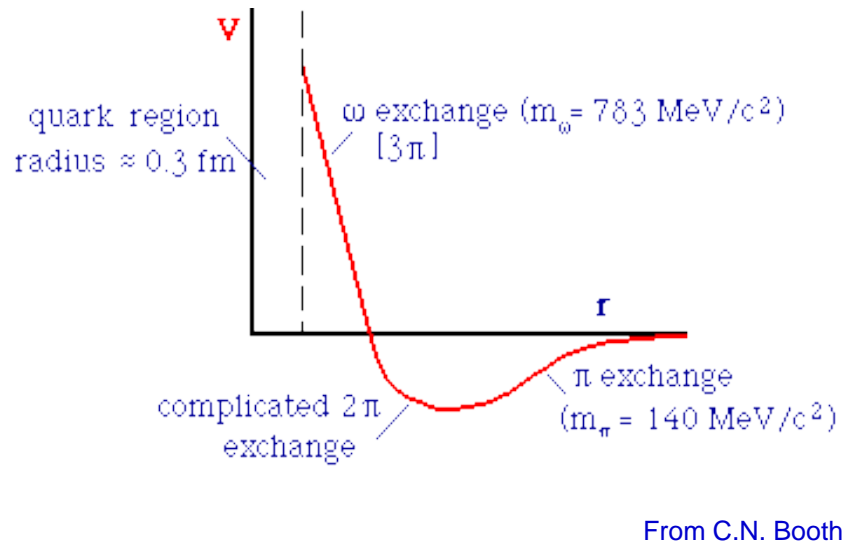
ii) My own

Probably closer to the dominant  
view in the community  
working on light scalars

# Motivation: The $f_0(600)$ or $\sigma$ , half a century around

- $I=0, J=0$   $\pi\pi$  exchange very important for nucleon-nucleon attraction!!

## Crude Sketch of NN potential:



Scalar-isoscalar field already proposed by Johnson & Teller in 1955

Soon interpreted within "Linear sigma model" (Gell-Mann) or Nambu Jona Lasinio - like models, in the 60's.

## The longstanding controversy ( situation ca. 2002)

- The  $\sigma$ , controversial since the 60's.

“not well established”  $0^+$  state in PDG until 1974

Removed from 1976 until 1994.

Back in PDG in 1996, renamed “ $f_0(600)$ ”



- The reason: The  $\sigma$  is EXTREMELY WIDE and has no “BW-resonance peak”.

Usually quoted by its pole:  $\sqrt{s_{pole}} \approx M - i\Gamma/2$

- The “kappa”: similar situation to the  $\sigma$ , but with strangeness.

Still out of PDG “summary tables”

- Narrower  $f_0(980)$ ,  $a_0(980)$  scalars well established but not well understood.

Even more states:  $f_0(1370)$ ,  $f_0(1500)$ ,  $f_0(1700)$

## Motivation: The role of scalars

- The  $f_0$ 's have the vacuum quantum numbers.  
Relevant for spontaneous chiral symmetry breaking.
- Glueballs: Feature of non-abelian QCD nature  
The lightest one expected with these quantum numbers
- Why lesser role in the saturation of ChPT parameters?
- SU(3) classification. How many multiplets? Inverted hierarchy?
- Non ordinary mesons? Tetraquarks, molecules, mixing...

First of all it is relevant to settle their existence, mass and width

1) Scalar Mesons: motivation & perspective

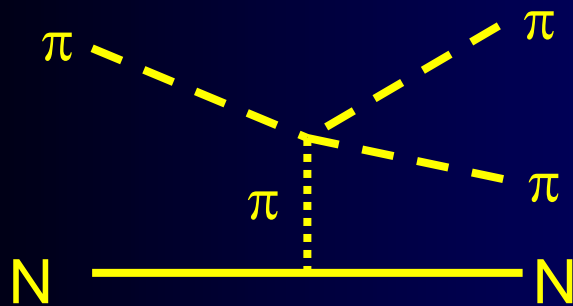
2) The  $\sigma$  or  $f_0(500)$

# The $\sigma$ until 2010: the data

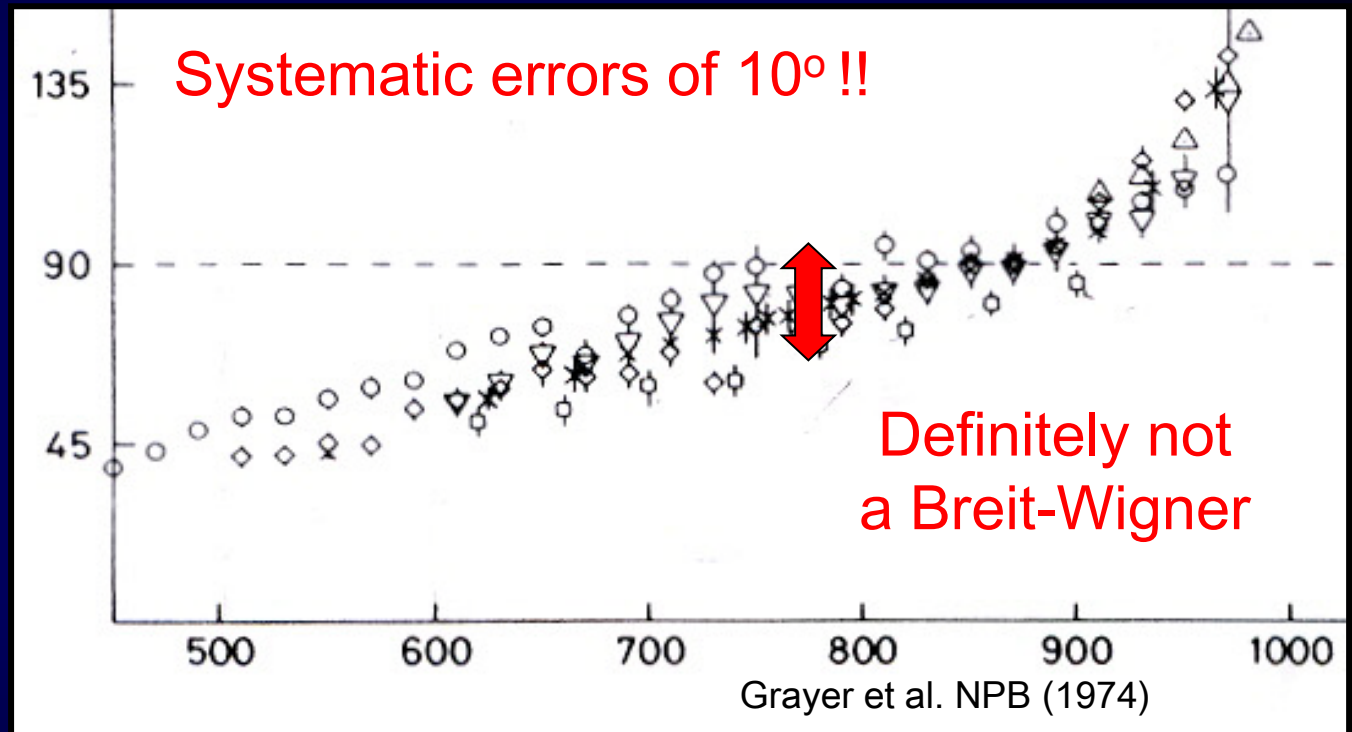
Unfortunately....NN insensitive to details... need other sources

## 1) From $\pi N$ scattering

Initial state not well defined, model dependent off-shell extrapolations (OPE, absorption,  $A_2$  exchange...) Phase shift ambiguities, etc...



Example: CERN-Munich  
5 different  $\pi\pi \rightarrow \pi\pi$   
analysis of same  
 $\pi p \rightarrow \pi\pi n$  data !!



## 2) From $K \rightarrow \pi\pi e\nu$ (" $K_{l4}$ decays")

Geneva-Saclay (77), E865 (01)

Pions on-shell. Very precise, but  $\delta_{00} - \delta_{11}$ .

**2010 NA48/2 data**

## 3) Decays from heavier mesons

Fermilab E791, Focus, Belle, KLOE, BES,...

“Production” from  $J/\Psi$ , B- and D- mesons, and  $\Phi$  radiative decays.

Very good statistics Clear initial states and different systematic uncertainties.

Strong experimental claims for wide and light  $\sigma$  around 500 MeV

“Strong” experimental claims for wide and light  $\kappa$  around 800 MeV

Very convincing for PDG, but personal caveats on parametrizations used, which may affect the precision and meaning of the pole parameters

PDG2002: “ $\sigma$  well established”

