Nucleon Polarizabilities in Compton Scattering

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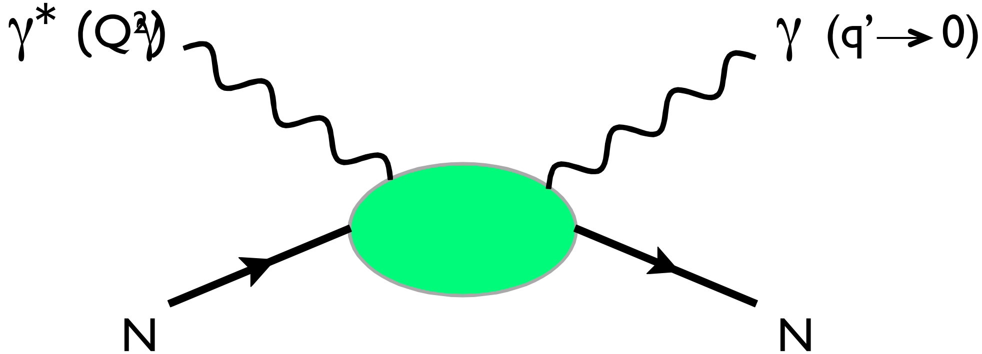


low energy outgoing photon plays
role of applied e.m. dipole field

Nucleon response:

POLARIZABILITIES

- Real Compton Scattering:
  - ⇒ global response of internal degrees of freedom
- Virtual Compton Scattering:
  - ⇒ local response on a distance scale depending on Q^2

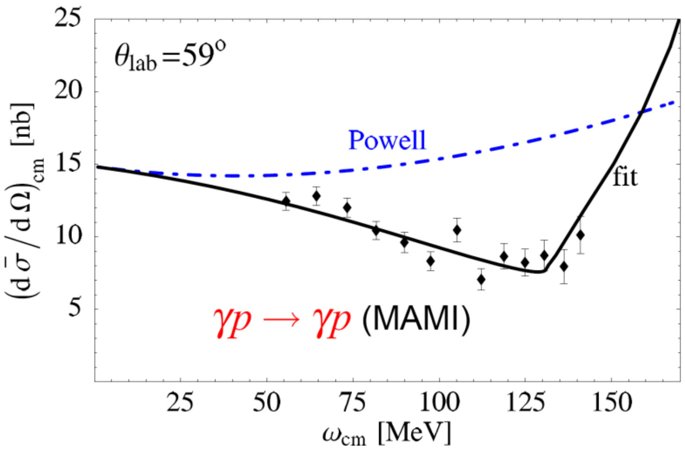
Polarizabilities in
Real and Virtual Compton Scattering

- What we learn about e.m. structure of the nucleon from Polarizabilities

- Status of theoretical and experimental analysis

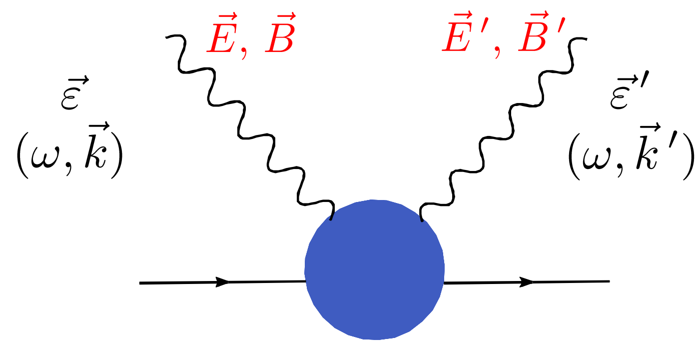
- Dispersion relation formalism as a tool to extract polarizabilities from data

Static polarizabilities in Real Compton Scattering



Powell cross section: photon scattering off a pointlike nucleon with anomalous magnetic moment

Static polarizabilities: response of the internal nucleon degrees of freedom to a static electric and magnetic field



$$H_{\rm eff}^{\rm pol} = -2\pi \Big\{ \omega^2 \left[\alpha_{E1}\, \vec{E}^2 + \beta_{M1}\, \vec{B}^2 \right]$$

spin-independent dipole

$$+\, \omega^3 \Big[\gamma_{E1E1}\, \vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}\, \vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})$$

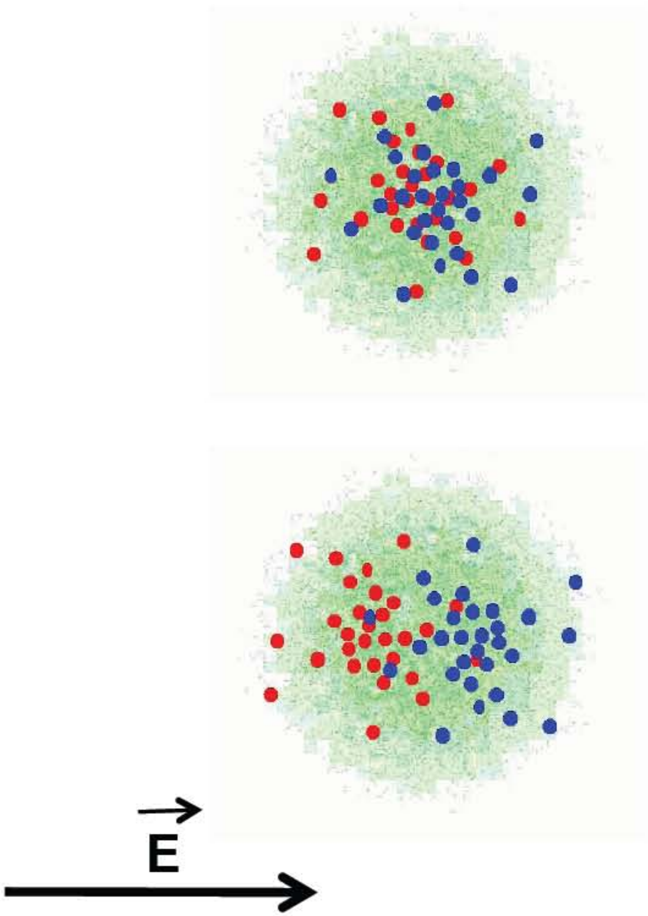
spin-dependent dipole

$$-2\, \gamma_{M1E2}\, \sigma_i B_j E_{ij} + 2\, \gamma_{E1M2}\, \sigma_i E_j B_{ij} \Big] + \mathcal{O}(\omega^3) \Big\}$$

spin-dependent dipole-quadrupole

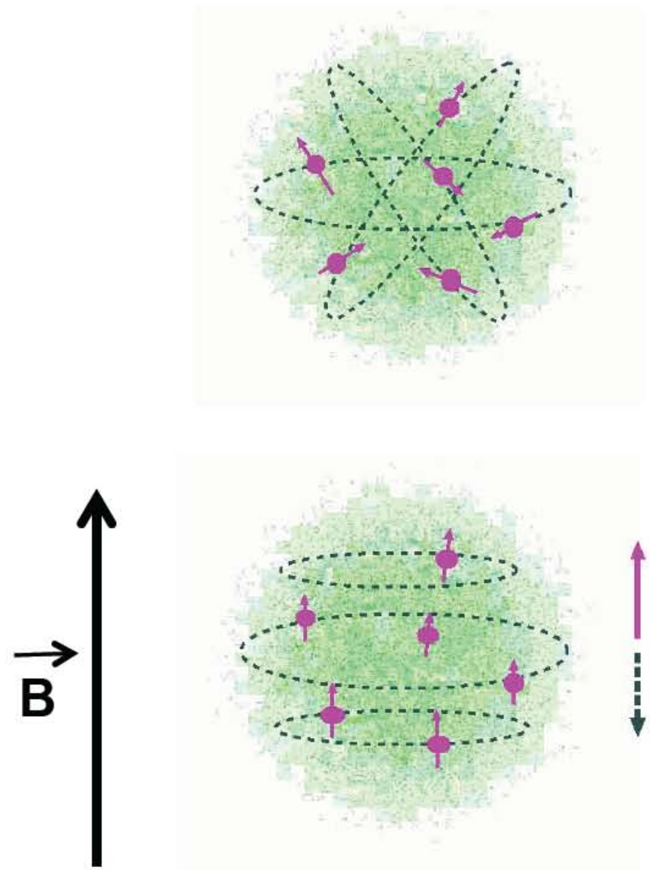
final — initial multipole

Electric polarizability:



$$\vec{d}_{induced} = \alpha \vec{E}$$

Magnetic polarizability:



$$\vec{\mu}_{induced} = \beta \vec{B}$$

Classical extended object

small dielectric and permeable sphere with radius a

$$\alpha_{E1} = \frac{\epsilon - 1}{\epsilon + 2} a^3 \qquad \beta_{M1} = \frac{\mu - 1}{\mu + 2} a^3$$

➡ for a perfectly conducting sphere $\epsilon \to \infty$ and $\mu \to 0$

$$\alpha_{E1} = a^3 \qquad \beta_{M1} = -\frac{1}{2} a^3$$

[Jackson, 1975]

Quantum mechanical system

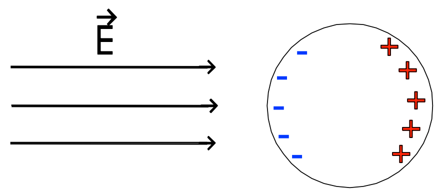
energy shift of the system under the influence of an external electric field

$$\Delta E = -\alpha_{em} \sum_{n>0} \frac{|\langle n|z|0\rangle|^2}{E_n - E_0} \vec{E}_0^2 = -\frac{1}{2} \alpha_{E1} \vec{E}_0^2$$

quadratic Stark effect

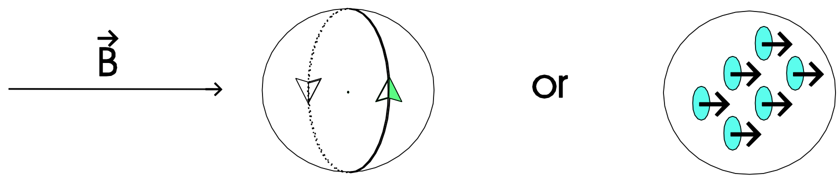
➡ for hydrogen atom $\alpha_{E1}/\text{volume} \approx \frac{1}{10}$ pretty good conductor!

Spin independent dipole polarizabilities



$$\alpha_{E1}^{exp} = (12.1 \pm 0.3 \mp 0.4) \times 10^{-4}\ \text{fm}^3$$

1000 times "stiffer" than hydrogen!



Diamagnetism — $\beta_{dia} < 0$

Paramagnetism — $\beta_{para} > 0$

$$\beta_{M1}^{exp} = (1.6 \pm 0.4 \mp 0.4) \times 10^{-4}\ \text{fm}^3$$

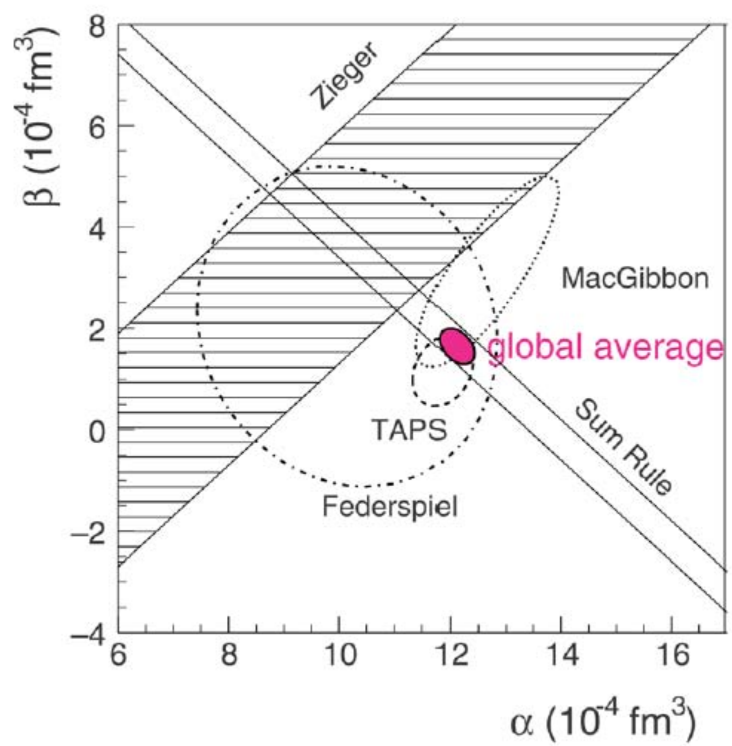
Baldin Sum Rule (1960)

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_{thr}}^{\infty} d\nu\, \frac{(\sigma_{1/2} + \sigma_{3/2})}{\nu^2}$$

$$\alpha_{E1} + \beta_{M1} = (13.8 \pm 0.4) \cdot 10^{-4}\ \text{fm}^3$$

Compton scattering

$$\alpha_{E1} - \beta_{M1} = (10.5 \pm 0.9) \cdot 10^{-4}\ \text{fm}^3$$



Olmos de Leon et al. (MAMI-TAPS), EPJ A10 (2001)

Spin polarizabilities

forward spin polarizability

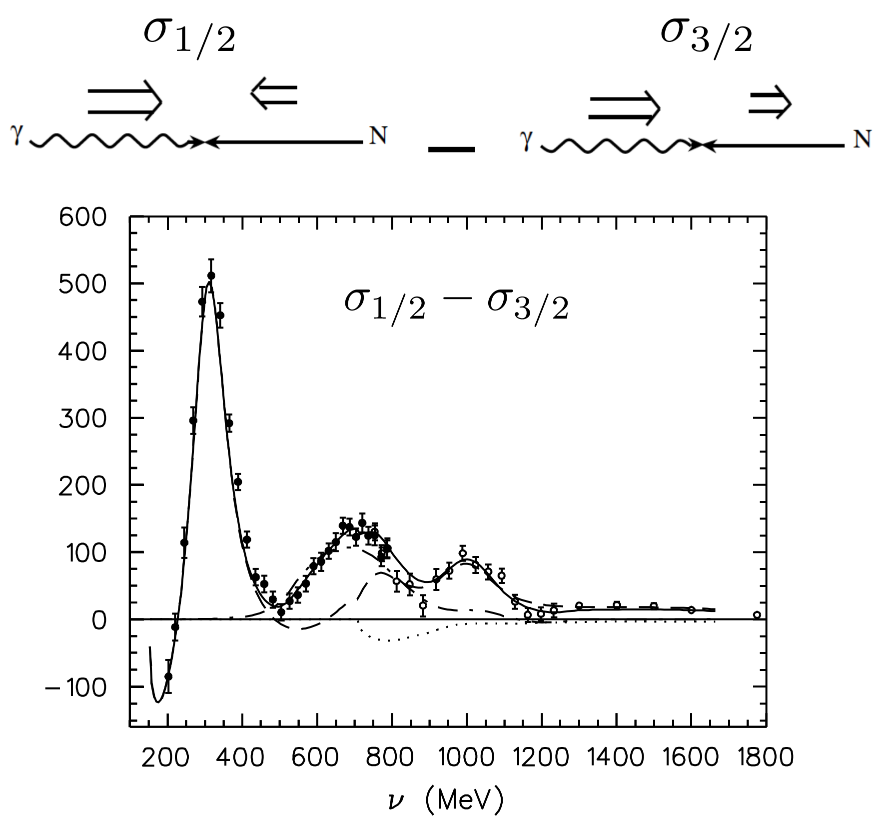
$$\gamma_0 = -\gamma_{E1E1} - \gamma_{M1M1} - \gamma_{E1M2} - \gamma_{M1E2}$$

$$\gamma_0 = \frac{1}{4\pi^2}\int_{\nu_{thr}}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu^3}\, d\nu$$

$$\gamma_0 = -(1.00 \pm 0.08 \pm 0.10) \times 10^{-4}\ \mathrm{fm}^4$$

GDH Coll. (MAMI & ELSA)

Ahrens et al., PRL87 (2001)
Dutz et al. PRL91 (2003)



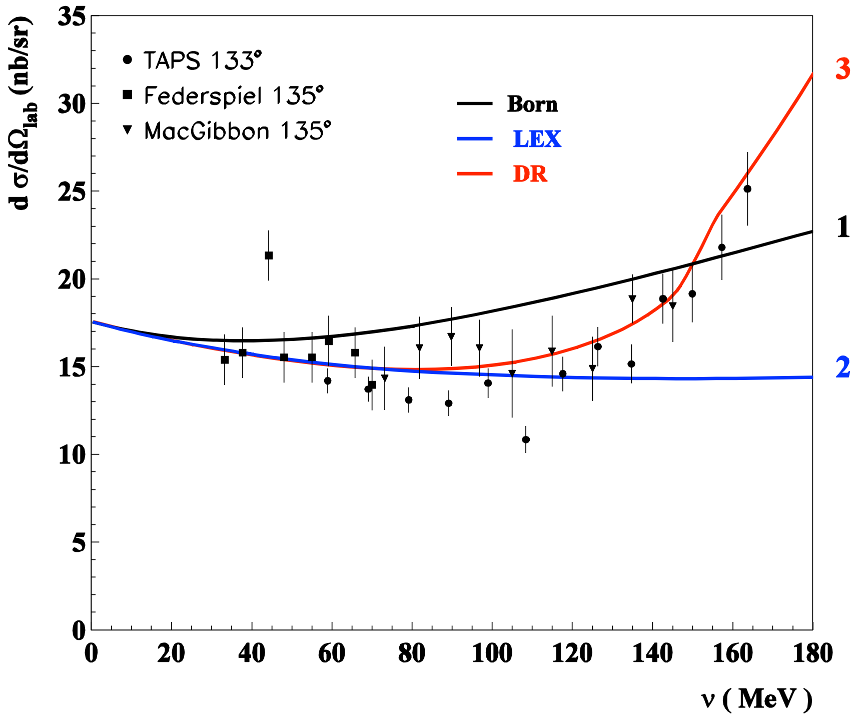
backward spin polarizability (unpolarized Compton scattering)

$$\gamma_\pi = \gamma_{E1E1} + \gamma_{M1M1} - \gamma_{E1M2} + \gamma_{M1E2}$$

$$\gamma_\pi = (-38.7 \pm 1.8) \cdot 10^{-4}\ \mathrm{fm}^4$$

TAPS, LARA, SENECA
Schumacher, Prog. Part. Nucl. Phys. 55(2005)

How to extract the RCS polarizabilities



1 — Born (anomalous magnetic moment)

2 — LEX (polarizabilities at leading order)

3 — Dispersion relations (full calculation)

❖ RCS below pion threshold

- ➢ effects of the polarizabilities rather small (~ 10 %) below threshold
- ➢ limitation in energy in order to apply low energy expansion (LEX)

❖ Dispersion relation formalism allows

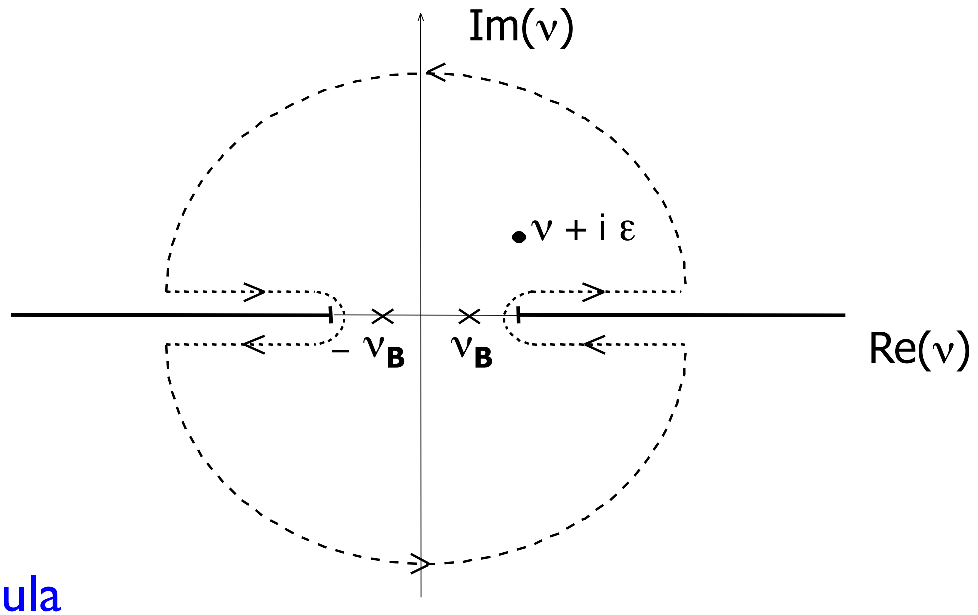
- ➢ to check validity of LEX
- ➢ to extract polarizabilities with a minimum of model dependence
- ➢ to go to higher energies to enhance the sensitivity to polarizabilities

Dispersion Relations at fixed t

$$\mathrm{T} = \varepsilon_\mu \varepsilon'^*_\nu \sum_{i=1}^{6} A_i(\nu,t) O_i^{\mu\nu}$$

$A_i(\nu,t)$: 6 Lorentz invariant functions of $\nu = E_\gamma + \frac{t}{4M}$ and $t = -2E'_\gamma E_\gamma(1-\cos\theta)$

analytical functions in the complex ν plane with cuts and poles on the real axis



❖ Cauchy integral formula

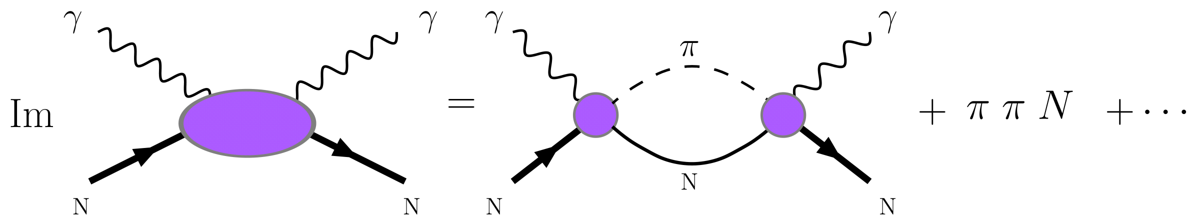
$$A_i(\nu,t) \;=\; A_i^B(\nu,t) + \oint_C \mathrm{d}\nu' \frac{A_i(\nu',t)}{\nu'-\nu}$$

❖ Crossing symmetry and analyticity

$$A_i(\nu,t) = A_i(-\nu,t) \qquad\qquad A_i(\nu^*,t) = A_i^*(\nu,t)$$

Unsubtracted Dispersion Relations

$$\mathrm{Re}A_i^{\mathrm{NB}} = \frac{2}{\pi}\int_{\nu_{thr}}^{\infty} \mathrm{Im}_s A_i(\nu' t)\frac{\nu' \mathrm{d}\nu'}{\nu'^2 - \nu^2} \qquad (i = 3, 4, 5, 6)$$

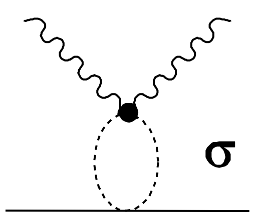


➡ input from phenomenological pion photoproduction amplitudes fitted to data and model for the two-pion contribution (suppressed for low energy RCS)

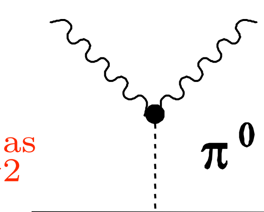
✧ Two amplitudes do not satisfy unsubtracted DRs ⟶ finite energy sum rule

$$\mathrm{Re}A_i^{\mathrm{NB}} = \frac{2}{\pi}\int_{\nu_{thr}}^{\nu_{max}} \mathrm{Im}_s A_i(\nu' t)\frac{\nu' \mathrm{d}\nu'}{\nu'^2 - \nu^2} + A_i^{\mathrm{as}} \qquad (i = 1, 2)$$

asymptotic contribution ⟶ meson exchange in the t-channel



A_1^{as} ➡ $\alpha_{E1} - \beta_{M1}$ **Fitted to Data** γ_π ⬅ A_2^{as}



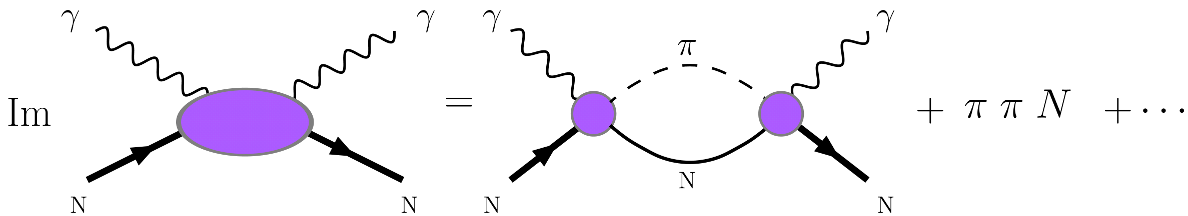
L'vov, Petrun'kin, Schumacher, PRC55 (1997)