However, since 1996 until 2010 still quoted as

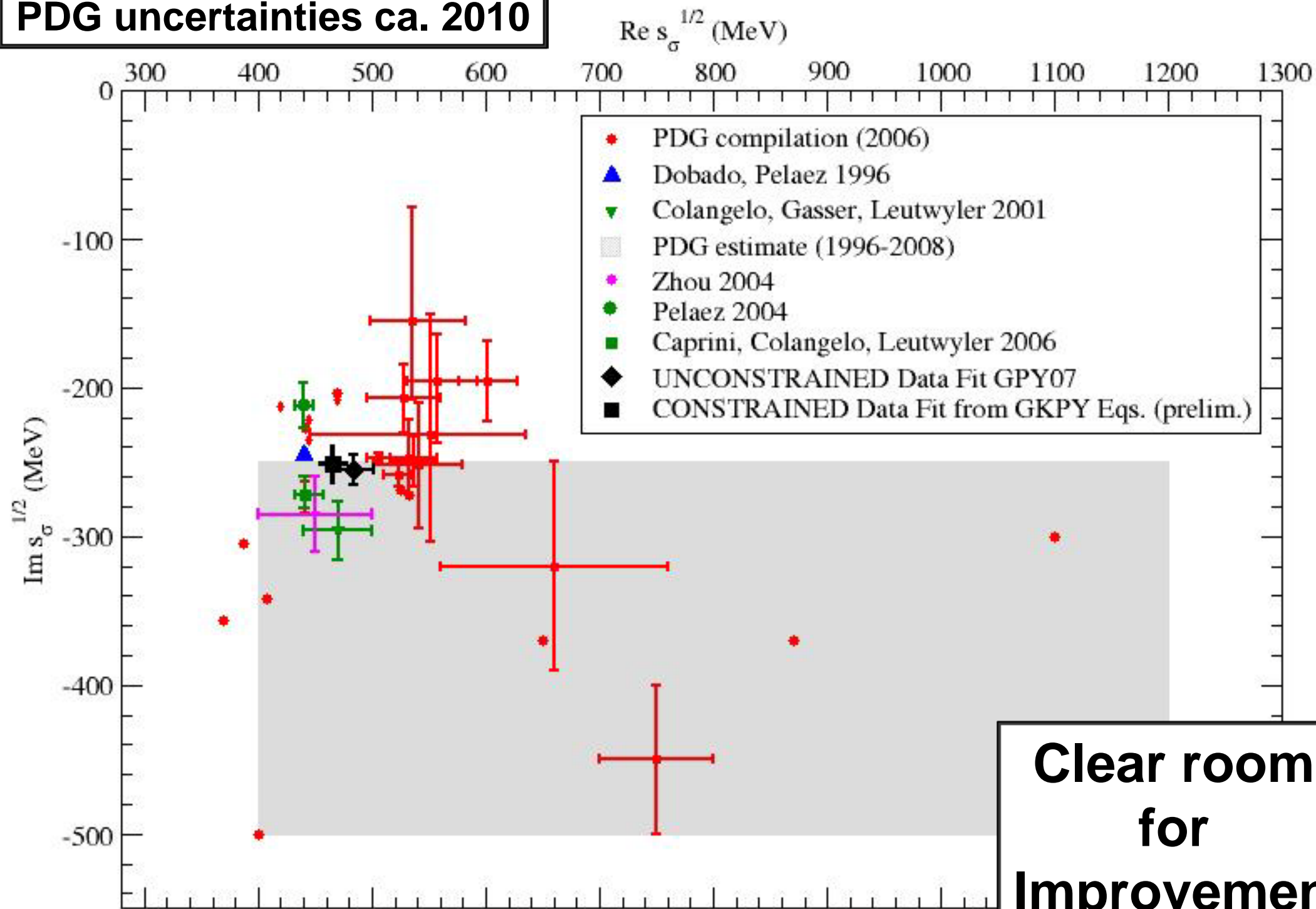
Mass= 400-1200 MeV

Width= 600-1000 MeV





# PDG uncertainties ca. 2010



## Part of the problem: The theory

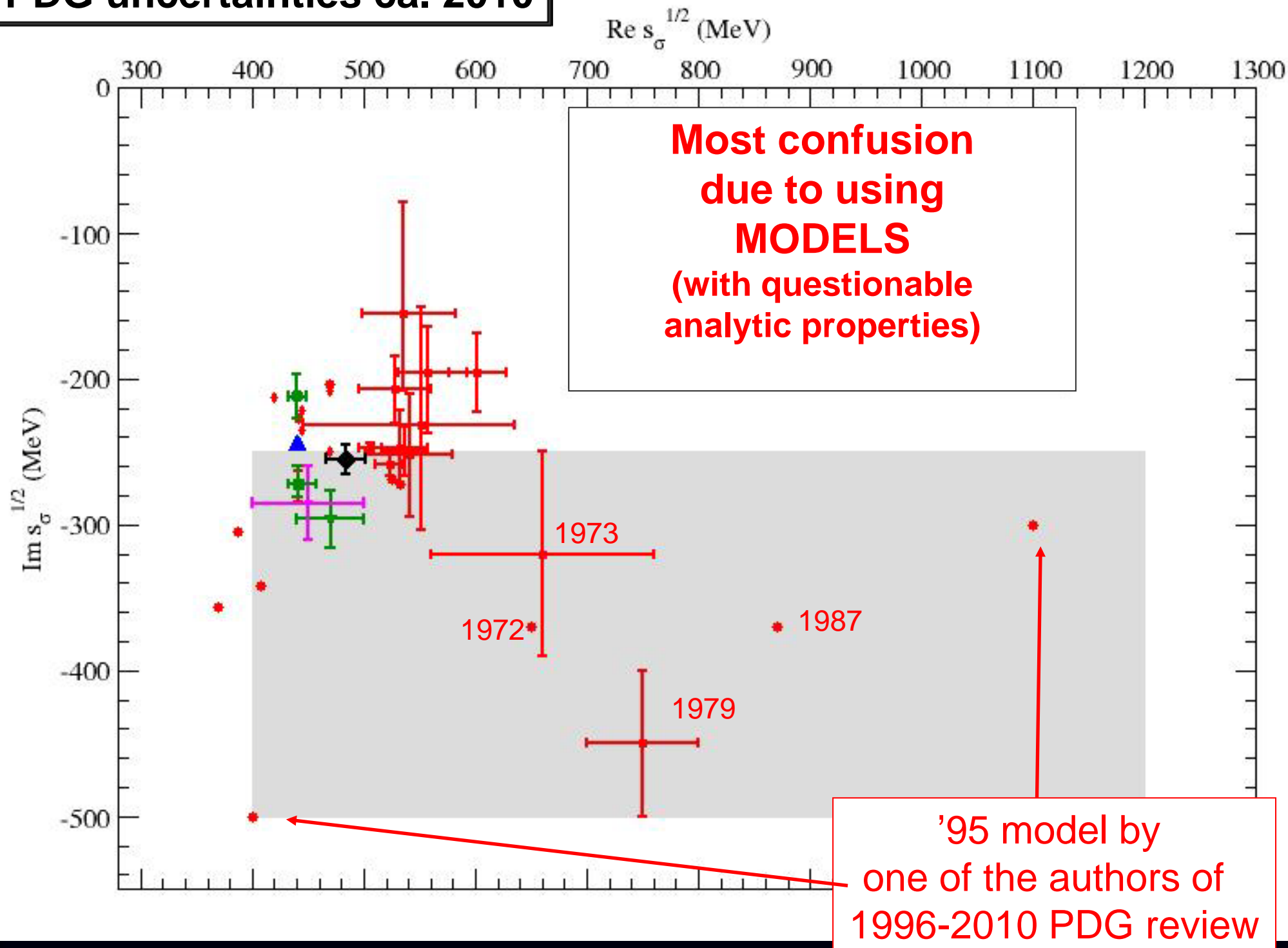
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Many old and new studies based on crude/simple models,

**Strong model dependences**

**Suspicion: What you put in is what you get out??**

# PDG uncertainties ca. 2010



## Part of the problem: The theory

Many old and new studies based on crude/simple models,

**Strong model dependences**

**Suspicion: What you put in is what you get out??**

Even experimental analysis using  
WRONG theoretical tools contribute to confusion  
(Breit-Wigners, isobars, K matrix, ....)

Lesson: For poles deep in the complex plane,  
the correct analytic properties are essential

Analyticity constraints more powerful in scattering

Dispersive formalisms are the most precise and reliable

**AND MODEL INDEPENDENT**

# The real improvement: Analyticity and Effective Lagrangians

- The 60's and early 70's: Strong constraints on amplitudes from ANALYTICITY in the form of dispersion relations

But poor input on some parts of the integrals and poor knowledge/understanding of subtraction constants = amplitudes at low energy values

- The 80's and early 90's: Development of Chiral Perturbation Theory (ChPT).  
(Weinberg, Gasser, Leutwyler)

It is the effective low energy theory of QCD. Provides information/understanding on low energy amplitudes

- The 90's and early 2000's: Combination of Analyticity and ChPT

(Truong, Dobado, Herrero, Donoghe, JRP, Gasser, Leutwyler, Bijnens, Colangelo, Caprini, Zheng, Zhou, Pennington...)

## ● Unitarized ChPT (Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner, ...)

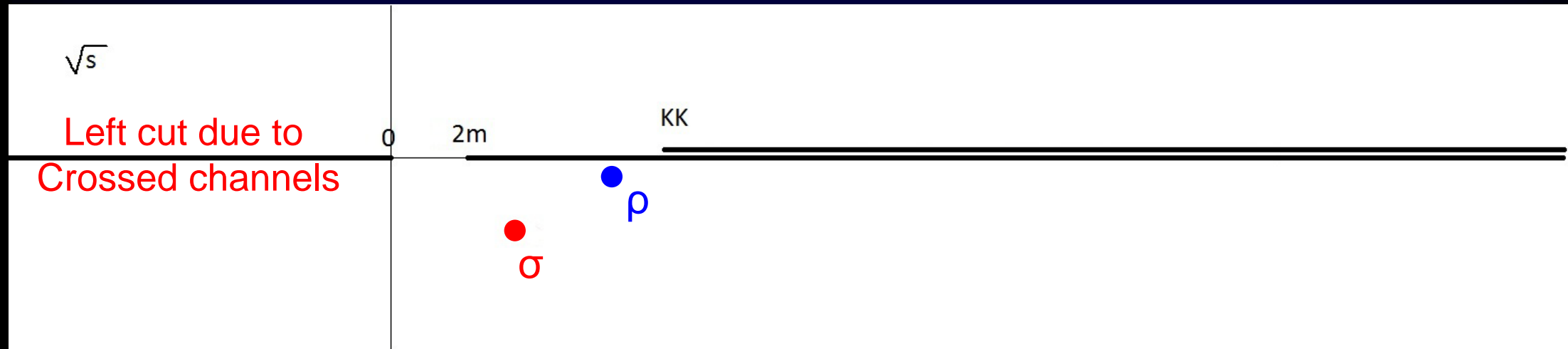
Use ChPT amplitudes inside left cut and subtraction constants of dispersion relation.

Relatively simple, although different levels of rigour. Generates all scalars

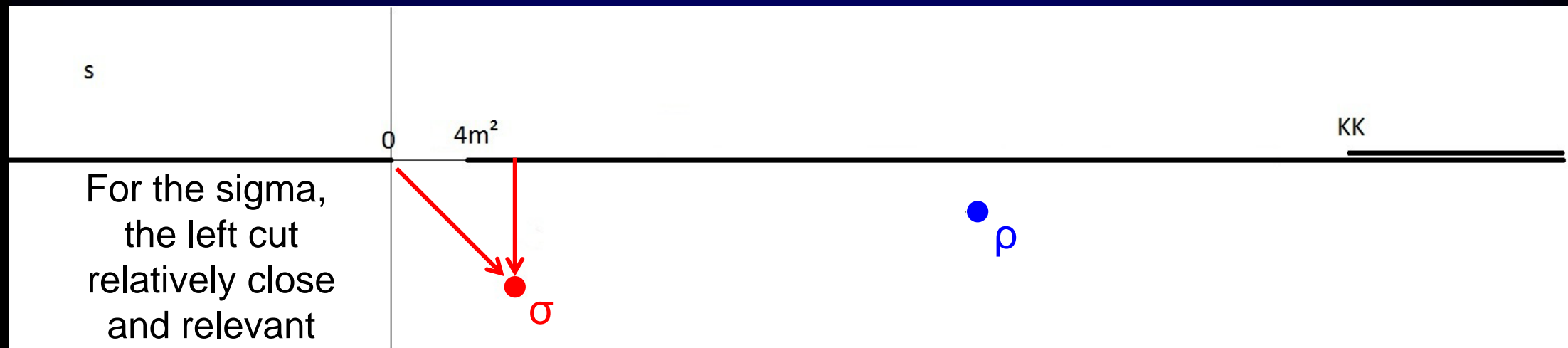
Crossing (left cut) approximated... so, not good for precision

# Why so much worries about “the left cut”?

It is wrong to think in terms of analyticity in terms of  $\sqrt{s}$



Since the partial wave is analytic in  $s$  ....