Subtracted Dispersion Relations

$$A_i(\nu,t) = A_i^s(\nu,0) + A_i^t(0,t) + A_i(0,0)$$

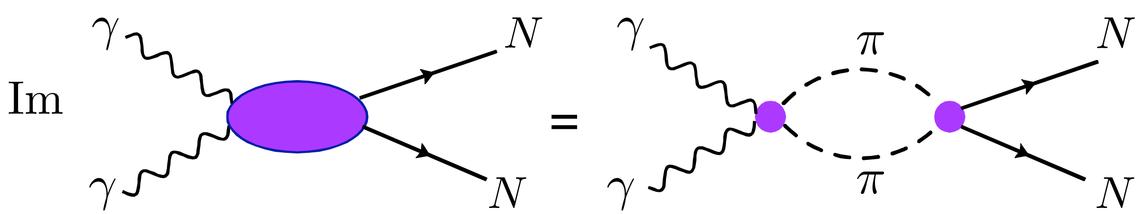
➢ subtracted dispersion relations in the s-channel

$$A_i^s(\nu,0) \;=\; \frac{2}{\pi}\,\nu^2\,\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im_s}\;A_i(\nu',t)\;\frac{d\nu'}{\nu'(\nu'^2-\nu^2)}$$



⟹ improved convergence of the integral

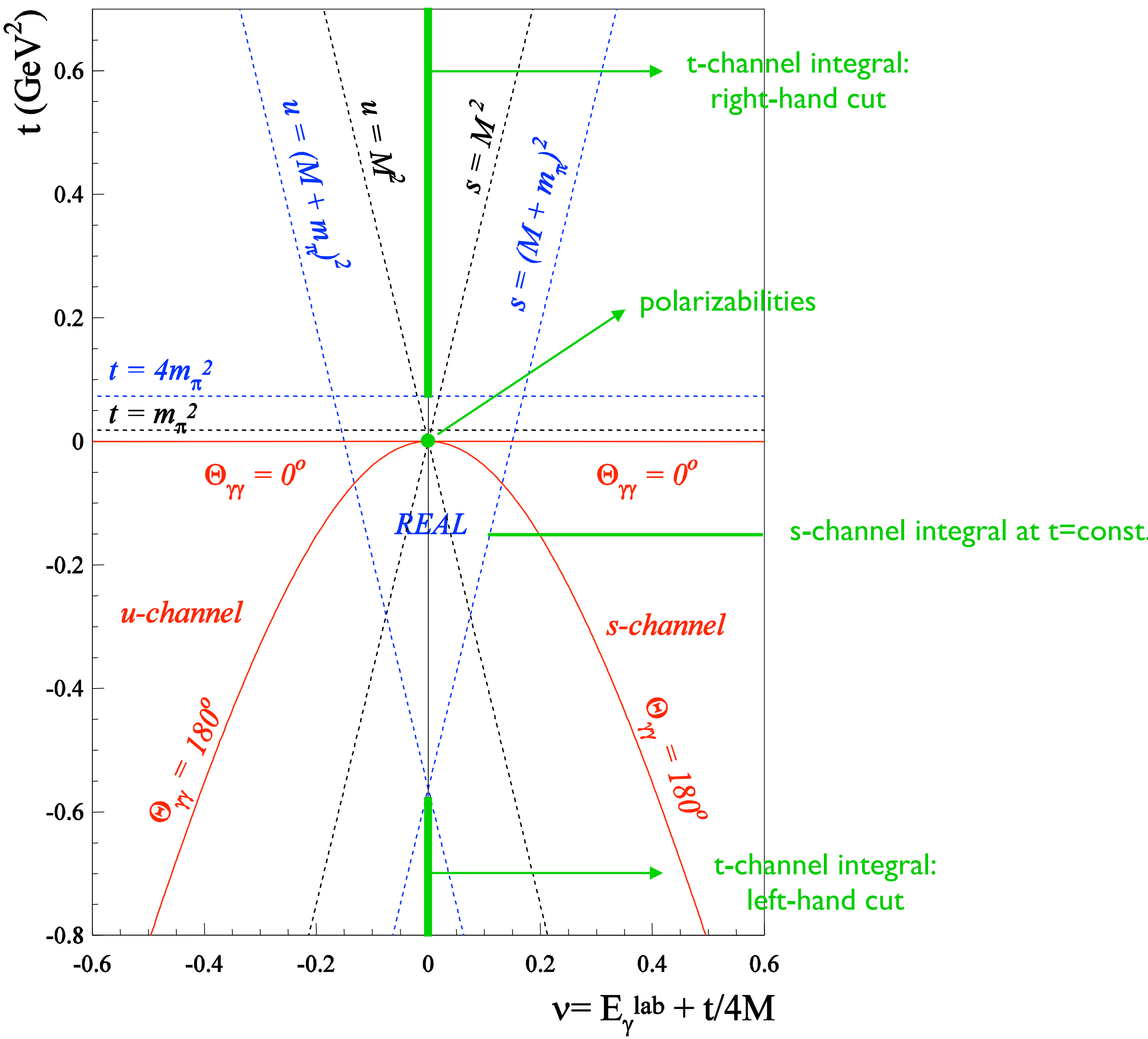
➢ $A_i^t(0,t)$: subtracted dispersion relations in the t-channel $\gamma\gamma \to N\bar{N}$



➢ $A_i(0,0) = a_i$ ⟹ polarizabilities: free parameters fitted to data

Gorchtein, Drechsel, B.P., Vanderhaeghen, PRC61 (1999); Drechsel, B.P., Vanderhaeghen, Phys. Rep. 378(2003)

Mandelstam plane for RCS



RCS below pion threshold: fit with fixed-t DRs

Baldin sum rule:

$$\alpha_{E1} + \beta_{M1} = (13.82 \pm 0.4) \times 10^{-4}\ \mathrm{fm}^3$$

Unsubtracted DRs

Olmos de Leon et al, EPJA100,2001

$$\alpha_{E1} - \beta_{M1} = (10.5 \pm 0.9) \times 10^{-4}\ \mathrm{fm}^3$$

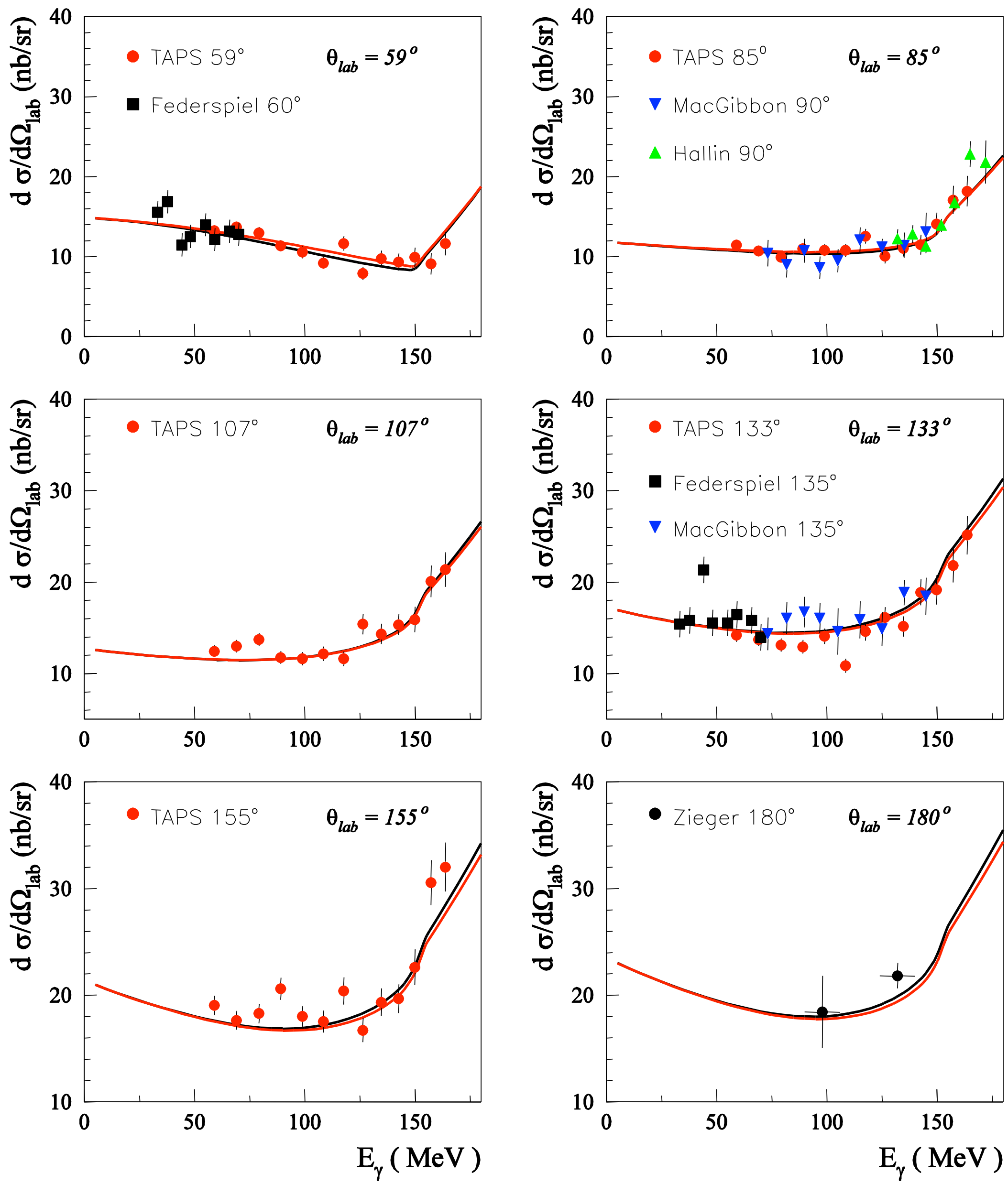
$$\gamma_\pi = (-36.1 \pm 2.1) \times 10^{-4}\ \mathrm{fm}^4$$

Subtracted DRs

Drechsel et al., Phys. Rep. 378 (2003)

$$\alpha_{E1} - \beta_{M1} = (11.3 \pm 1.1) \times 10^{-4}\ \mathrm{fm}^3$$

$$\gamma_\pi = (-35.9 \pm 1.8) \times 10^{-4}\ \mathrm{fm}^4$$



Chiral Effective Field Theories

- Two main variants of Chiral EFT:
  - -Baryon Chiral Perturbation Theory (BChPT)
    → expansion in q/Λ, fully relativistic
  - -Heavy Baryon Chiral Perturbation Theory (HBChPT)
    → expansion in q/Λ and 1/$M_{N,}$ non relativistic

- Inclusion of Δ as explicit degree of freedom:

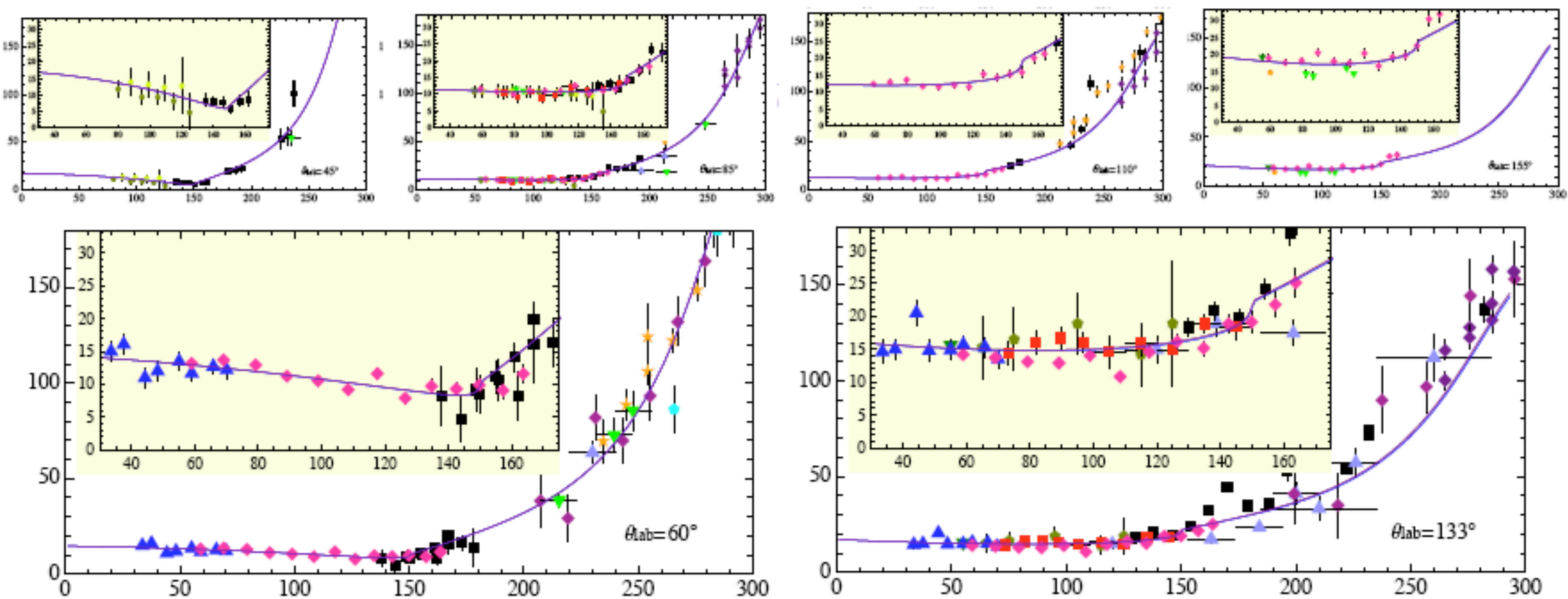
  - BChPT: δ - expansion: $\delta = \left(\frac{m_\pi}{\Delta M}, \frac{\Delta M}{\Lambda}\right)$ with $\Delta M = M_\Delta - M_N$
    [Pascalutsa and Phillips, PRC67 (2003)]

  - HBChPT: small-scale expansion (SSE) or ϵ expansion: $\epsilon = \left(\frac{m_\pi}{\Lambda}, \frac{p}{\Lambda}, \frac{\Delta M}{\Lambda}\right)$
    [Hemmert, Holstein and Kambor, PLB395 (1997)]

- Different power counting scheme

- Unknown LECs enter in the polarizabilities at NLO → parameters to fit to RCS data

Global Fit using BChPT with Δ



McGovern, Phillips, Griesshammer, EPJ A49 (2013)

NLO + Δ
Baldin constrained

$\alpha_{E1} = 10.7 \pm 0.3_{\text{stat.}} \pm 0.2_{\Sigma} \pm 0.8_{\text{th.}}$ $\beta_{M1} = 3.1 \mp 0.3_{\text{stat.}} \pm 0.2_{\Sigma} \pm 0.8_{\text{th.}}$

NLO + Δ
δ expansion
Lensky, Pascalutsa, EPJC65 (2010)

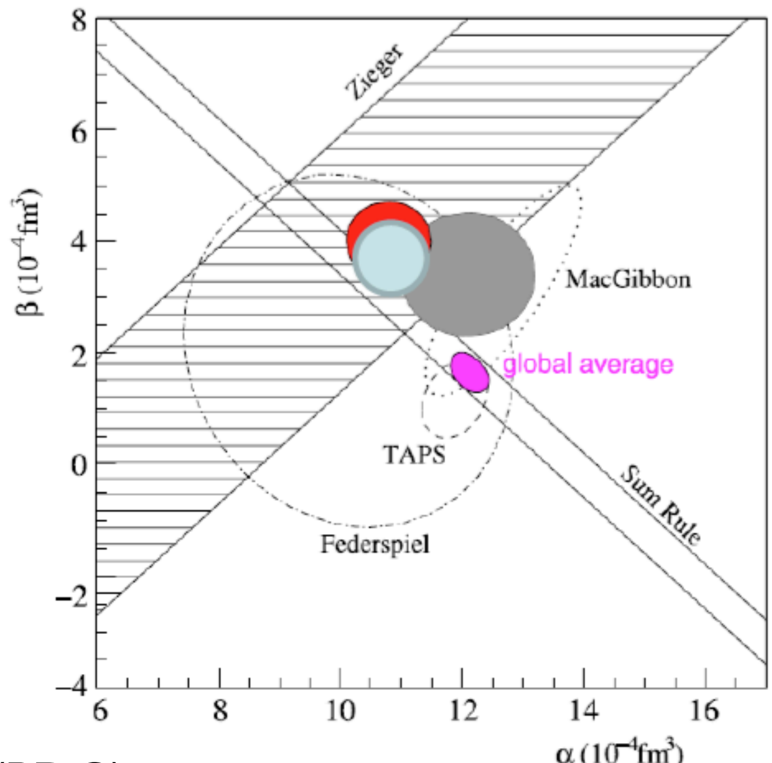
$\alpha_{E1} = 10.8 \pm 0.7$ $\beta_{M1} = 4.0 \pm 0.7$

MAMI/TAPS

$\alpha_{E1} = 12.1 \pm 1.2_{\text{stat.+model}} \pm 0.4_{\Sigma}$ $\beta_{M1} = 1.6 \mp 1.2_{\text{stat.+model}} \pm 0.4_{\Sigma}$

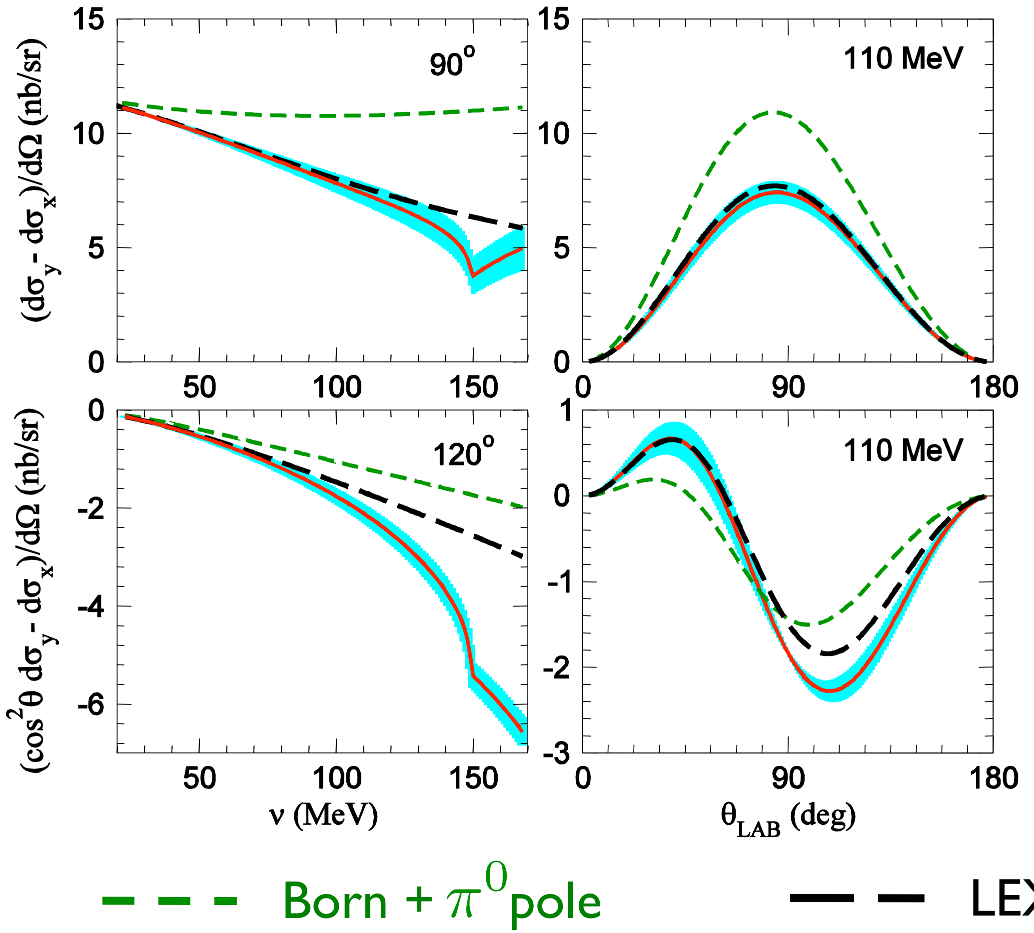
EFTs versus DRs

Fit to UNPOLARIZED cross section → sensitivity to $\alpha_{E1} - \beta_{M1}$ and $\alpha_{E1} + \beta_{M1}$



- Particle Data Group (PDG) [analysis based on DRs]
- Heavy Baryon ChPT [Beane, Malheiro, McGovern, Phillips, van Kolck, NPA747 (2005)]
- Baryon ChPT with Δ [Lensky and Pascalutsa, EPJC65 (2010)]
- Partially Covariant Baryon ChPT with Δ [McGovern, Phillips, Griesshammer, EPJA49 (2013)]

how to obtain more accurate values for scalar polarizabilities?



$$\frac{d\sigma^{\perp} - d\sigma^{\parallel}}{d\Omega} \Rightarrow \alpha_{E1}$$

$$\frac{(\cos^2\theta) d\sigma^{\perp} - d\sigma^{\parallel}}{d\Omega} \Rightarrow \beta_{M1}$$

[Lensky, Pascalutsa, EPJC (2010)]

- combination of cross sections with linearly pol. photons in the transverse direction to the photon momentum ➡ independent extraction of α_{E1} and β_{M1}
- proposal of measurements for the neutron at HIgS and for the proton at MAMI
  MAMI proposal: A2/06-2012

Spin Polarizabilities

	HB^3	HB^4	SSE	LC^3	$LC^3 + \Delta$	LC^4	DRs	Exp.
γ_{E1E1}	-5.7	-1.4	-5.4	-3.3	-4.5	-2.8	-4.3	no data
γ_{M1M1}	-1.1	3.3	1.4	-0.1	3.7	-3.1	2.9	no data
γ_{E1M2}	1.1	0.2	1.0	0.5	-0.9	0.8	0.0	no data
γ_{M1E2}	1.1	1.8	1.0	0.8	2.2	0.3	2.1	no data
γ_0	4.6	-3.9	2.0	1.8	-0.7	4.8	-0.7	-0.9 ± 0.08 ± 0.11
γ_π	4.6	6.3	6.8	3.5	11.3	-0.8	9.3	8.0 ± 1.8

HB^3: Heavy Baryon ChPT at $O(p^3)$ [Hemmert et al, 1998]

HB^4: Heavy Baryon ChPT at $O(p^4)$ [Kumar et al, 2000]

SSE: Heavy Baryon with Δ at $O(p^3)$ [Hemmert et al, 1998]

$LC^3 + \Delta$: [Lensky, Pascalutsa, EPJC (2010); Lensky, Pascalutsa, to be published]

LC^4: Lorentz covariant ChPT [Djukanovic, PhD Thesis, Mainz, 2008]

DRs: Dispersion Relations [Drechsel et al., 2003]

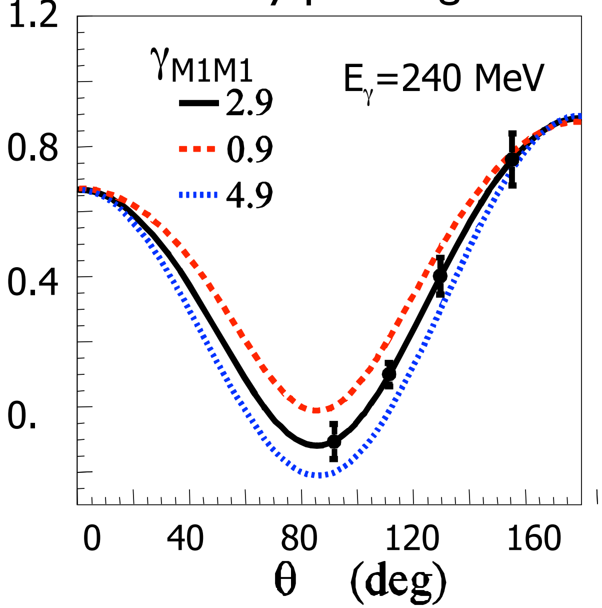
** values WITHOUT pion-pole contribution