# Analyticity and Effective Lagrangians: two approaches

## Unitarized ChPT

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,...

Use ChPT amplitudes inside left cut and subtraction constants of dispersion relation.

Relatively simple, although different levels of rigour. Generates all scalars

Crossing (left cut) approximated... , not good for precision

## Roy and GKPY equations.

70's Roy, Basdevant, Pennington, Petersen...

00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski, JRP...

Left cut implemented with precision . Use data on all waves + high energy .

Optional: ChPT predictions for subtraction constants

The most precise and model independent pole determinations

$f_0(600)$  and  $\kappa(800)$  existence,  
mass and width

firmly established with precision

For long, well known  
for the “scalar community”

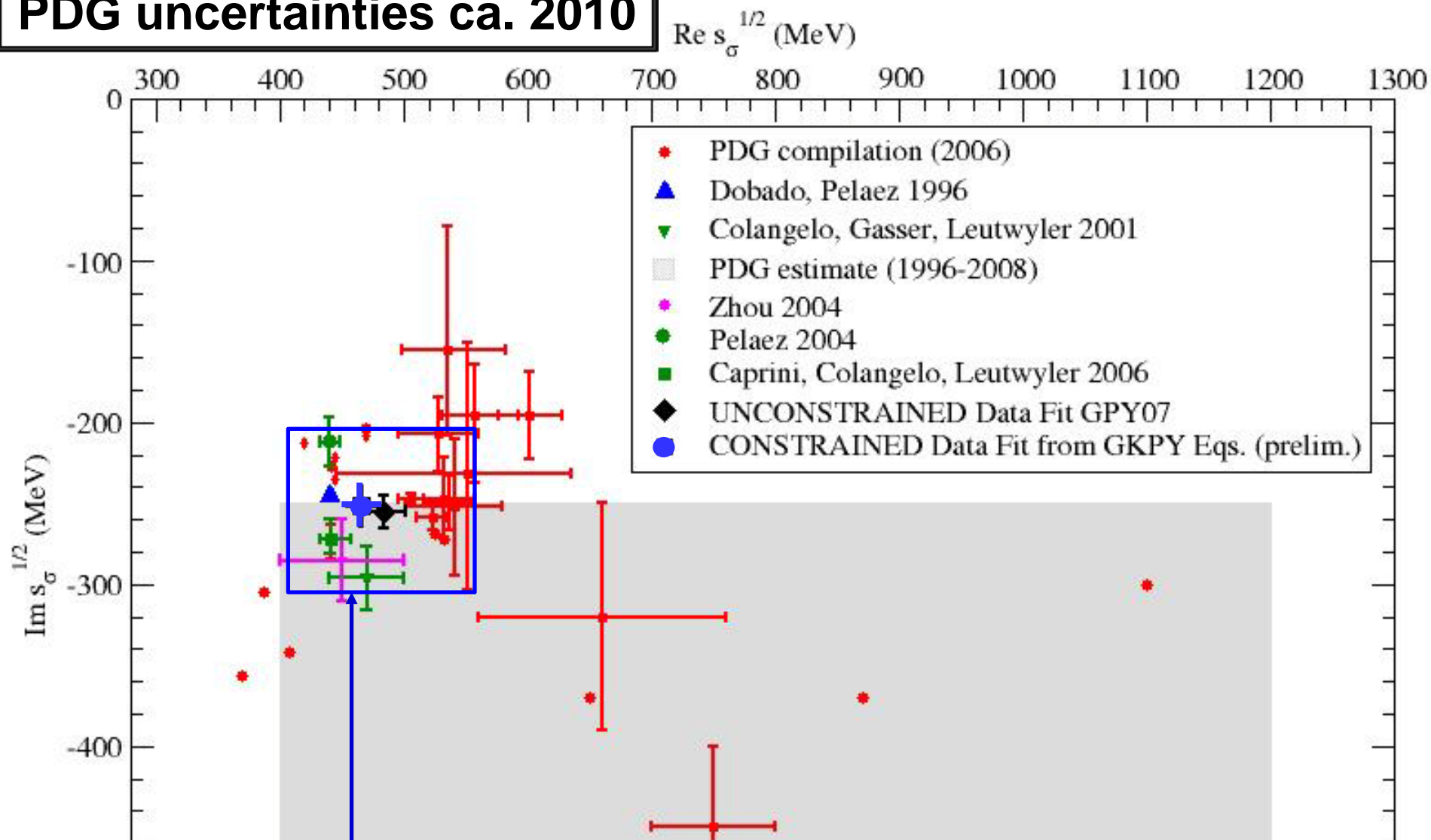
Yet to be acknowledged by PDG....

By 2006 very precise Roy Eq.+ChPT pole determination

Caprini,Gaser, Leutwyler



# PDG uncertainties ca. 2010



Data after 2000, both scattering and production  
**Dispersive- model independent** approaches  
Chiral symmetry correct

Yet to be  
acknowledged by  
PDG....

# Some relevant DISPERSIVE POLE Determinations

(after 2009, also “according” to PDG)

## GKPY equations = Roy like with one subtraction

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).

Includes latest NA48/2 constrained data fit .One subtraction allows use of data only

NO ChPT input but good agreement with previous Roy Eqs.+ChPT results.

$$(457_{-15}^{+14}) - i(279_{-7}^{+11})\text{MeV}$$

## Roy equations

B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

An S0 Wave determination up to KK threshold with input from previous Roy Eq. works

$$(442_{-8}^{+5}) - i(274_{-5}^{+6})\text{MeV}$$

## Analytic K-Matrix model

G. Mennesier et al, PLB696, 40 (2010)

$$(452 \pm 13) - i(259 \pm 16)\text{MeV}$$

The consistency of dispersive approaches, and also with previous results implementing UNITARITY, ANALTICITY and chiral symmetry constraints by many other people ...

(Ananthanarayan, Caprini, Bugg, Anisovich, Zhou, Ishida Surotsev, Hannah, JRP, Kaminski, Oller, Oset, Dobado, Tornqvist, Schechter, Fariborz, Saninno, Zoou, Zheng, etc....)

Has led the PDG to neglect those works not fulfilling these constraints also restricting the sample to those consistent with NA48/2, Together with the latest results from heavy meson decays Finally quoting in the 2012 PDG edition...

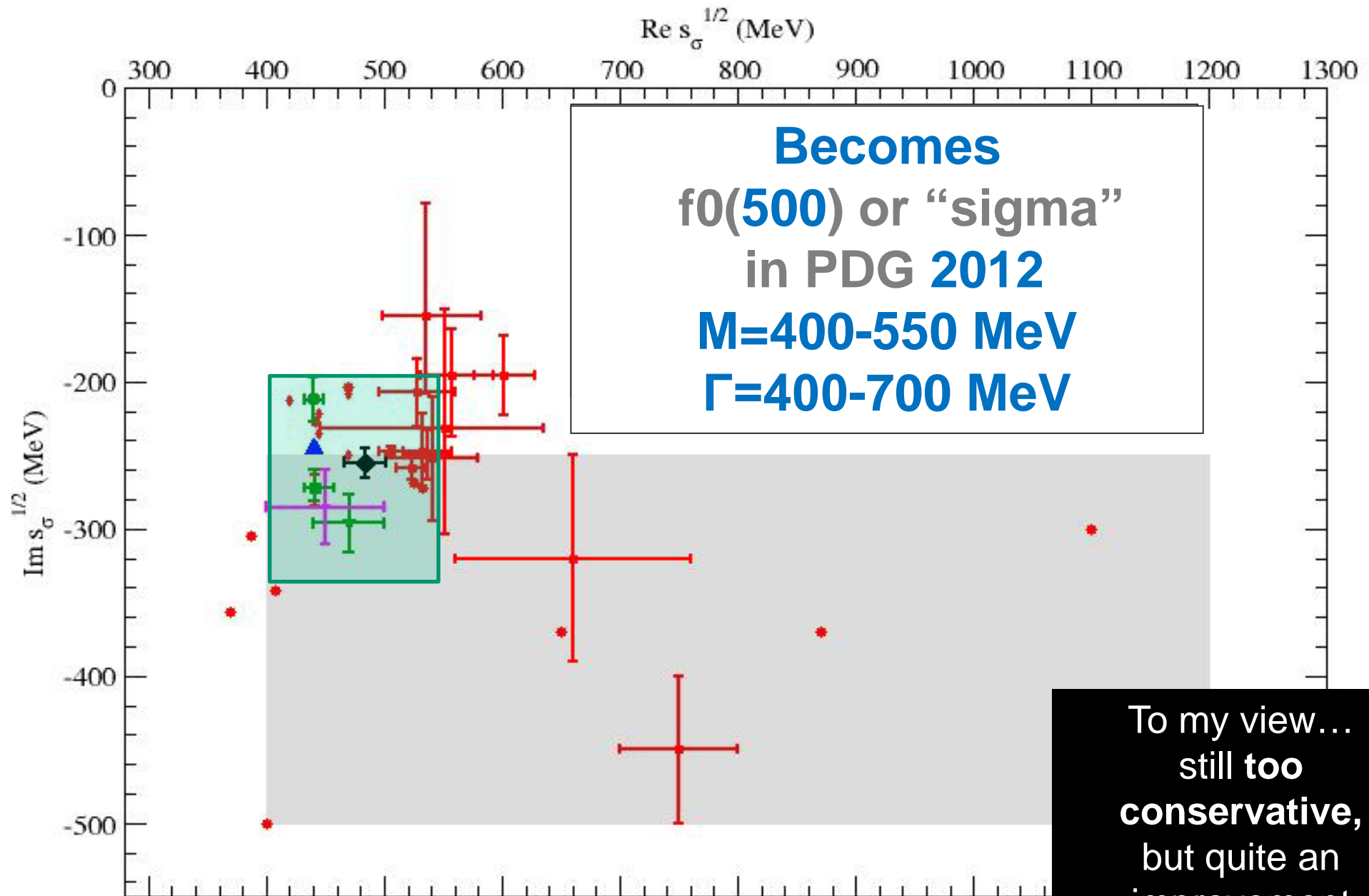
**$M=400-550$  MeV  
 $\Gamma=400-700$  MeV**

More than 5 times reduction in the mass uncertainty and 40% reduction on the width uncertainty

Accordingly THE NAME of the resonance is changed to...