Spin Polarizabilities from Double and single polarization experiments at MAMI

(MAMI proposal A2/11-2009)

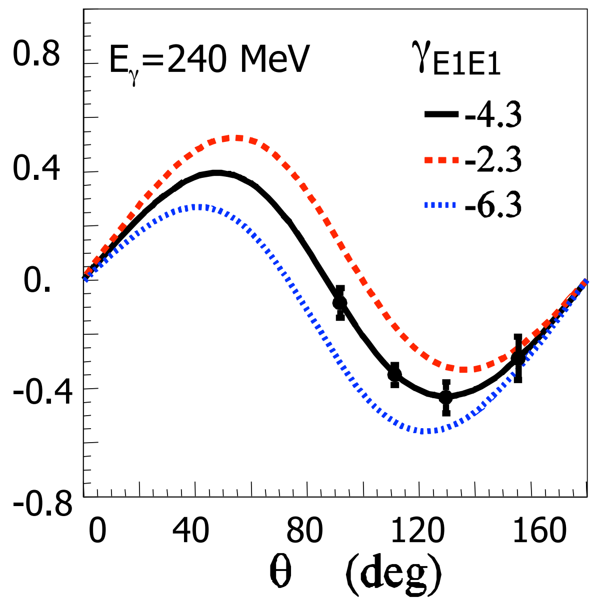
circularly pol. photons

linearly pol. target

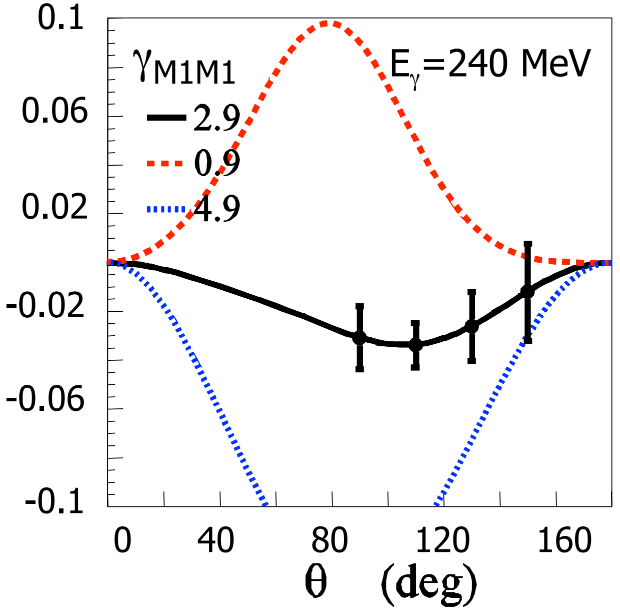


circularly pol. photons

transversely pol. target

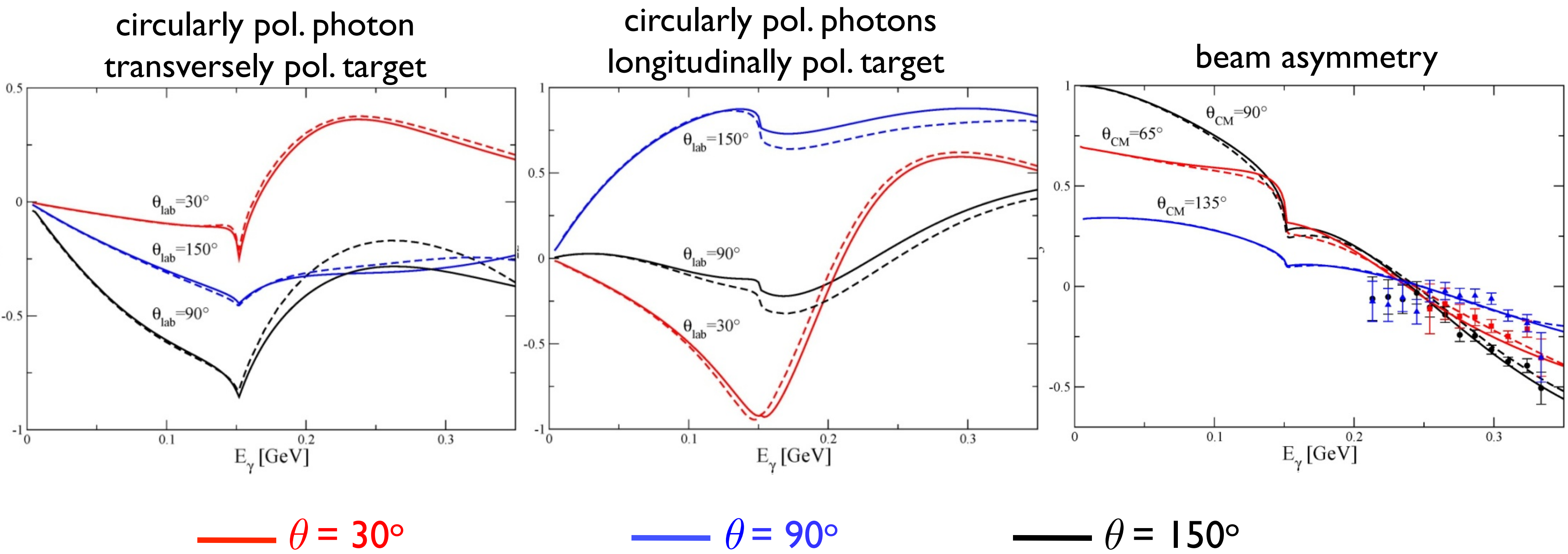


beam asymmetry



- leading spin polarizabilities are treated as free parameters
- α_{E1} and β_{M1} are fixed to central exp. value
- higher-order polarizabilities are fixed by subtracted dispersion relations based on pion-photoproduction multipoles

cross-check with alternative models



- ❖ solid curves: unitary and casual effective field theory based on chiral Lagrangian

⟶ parameters fitted to pion-photoproduction multipoles

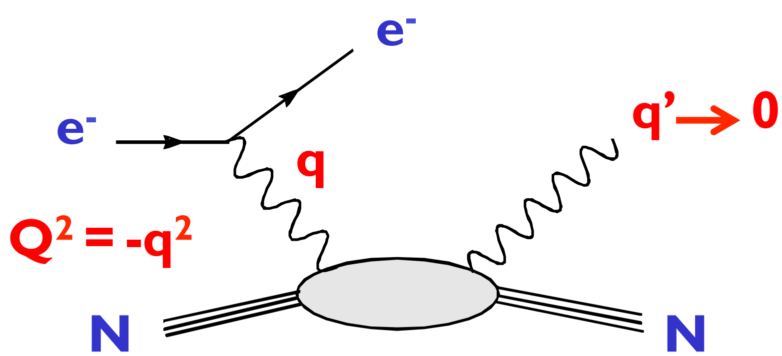
⟶ description of pion-photoproduction and Compton scattering up to W=1.3 GeV

[Gasparyan, Lutz, NPA848 (2010); Gasparyan, Lutz, BP, Nucl.Phys. A866(2011)]

- ❖ dashed curves: dispersion relations using the same values for polarizabilities

difference between dashed and solid curves due to higher-order terms

Virtual Compton Scattering on proton



low energy outgoing photon plays
role of applied e.m. dipole field

⬇

nucleon response :

Guichon et al. (1995)

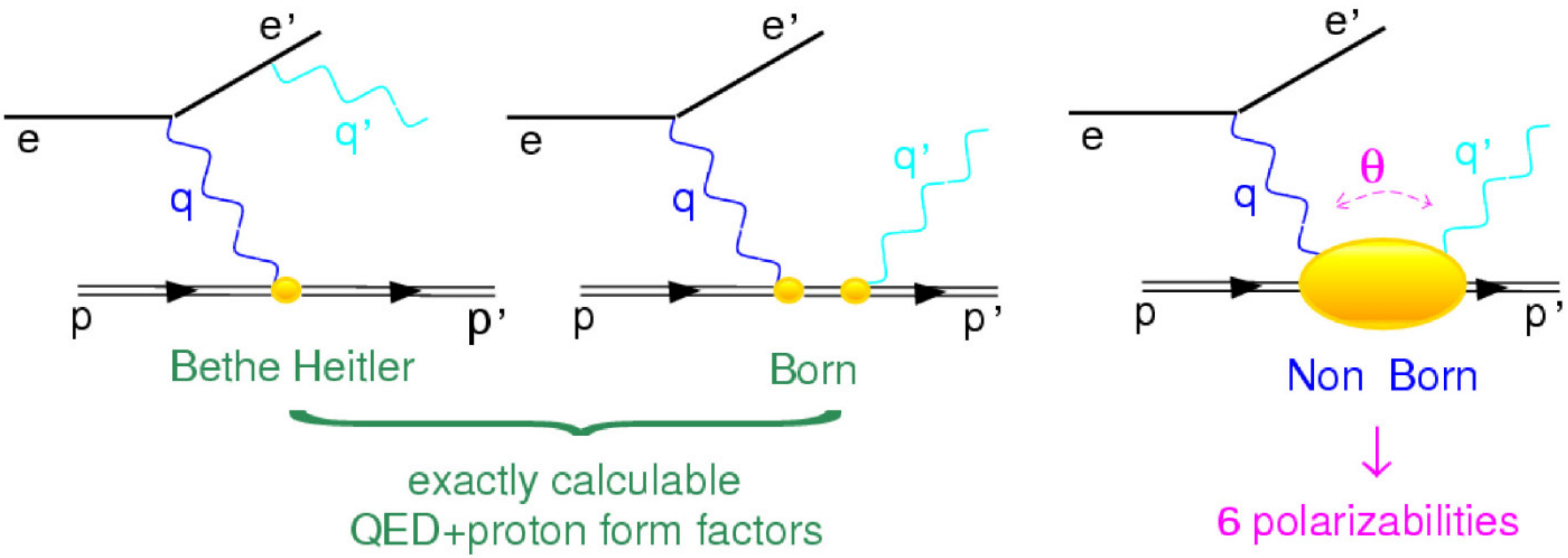
Generalized Polarizabilities (GPs)

Drechsel et al. (1998)

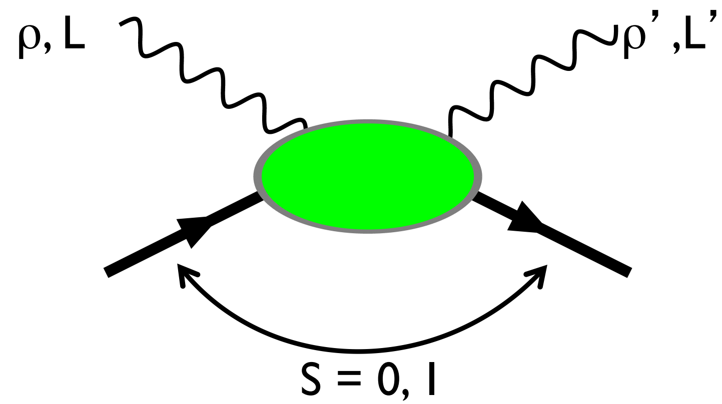
$\alpha(Q^2)$, $\beta(Q^2)$, and 4 spin GPs
describe the spatial distributions of induced polarizations

e p ⟶ e`p γ

$$T^{\mathrm{ep}\to\mathrm{e}'\mathrm{p}\gamma} = T^{\mathrm{BH}} + T^{\mathrm{VCS}}$$



➢ Multipole expansion of T_{NB}



ρ, ρ' { 2 Coulomb; 1 Magnetic; 0 Electric }

Angular momentum conservation: $|L-L'| \leq S \leq L+L'$

Parity conservation: $(-1)^{L+\rho} = (-1)^{L'+\rho'}$

➢ Generalized Polarizabilities (GPs)

(using charge conjugation and nucleon crossing symmetry)

final γ	initial γ^*	S	$P^{(\rho' L', \rho L) S}$	RCS limit
E1	C1	0	$P^{(01,01)0}$	α
	C1	1	$P^{(01,01)1}$	0
	M2	1	$P^{(01,12)1}$	γ_{E1M2}
M1	M1	0	$P^{(11,11)0}$	β
	M1	1	$P^{(11,11)1}$	0
	E2	1	$P^{(11,02)1}$	γ_{M1E2}

2 scalar GPs
+
4 spin GPS

Methods of experimental analysis

❖ LEX

o expansion in q'

o valid only at low energies (below pion threshold)

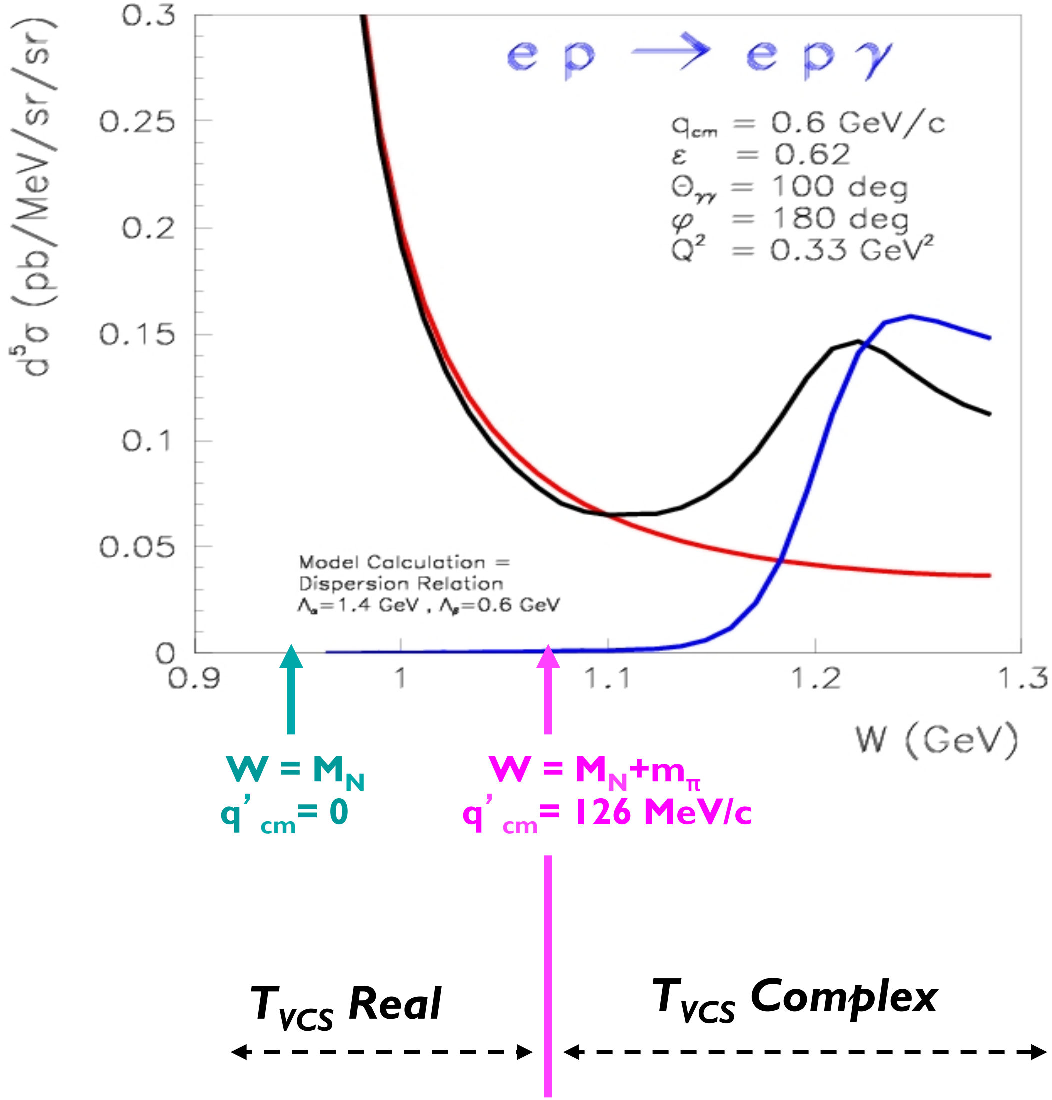
o extraction of structure functions which contain linear combinations of 6 GPs
⟶ model input for the spin-flip GPs to isolate $\alpha(Q^2)$ and $\beta(Q^2)$

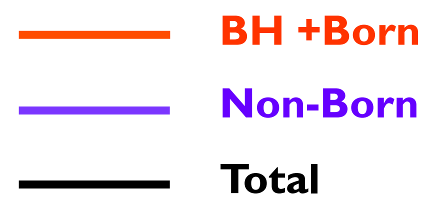
❖ DR

o full energy dependence

o valid below and above pion threshold
⇨ consistency check of LEX below pion threshold
it allows to go at higher energies where the effects of GPs are enhanced

o direct extraction of $\alpha(Q^2)$ and $\beta(Q^2)$





Analysis methods :

Low Energy Theorem (LET)

P.Guichon and M.Vanderhaeghen,
Prog.Part.Nucl.Phys. 41 (1998) 125.

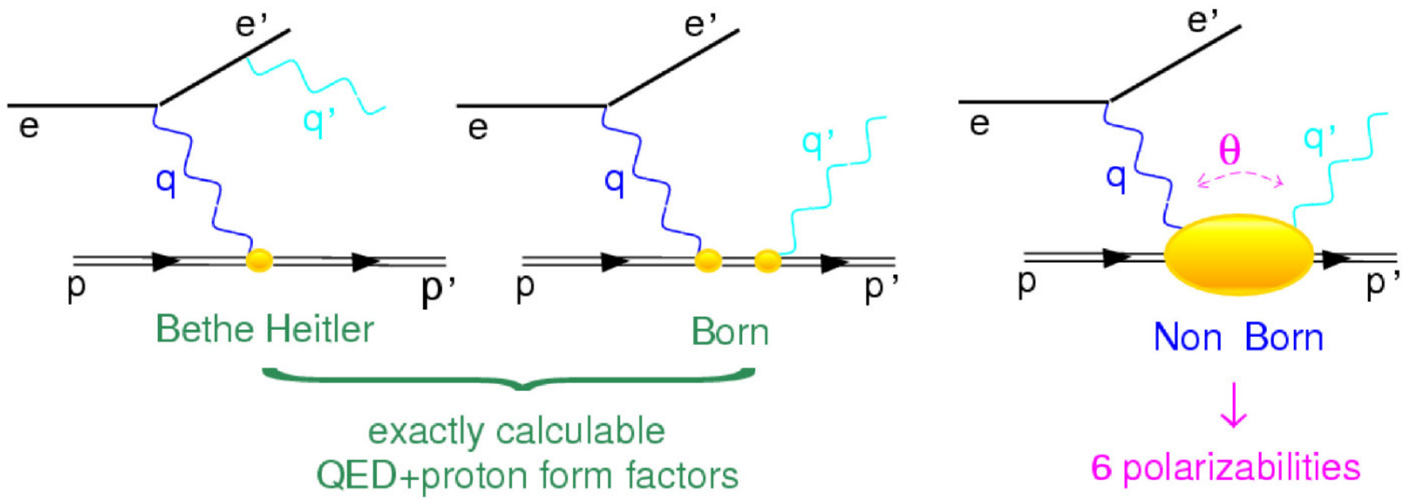
Dispersion Relations (DR)

B.Pasquini et al., Eur.Phys.J.A 11 (2001) 185
D.Drechsel et al., Phys.Rept. 378 (2003) 99

The Unpolarized VCS experiments in the World

	Q^2 (GeV^2)	ε	Method of analysis	
MAMI – A1 -1	0.33	0.62	LEX	Roche, et al, PRL 85 (2000)
JLab E93050	0.9, 1.8	0.95, 0.88	LEX and DRs	Laveissiere, et al, PRL 93 (2004)
Bates E9703	0.057	0.90	LEX and DRs	Bourgeois et al., PRL97 (2006)
MAMI – A1 -2	0.33	0.48	DRs	Bensafa et al., EPJA32 (2007)
MAMI – A1 -3	0.33	0.64	LEX	Janssens et al., EPJA37 (2008)

Low Energy Expansion (below π threshold)



- Low energy expansion in q'

$$d^5\sigma^{e\,p\to e'\,p'\,\gamma}(q,q',\theta,\epsilon,\phi) = d^5\sigma^{\mathrm{BH+Born}} + q'\ \Phi\,\Psi_0(q,\theta,\epsilon,\phi) + \mathcal{O}(q'^2)$$

$$\Psi_0(q,\theta,\epsilon,\phi) = v_1(\theta,\phi)\left[P_{LL}(Q^2) - \frac{1}{\epsilon}P_{TT}\right] + v_2(\theta,\phi)\,P_{LT}(Q^2)$$

unpolarized experiment ➡ 2 VCS structure functions
linear combinations of 6 GPs

The VCS Structure Functions

- $P_{LL} - \frac{1}{\epsilon} P_{TT}$

$$P_{LL}(Q^2) = -2\sqrt{6} M G_E(Q^2) P^{(01,01)0}(Q^2) \longrightarrow \alpha(Q^2)$$

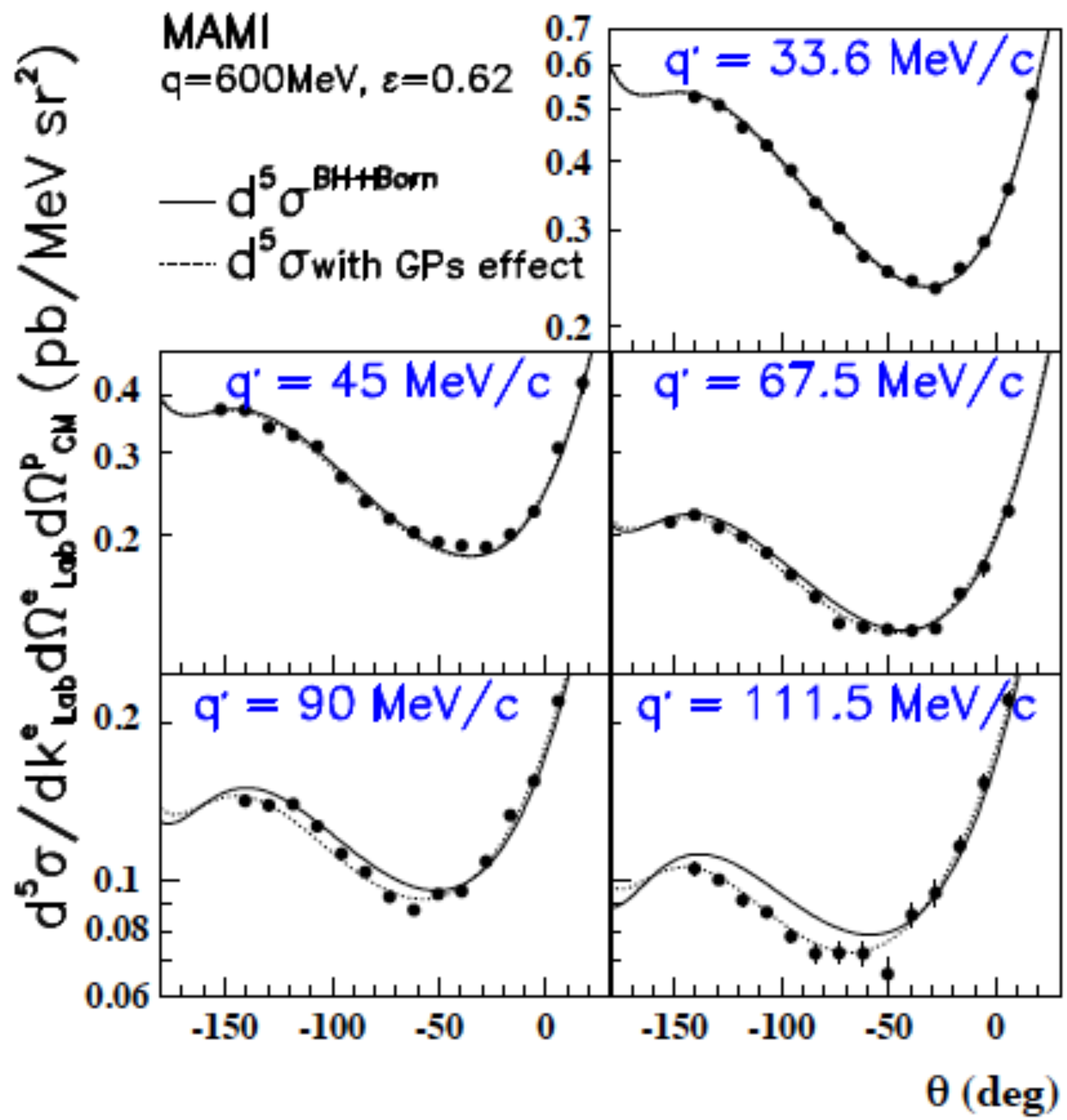
$$P_{TT}(Q^2) = -3 G_M(Q^2) \frac{q^2}{\omega^2} [P^{(11,11)1}(Q^2) - \sqrt{2}\omega P^{(01,12)1}(Q^2)] \longrightarrow \text{spin flip polariz.}$$

- $$P_{LT}(Q^2) = -\sqrt{\frac{3}{2}} \frac{Mq}{Q} G_E(Q^2) P^{(11,11)0}(Q^2) + \frac{3}{2}\frac{Qq}{\omega} G_M P^{(01,01)1}(Q^2)$$

$P^{(11,11)0}(Q^2) \downarrow$ $\beta(Q^2)$

$G_M P^{(01,01)1}(Q^2) \downarrow$ spin flip polariz.