**$f_0(500)$**

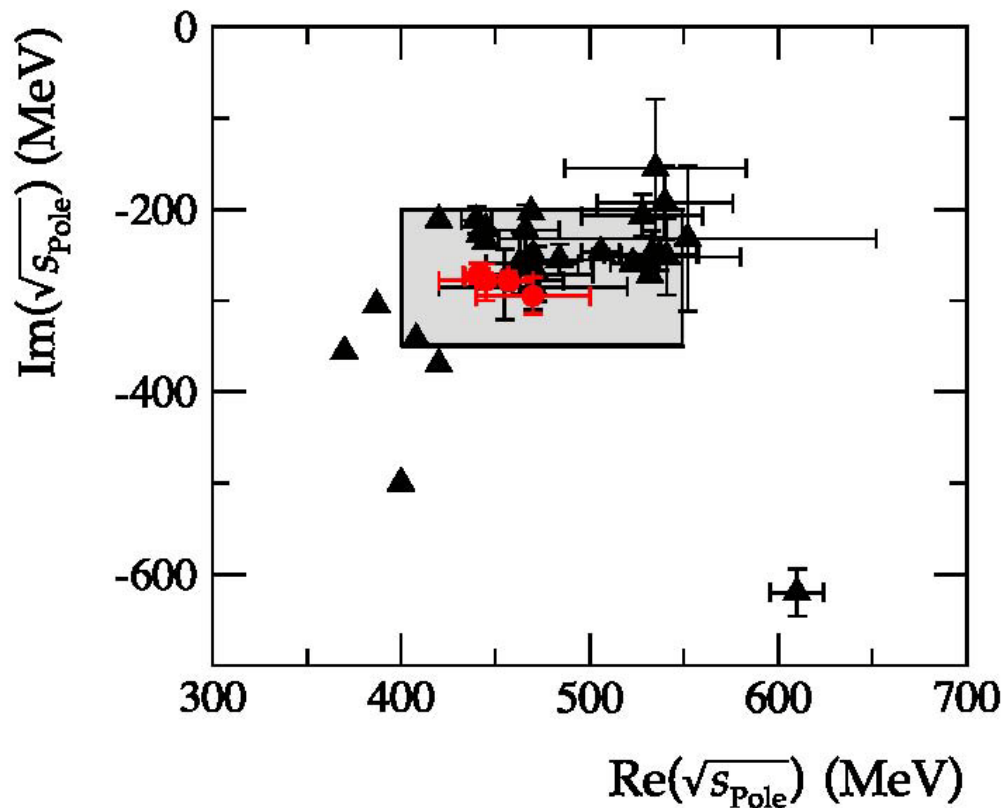
# DRAMMATIC AND LONG AWAITED CHANGE ON “sigma” RESONANCE @ PDG!!



Actually, in  
PDG 2012:  
“Note on  
scalars”

One might also take the more radical point of view and just average the most advanced dispersive analyses, Refs. [8–11], shown as solid dots in Fig. 1, for they provide a determination of the pole positions with minimal bias. This procedure leads to the much more restricted range of  $f_0(500)$  parameters

$$\sqrt{s_{\text{Pole}}} = (446 \pm 6) - i(276 \pm 5) \text{ MeV} .$$



And, at the risk of being annoying....

Now I find somewhat bold to average those results, particularly the uncertainties

8. G. Colangelo, J. Gasser, and H. Leutwyler, NPB603, 125 (2001).
9. I. Caprini, G. Colangelo, and H. Leutwyler, PRL 96, 132001 (2006).
10. R. Garcia-Martin, R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).
11. B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

Unfortunately, to keep the confusion  
the PDG still quotes a “Breit-Wigner mass” and width...



I have no words...

The dispersive approach is model independent.

Just analyticity and crossing properties

- Determine the amplitude at a given energy even if there were no data precisely at that energy.
- Relate different processes
- Increase the precision
- The actual parametrization of the data is irrelevant once inside integrals.

A precise  $\pi\pi$  **scattering analysis** helps determining the  $\sigma$  and  $f_0(980)$  parameters and is useful for any hadronic process containing several pions in the final state

Conformal expansion, 4 terms are enough. First, Adler zero at  $m_\pi^2/2$

Average of  $\pi N \rightarrow \pi\pi N$  data sets with enlarged errors, at 870- 970 MeV, where they are consistent within  $10^\circ$  to  $15^\circ$  error.

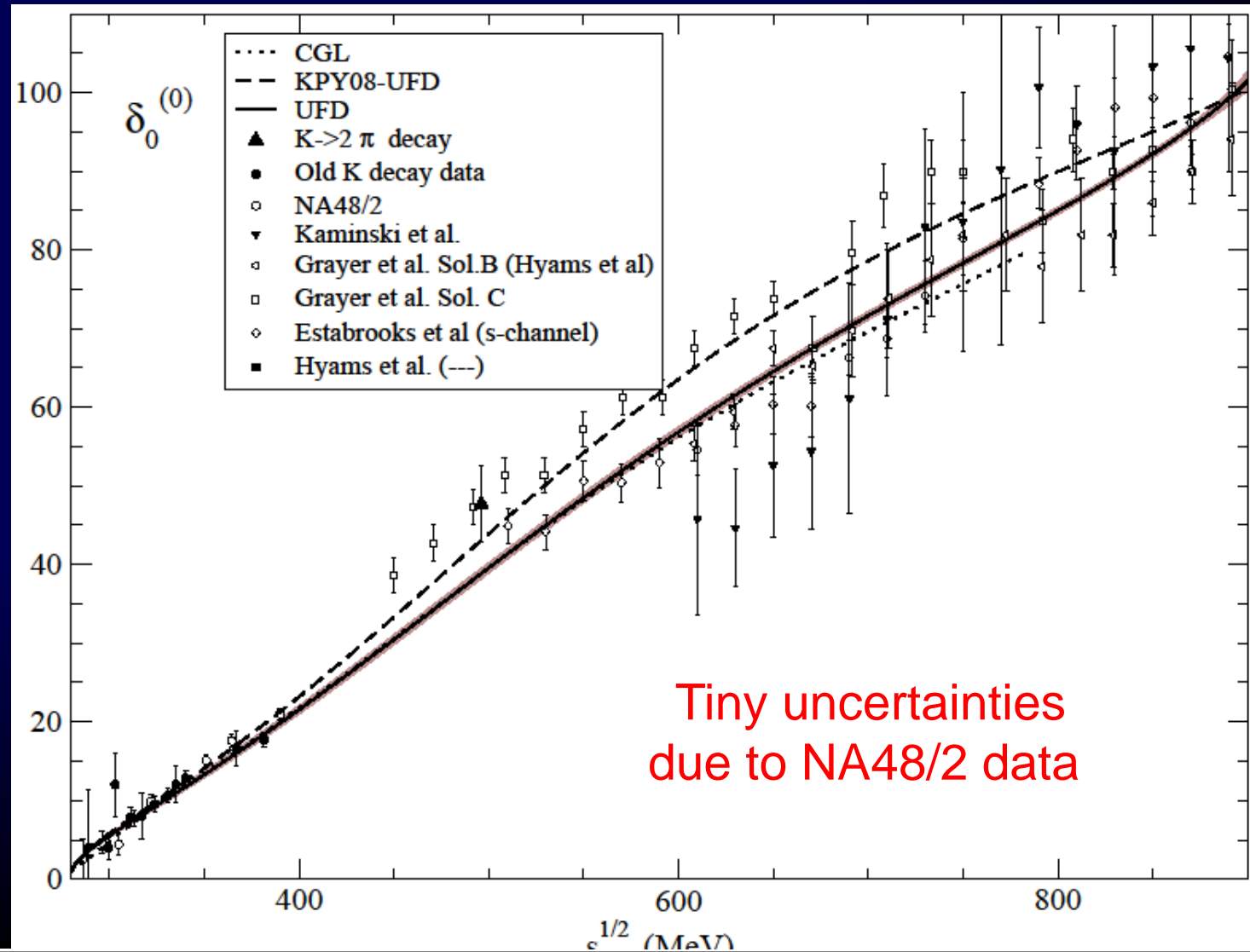
We use data on KI4 including the NEWEST:

NA48/2 results

Get rid of  $K \rightarrow 2\pi$

Isospin corrections from Gasser to NA48/2

It does **NOT** HAVE  
A BREIT-WIGNER  
SHAPE





Structure of calculation: Example Roy and GKPY Eqs.

Both are coupled channel equations for the infinite partial waves:

$I$ =isospin 0,1,2 ,  $\ell$  =angular momentum 0,1,2,3....

$$\text{Re } t_{\ell}^{(I)}(s) = ST_{\ell}^{(I)}(s) + \sum_{I'=0}^2 \sum_{\ell'=0}^1 PP \int_{4M_{\pi}^2}^{s_{\max}} ds' K_{\ell\ell'}^{II'}(s') \text{Im } t_{\ell}^{(I)}(s') + DT_{\ell}^{(I)}(s)$$

SUBTRACTION  
TERMS  
(polynomials)

ROY: 2nd order

GKPY: 1st order

KERNEL TERMS  
known

More energy suppressed

Less energy suppressed

DRIVING  
TERMS  
(truncation)  
Higher waves  
and High energy

Very small

small

Partial wave  
on  
real axis

“OUT”

=?

“IN (from our data parametrizations)”

Similar  
Procedure  
for FDRs

# Imposing FDR's , Roy Eqs and GKPY as constraints

To improve our data fits, we can IMPOSE FDR's, Roy Eqs. GKPY Eqs.

We obtain CONSTRAINED FITS TO DATA (CFD) by minimizing:

$$\chi^2 \equiv \underbrace{\{\bar{d}_{00}^2 + \bar{d}_{0+}^2 + \bar{d}_{It=1}^2\}}_{3 \text{ FDR's}} + \underbrace{\{\bar{d}_{S0_{roy}}^2 + \bar{d}_{P_{roy}}^2 + \bar{d}_{S2_{roy}}^2\}}_{3 \text{ Roy Eqs.}} + \underbrace{\{\bar{d}_{S0_{GKPY}}^2 + \bar{d}_{P_{GKPY}}^2 + \bar{d}_{S2_{GKPY}}^2\}}_{3 \text{ GKPY Eqs.}} \}W +$$

$$\underbrace{\bar{d}_{SR1}^2 + \bar{d}_{SR2}^2}_{\text{Sum Rules for crossing}} + \underbrace{\sum_k \frac{(p_k - p_k^{\text{exp}})^2}{\delta p_k^2}}_{\text{Parameters of the unconstrained data fits}}$$

W roughly counts the number of effective degrees of freedom  
(sometimes we add weight on certain energy regions)

The resulting fits differ by less than  $\sim 1\sigma$  -  $1.5\sigma$  from original unconstrained fits

We impose 3 independent FDR's, 3 Roy Eqs + 3 GKPY Eqs.  
Very well satisfied at the end

# UNCERTAINTIES IN Standard ROY EQS. vs GKPY Eqs

Why are GKPY Eqs. relevant?

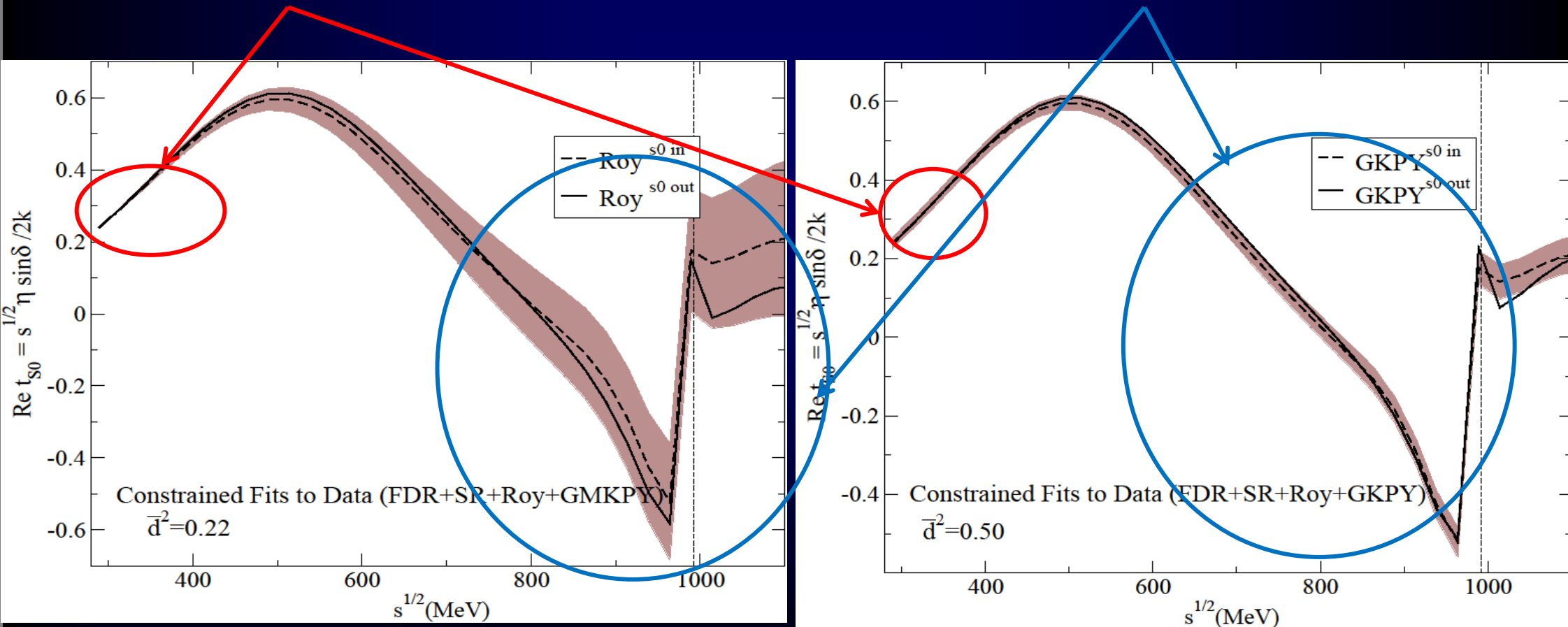
One subtraction yields better accuracy in  $\sqrt{s} > 400$  MeV region

Roy Eqs.

smaller uncertainty below  $\sim 400$  MeV

GKPY Eqs,

smaller uncertainty above  $\sim 400$  MeV



# DIP vs NO DIP inelasticity scenarios

Now we find large differences in GKPY S0 wave  $d^2$

## UFD

992MeV < e < 1100MeV

Dip 6.15  
No dip 23.68

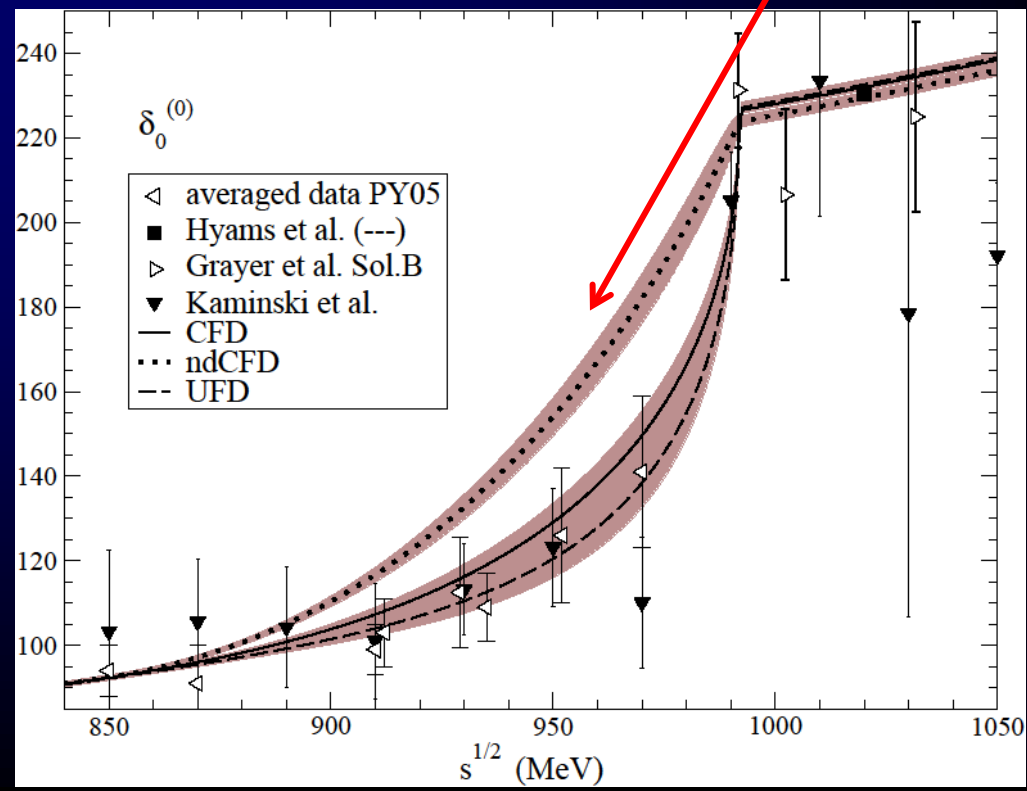
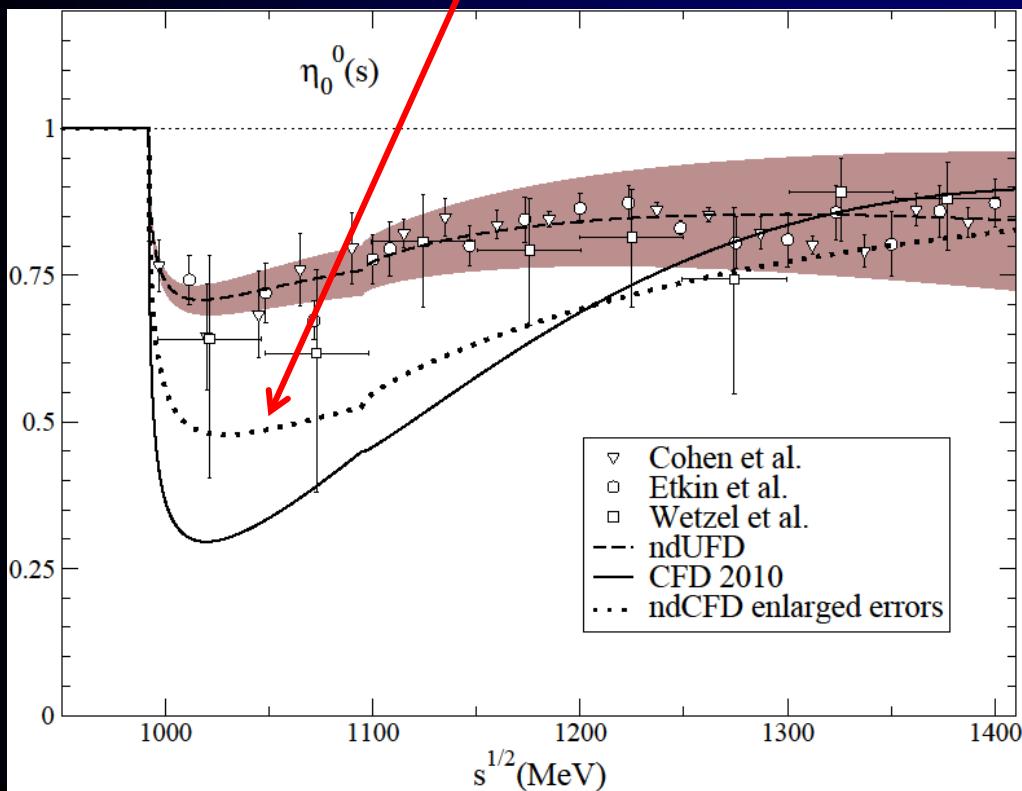
## CFD