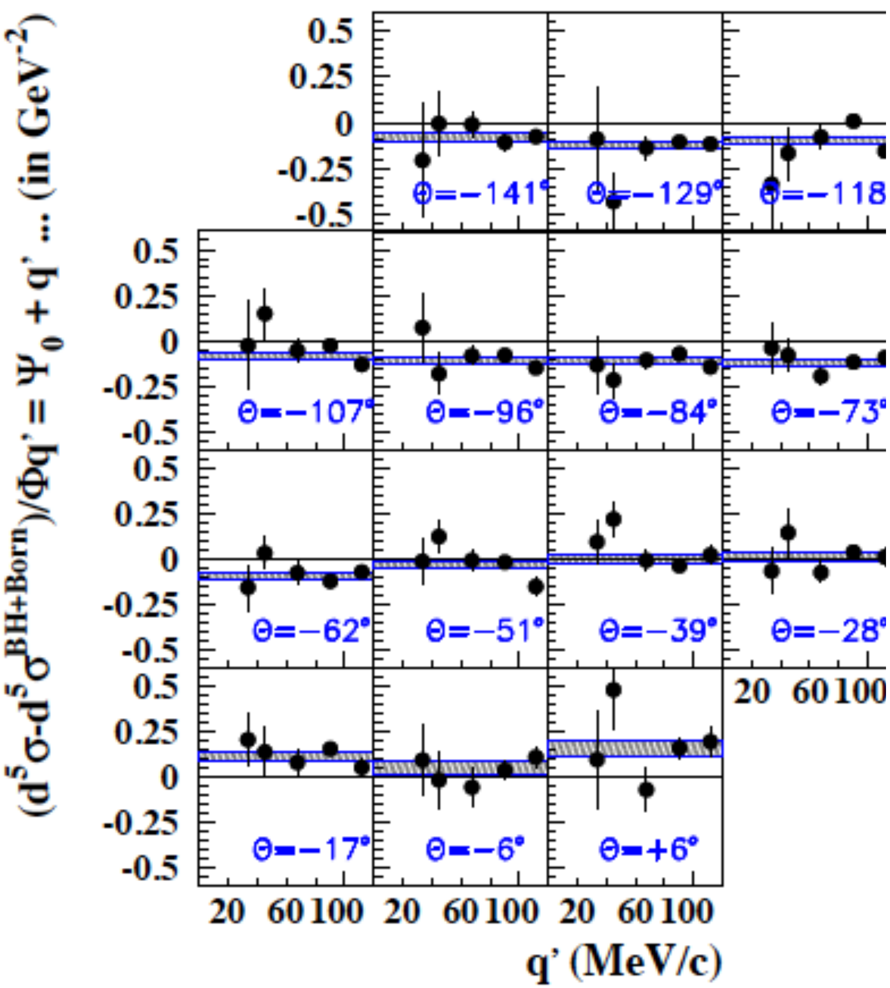
Measured ep $\longrightarrow$ e'p γ cross sections at MAMI below threshold

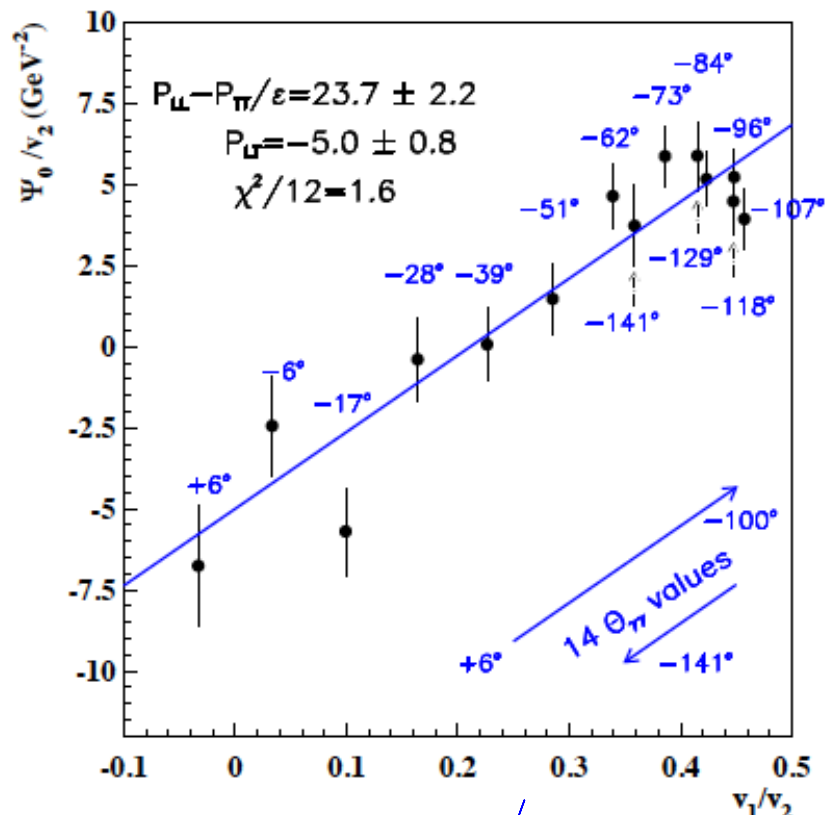


LEX analysis at MAMI

$$(\mathrm{d}\sigma_{\mathrm{exp}} - \mathrm{d}\sigma_{\mathrm{BH+Born}})/\Phi/q' = \Psi_0(q,\theta) + q' \cdots$$

$$\Psi_0 = P_{LT} + v_1/v_2(P_{LL} - P_{TT}/\epsilon)$$

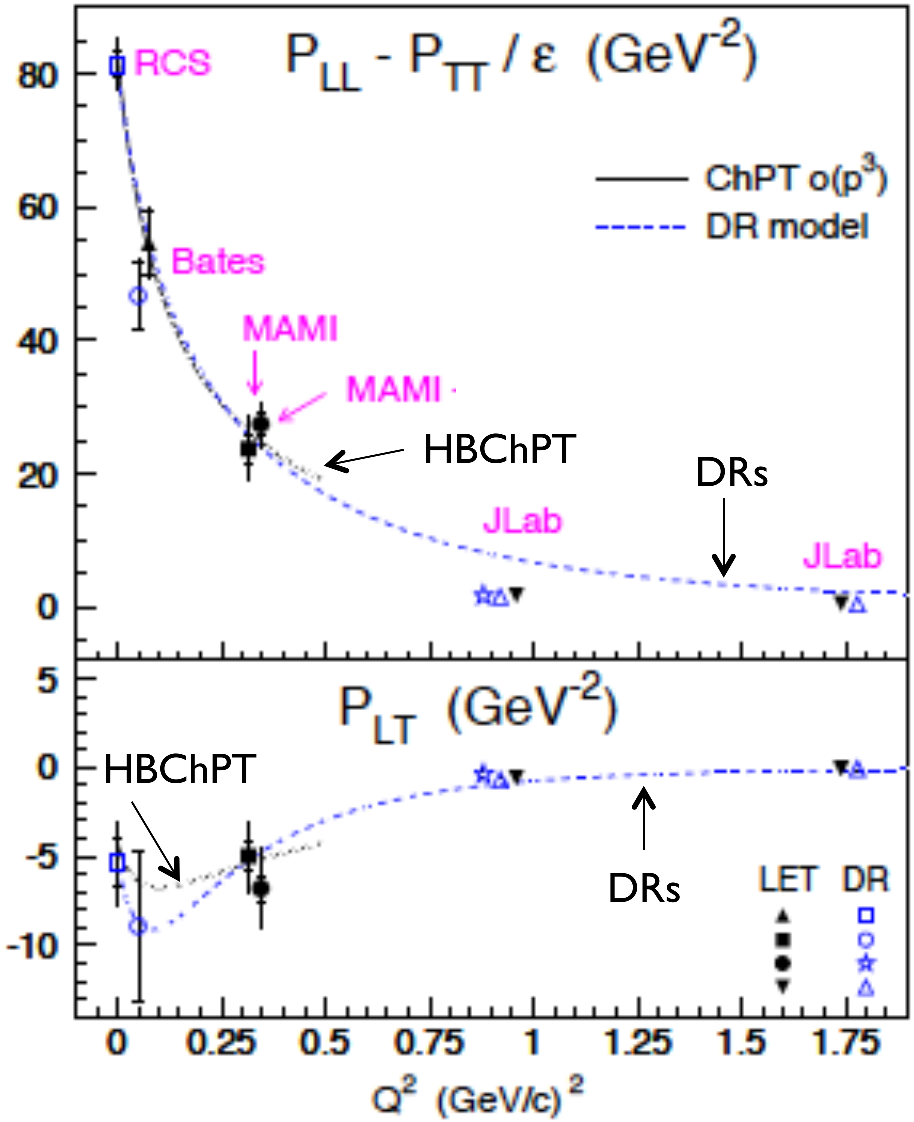




v_1/v_2

Analogous analysis at JLab and MIT-Bates

Results for Structure Funtions



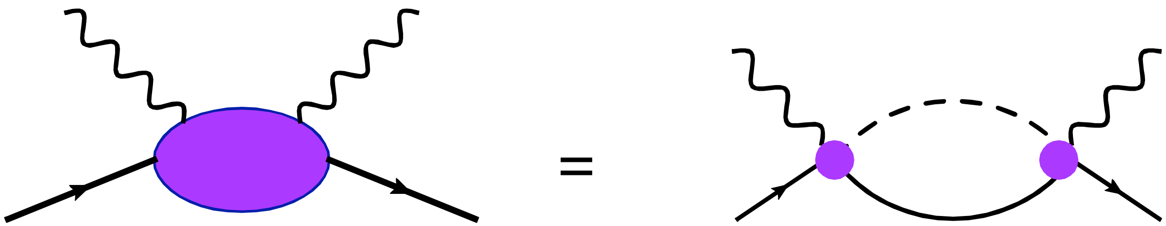
→ related to $\alpha(Q^2)$

→ related to $\beta(Q^2)$

Dispersion Relations

- VCS described in terms of 12 Lorentz invariant amplitudes F_i (ν, t, Q^2)
- unsubtracted dispersion relations in the s-channel at fixed t and Q^2

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}F_i^{\mathrm{NB}}(Q^2,\nu',t)\frac{\nu'\,\mathrm{d}\nu'}{\nu'^2-\nu^2}$$



$$\mathrm{Im}F_i \Longrightarrow \gamma^* N \to \pi N \quad \text{from MAID}$$

MAID: Drechsel, Hanstein, Kamalov, Tiator, NPA 645 (1999)