850MeV < e < 1050MeV

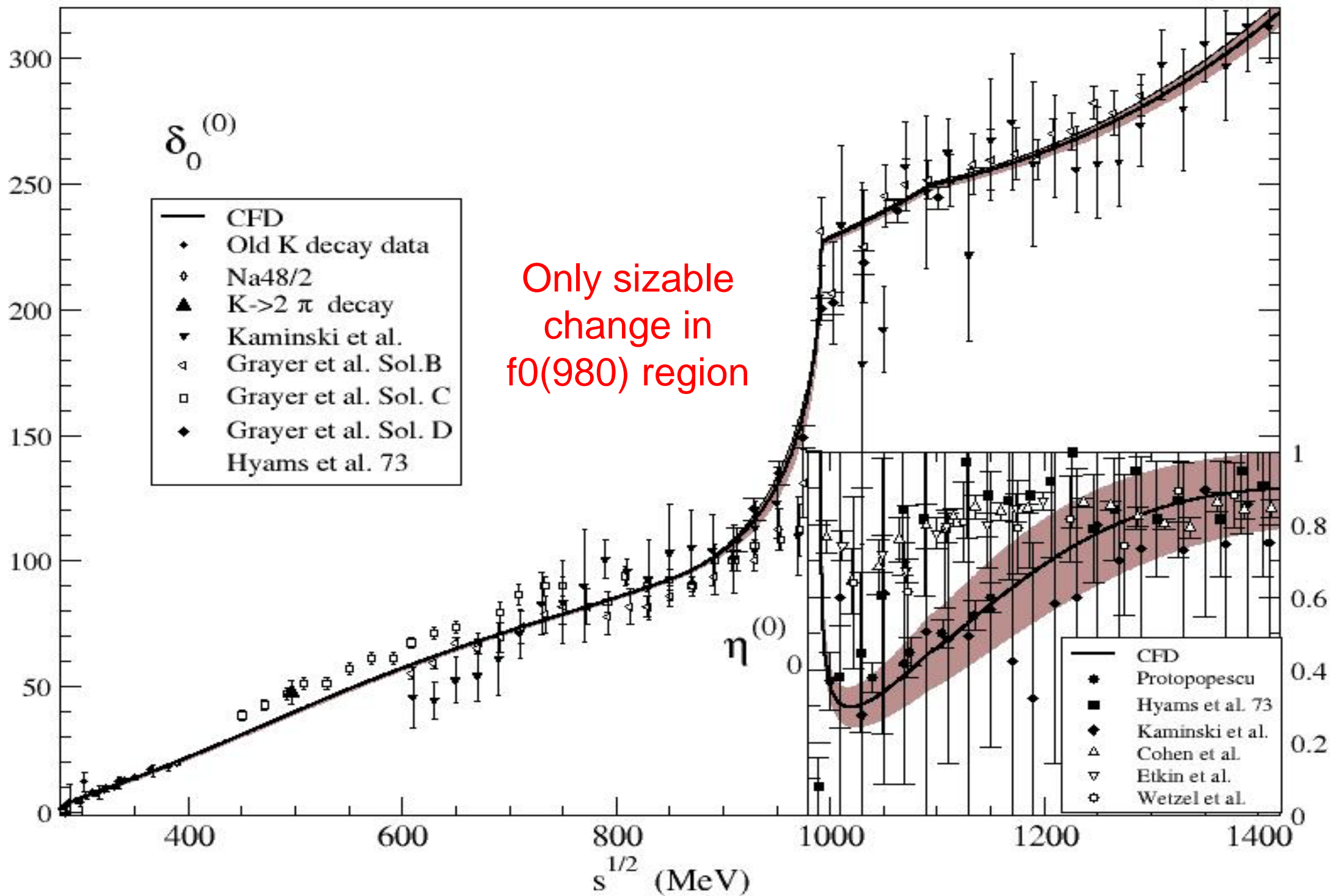
Dip 1.02  
No dip 3.49  
Improvement possible?  
No dip (enlarged errors) 1.66  
No dip (forced) 2.06

But becomes the "Dip" solution

Other waves worse and data on phase NOT described



# S0 wave: from UFD to CFD



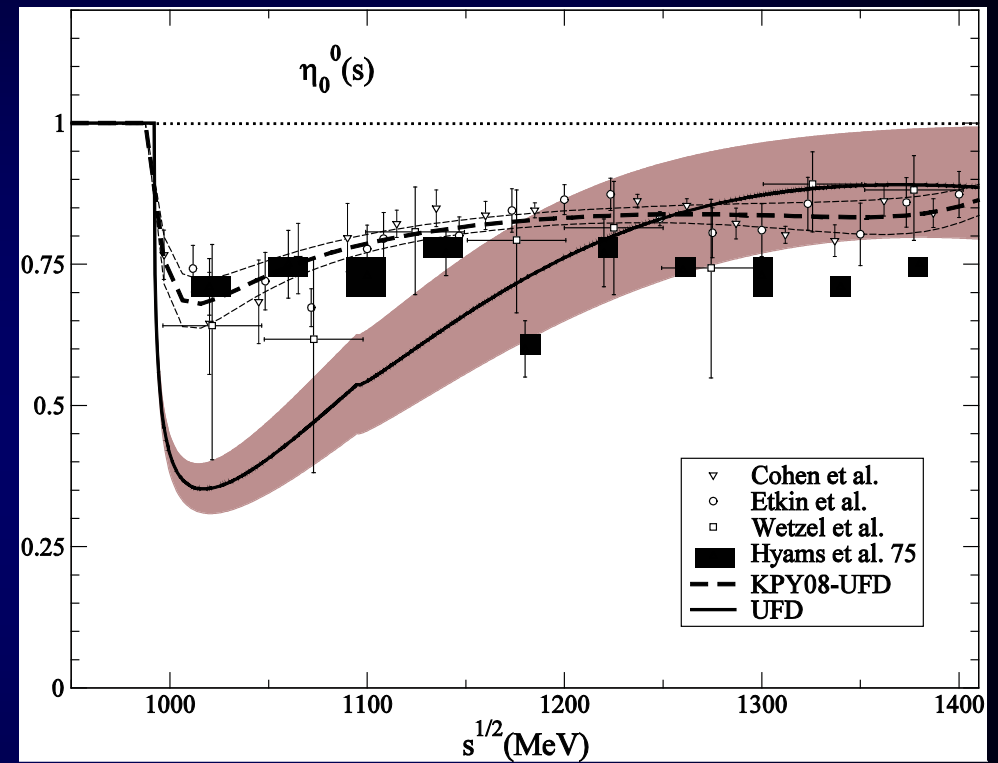
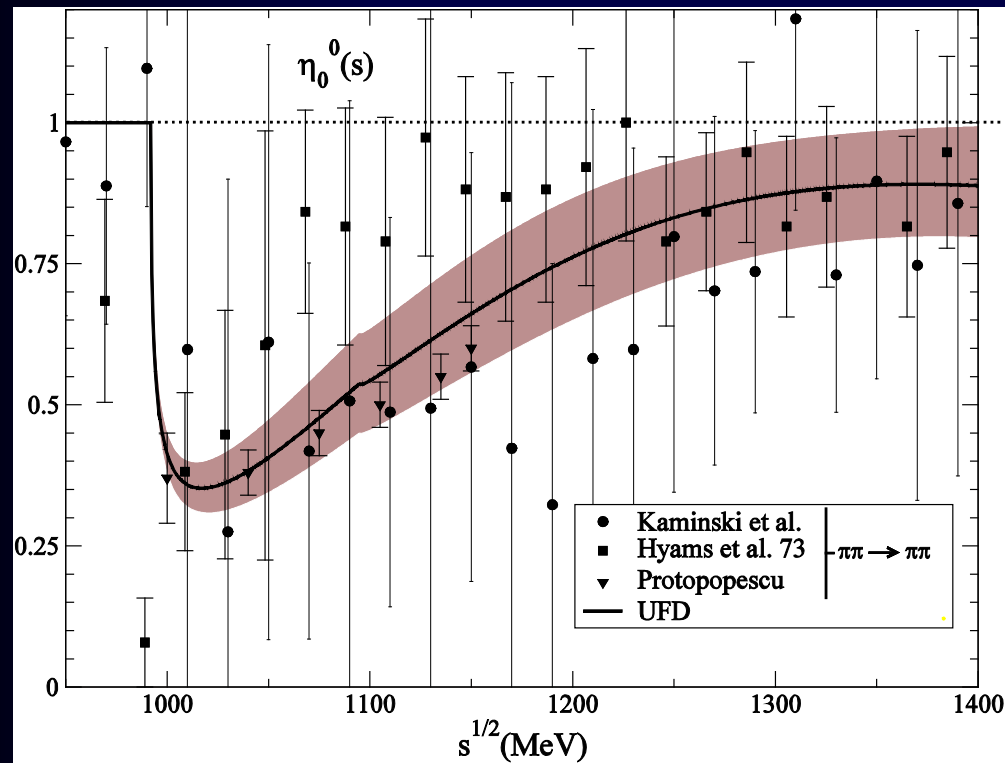
- 1) Scalar Mesons: motivation & perspective
- 2) The  $\sigma$  or  $f_0(500)$
- 3) The  $f_0(980)$

# DIP vs NO DIP inelasticity scenarios

Longstanding controversy between inelasticity data sets : (Pennington, Bugg, Zou, Achasov....)

Some of them prefer a “dip” structure...

... whereas others do not



GKPY Eqs. disfavors the non-dip solution

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

Garcia-Martin , Kaminski, JRP, Ruiz de Elvira, PRL107, 072001(2011)

Confirmation from Roy Eqs.

B. Moussallam, Eur. Phys. J. C71, 1814 (2011)

# Some relevant recent DISPERSIVE POLE Determinations of the $f_0(980)$ (after CD2009, also “according” to PDG)

## ● GKPY equations = Roy like with one subtraction

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

Garcia-Martin , Kaminski, JRP, Ruiz de Elvira, PRL107, 072001(2011)

$$(996 \pm 7) - i(25_{-6}^{+10}) \text{ MeV}$$

## ● Roy equations

$$(996_{-14}^{+4}) - i(24_{-3}^{+11}) \text{ MeV}$$

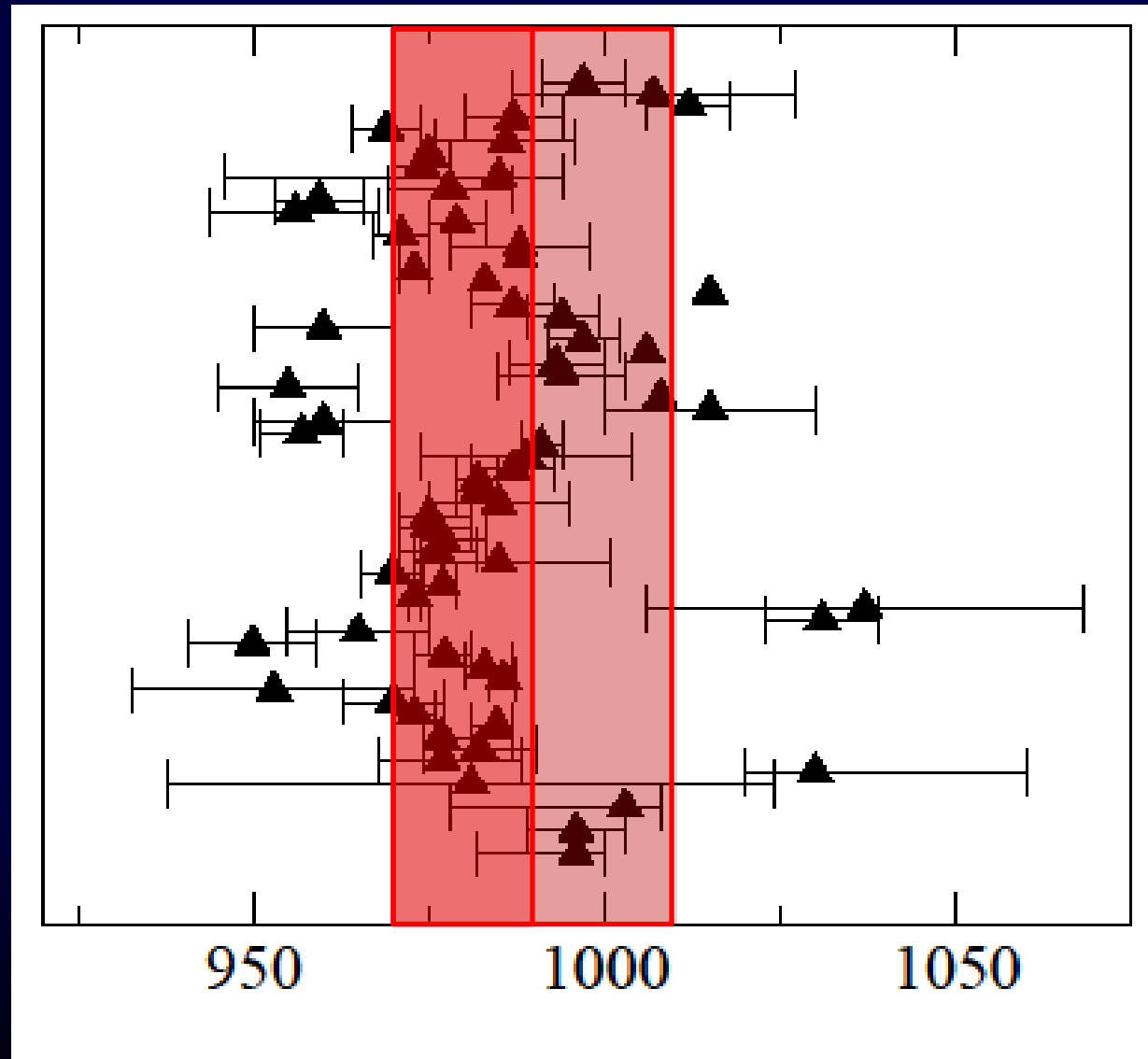
B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

The dip solution favors somewhat higher masses slightly above KK threshold  
and reconciles widths from production and scattering



Thus, PDG12 made a small correction for the  $f_0(980)$  mass  
& more conservative uncertainties

$$M = 980 \pm 10 \text{ MeV} \rightarrow M = 990 \pm 20 \text{ MeV}$$



1) Scalar Mesons: motivation & perspective

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3) The  $f_0(980)$

4) The  $\kappa$  or  $K(800)$  and  $a_0(980)$

No changes on the  $a_0$  mass and width at the PDG for the  $a_0(980)$

- Still “omitted from the summary table” since, “needs confirmation”

But, all sensible implementations of unitarity, chiral symmetry, describing the data find a pole between 650 and 770 MeV with a 550 MeV width or larger.

As for the sigma, and the most sounded determination comes from a Roy-Steiner dispersive formalism, consistent with UChPT

Decotes Genon et al 2006

Since 2009 two EXPERIMENTAL results are quoted from D decays @ BES2

Surprisingly BES2 gives a pole position of  $(764 \pm 63_{-54}^{+71}) - i(306 \pm 149_{-85}^{+143}) \text{MeV}$

But AGAIN!! PDG goes on giving Breit-Wigner parameters!! More confusion!!

Fortunately, the PDG mass and width averages are dominated by the Roy-Steiner result

$$(682 \pm 29) - i(273 \pm 22) \text{MeV}$$

## Summary of the mini-review

For quite some time now the use of analyticity, unitarity, chiral symmetry, etc... to describe scattering and production data has allowed to establish the existence of light the  $\sigma$  and  $\kappa$

These studies, together with more reliable and precise data, have allowed for PRECISE determinations of light scalar pole parameters

The PDG 2012 edition has FINALLY acknowledged the consistency of theory and experiment and the rigour and precision of the latest results, fixing, to a large extent, the very unsatisfactory compilation of  $\sigma$  results

Unfortunately, some traditional but inadequate parametrizations, long ago discarded by the specialists, are still being used in the PDG for the  $\sigma$  and the  $\kappa$

I expect a more “cleaning up” in the PDG for other scalar resonances soon

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5) Nature and classification.

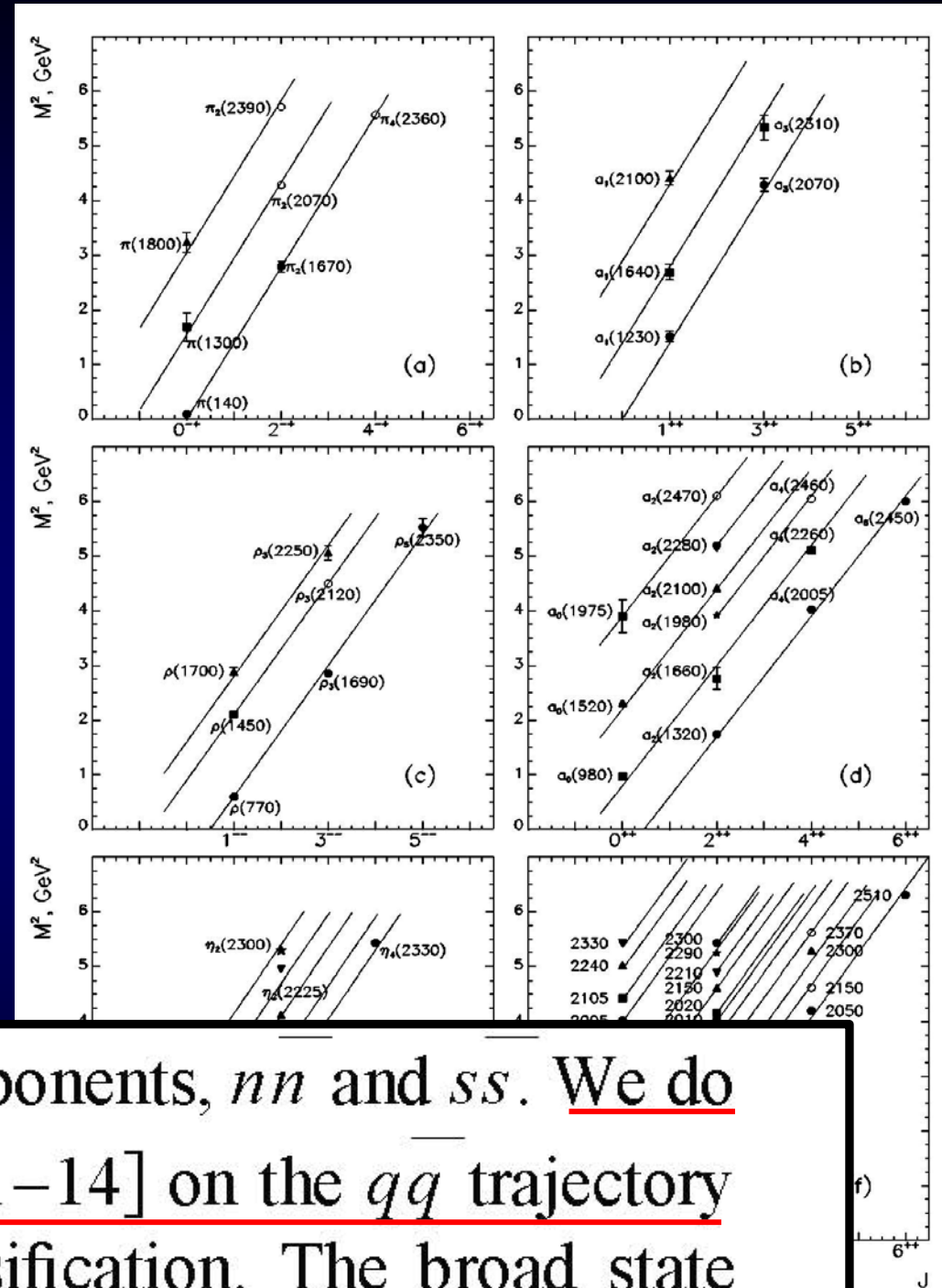
Regge trajectory of the  $f_0(500)$

Another feature of QCD as a confining theory is that hadrons are classified in almost linear  $(J, M^2)$  trajectories

Roughly, this can be explained by a quark-antiquark pair confined at the ends of a string-like/flux-tube configuration.

The trajectories can also be understood from the analytic extension to the complex angular momentum plane (Regge Theory)

However, light scalars, and particularly the  $f_0(500)$  do not fit in.



are doubled due to two flavor components,  $nn$  and  $ss$ . We do not put the enigmatic  $\sigma$  meson [11–14] on the  $qq$  trajectory supposing  $\sigma$  is alien to this classification. The broad state

An elastic partial wave amplitude near a Regge pole reads

Where  $\alpha$  is the “trajectory” and  $\beta$  the “residue”

$$t_l(s) = \frac{\beta(s)}{l - \alpha(s)} + f(l, s)$$

If the amplitude is dominated by the pole, unitarity implies:

$$\text{Im } \alpha(s) = \rho(s)\beta(s).$$

Imposing the threshold behavior  $q^{2l}$  and other constraints from the analytic extension to the complex plane,

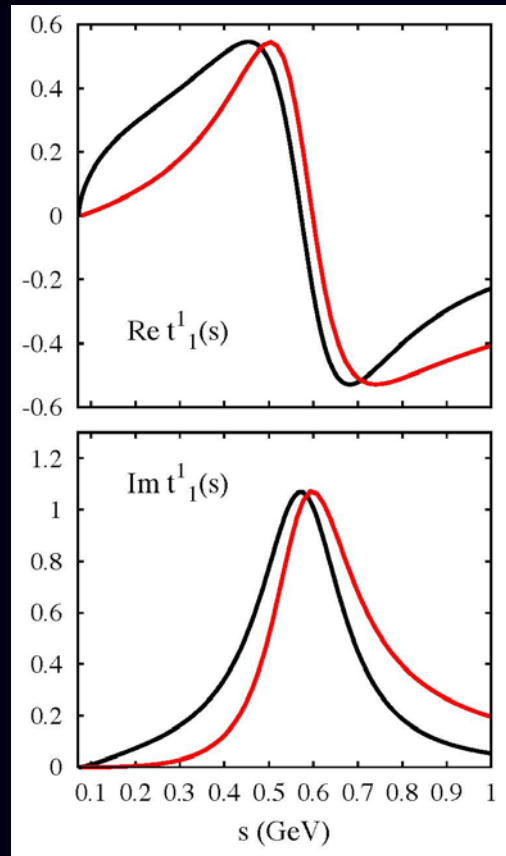
$$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})} \gamma(s)$$

This leads to a set of dispersion relations constraining the trajectory and residue

$$\text{Re}\alpha(s) = \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\alpha(s')}{s'(s' - s)}$$

$$\text{Im}\alpha(s) = \rho(s)b_0 \frac{\hat{s}^{\alpha_0 + \alpha' s}}{|\Gamma(\alpha(s) + \frac{3}{2})|} \exp\left(-\alpha' s [1 - \log(\alpha' s_0)] + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\alpha(s') \log \frac{\hat{s}}{\hat{s}'} + \arg \Gamma(\alpha(s') + \frac{3}{2})}{s'(s' - s)}\right)$$

The scalar case requires a small modification to include the Adler zero



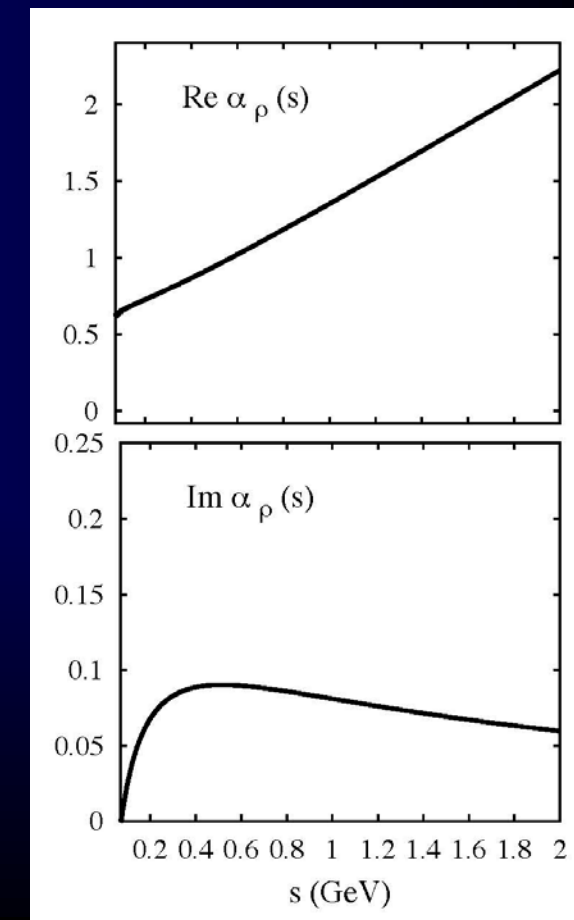
When we iteratively solve the previous equations fitting only the pole and residue of the  $\rho(770)$  obtained from the model independent GKPY approach...

We recover a fair representation of the amplitude

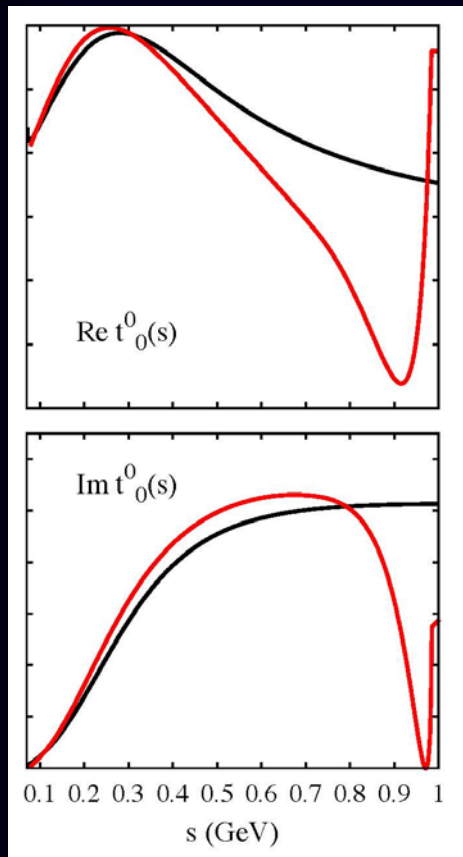
But we also obtain a “prediction” for the Regge rho trajectory, which is:

- 1) Almost real
- 2) Almost linear:  $\alpha(s) \sim \alpha_0 + \alpha' s$
- 3) The intercept  $\alpha_0 = 0.53$
- 4) The slope  $\alpha' = 0.895 \text{ GeV}^{-2}$

Remarkably consistent with the literature, taking into account our approximations







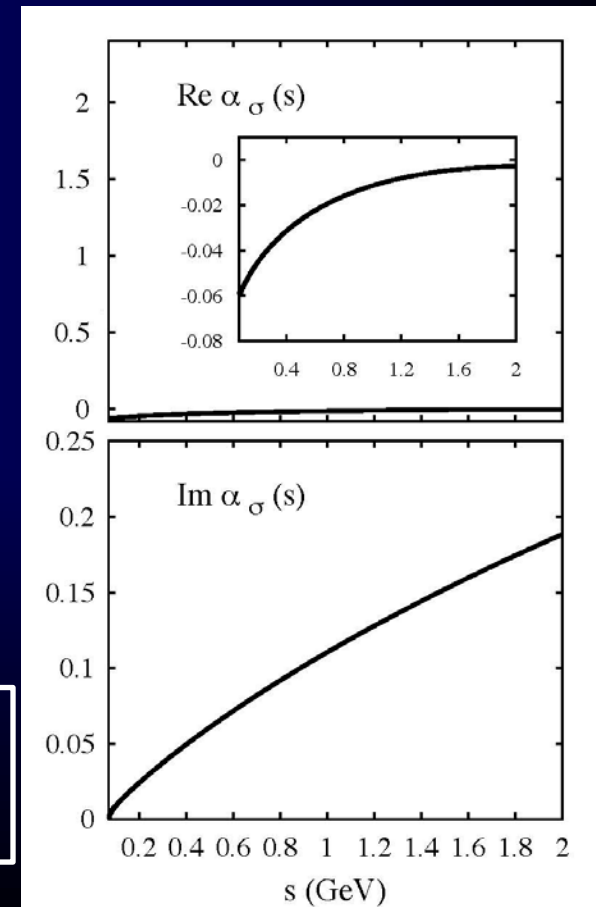
Since the approach works remarkably well for the rho, we repeat it for the f0(500). We fit the pole obtained from GKPY to a single pole-Regge like amplitude

Again we recover a fair representation of the amplitude, even better than for the rho

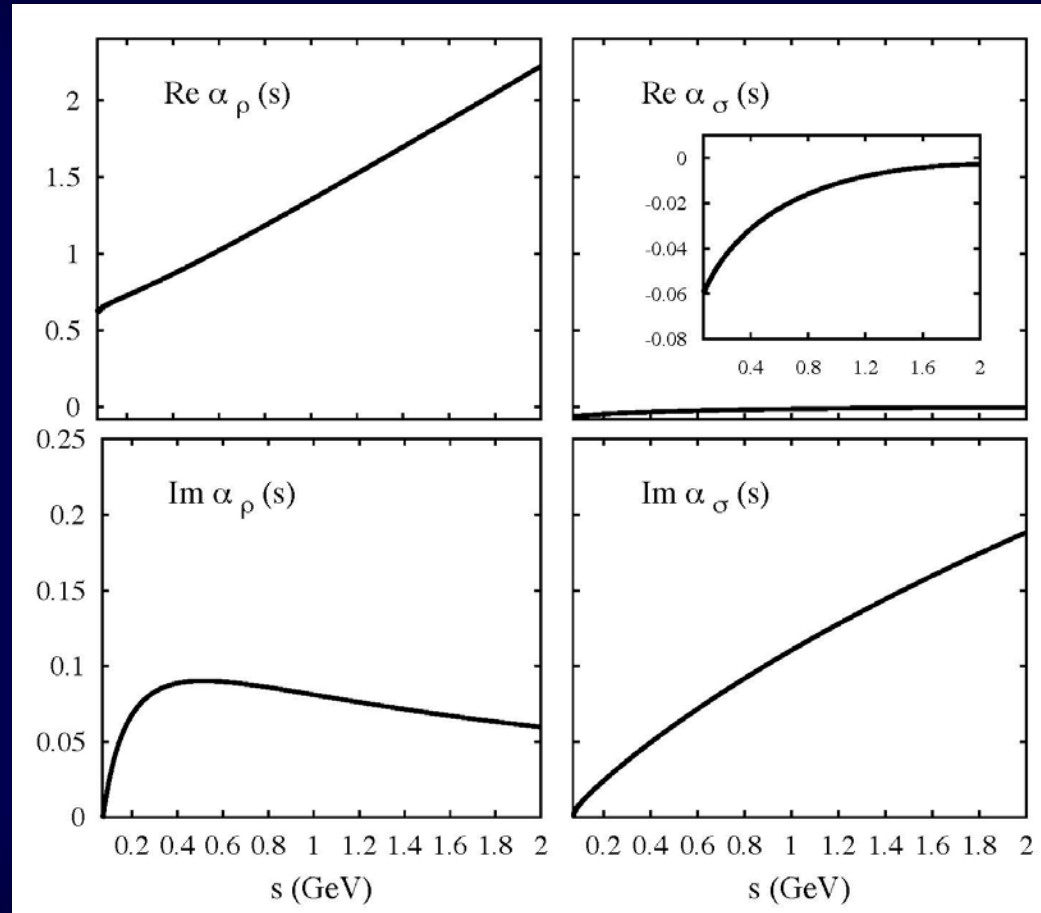
And we obtain a “prediction” for the Regge sigma trajectory, which is:

- 1) NOT real
- 2) NOT evidently linear
- 3) The intercept  $\alpha_0 = -0.08$
- 4) The slope  $\alpha' = 0.004 \text{ GeV}^{-2}$

**Two orders of magnitude flatter than other hadrons**  
**The sigma does NOT fit the usual classification**



## Comparison of sigma vs. Rho trajectories



**Two orders of magnitude flatter than other hadrons**  
**The sigma does NOT fit the usual classification**

## Summary

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Existence and properties of lightest scalars settled (and finally acknowledged) with precision thanks to model independent dispersive approaches

Emerging picture with two nonets. One non-ordinary below 1GeV.

Support for non-ordinary nature from :

Estimates of sigma Regge trajectory. It does not fit ordinary classification