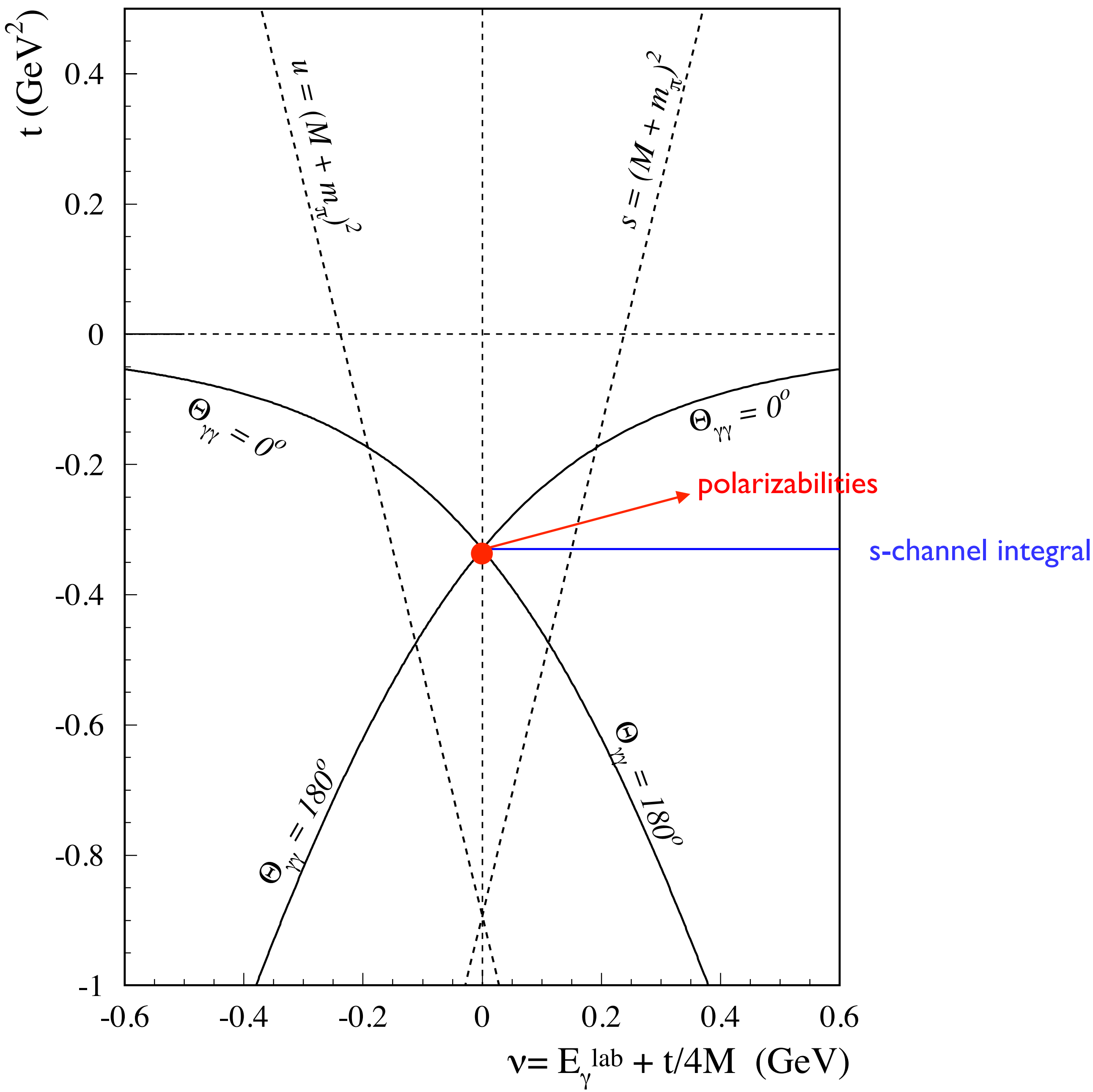
- two amplitudes do not satisfy unsubtracted DRs ⟶ finite energy sum rule

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\nu_{max}} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu'\mathrm{d}\nu'}{\nu'^{\,2}-\nu^2} + F_i^{\mathrm{as}}(Q^2,\nu,t)$$

⟹ asymptotic parts contain free parameters fitted to data at fixed Q^2

$$\alpha(Q^2) = \alpha^{\mathrm{DISP}}(Q^2) + \alpha^{\mathrm{ASY}}(Q^2) \qquad \beta(Q^2) = \beta^{\mathrm{disp}}(Q^2) + \beta^{\mathrm{ASY}}(Q^2)$$

Mandelstam plane for RCS



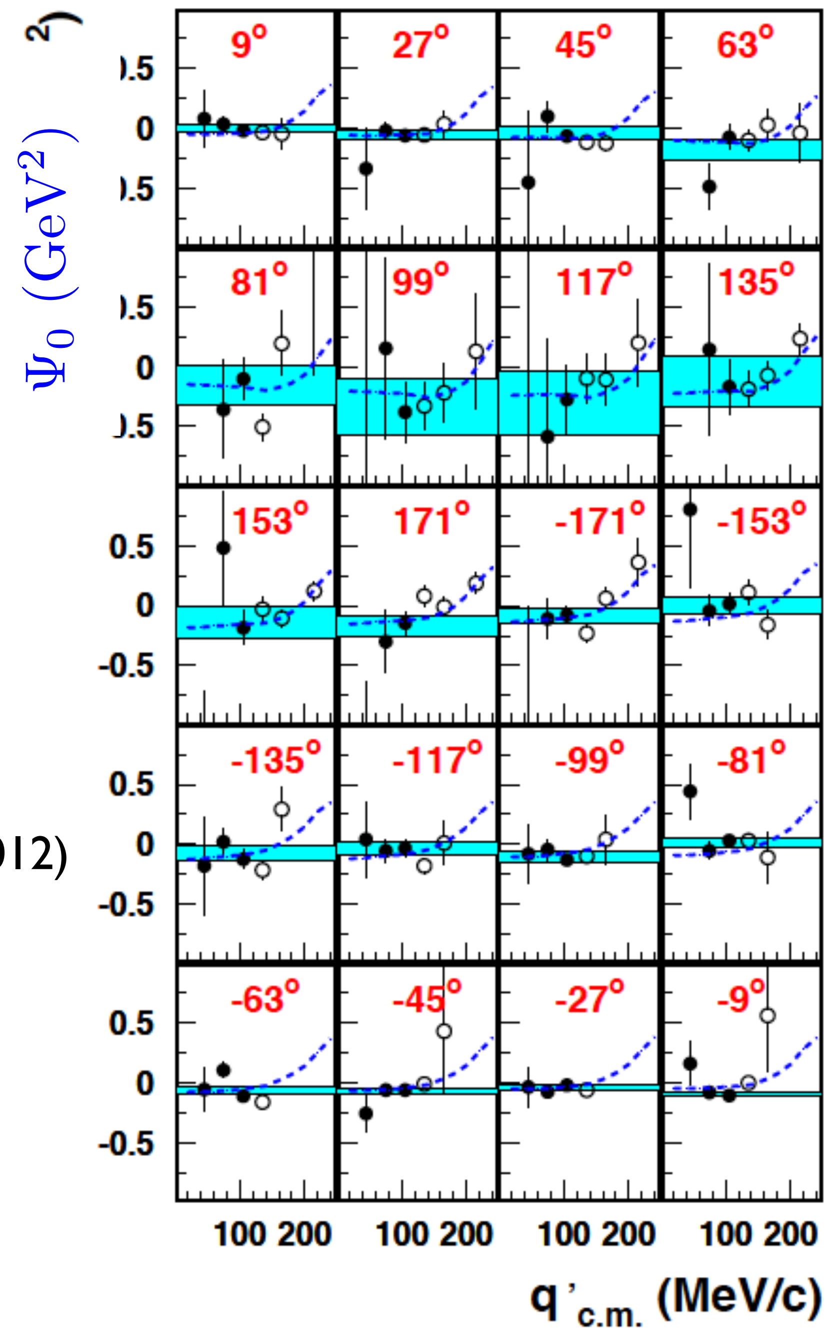
$$(d\sigma_{\mathrm{exp}} - d\sigma_{\mathrm{BH+Born}})/\Phi/q' = \Psi_0(q,\theta) + q' \cdots$$

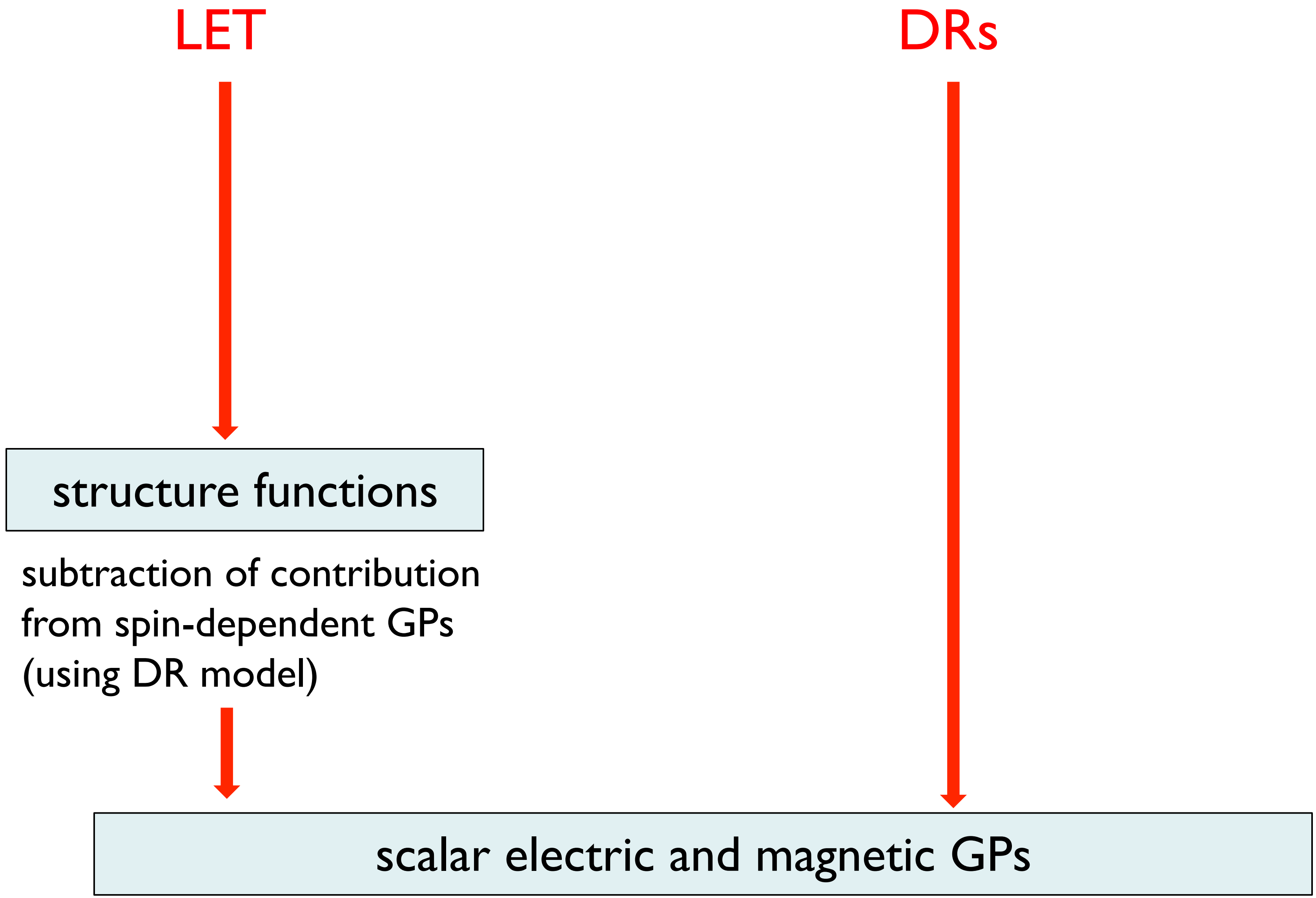
$$Q^2 = 0.92\ \mathrm{GeV}^2$$

data below threshold

data above threshold

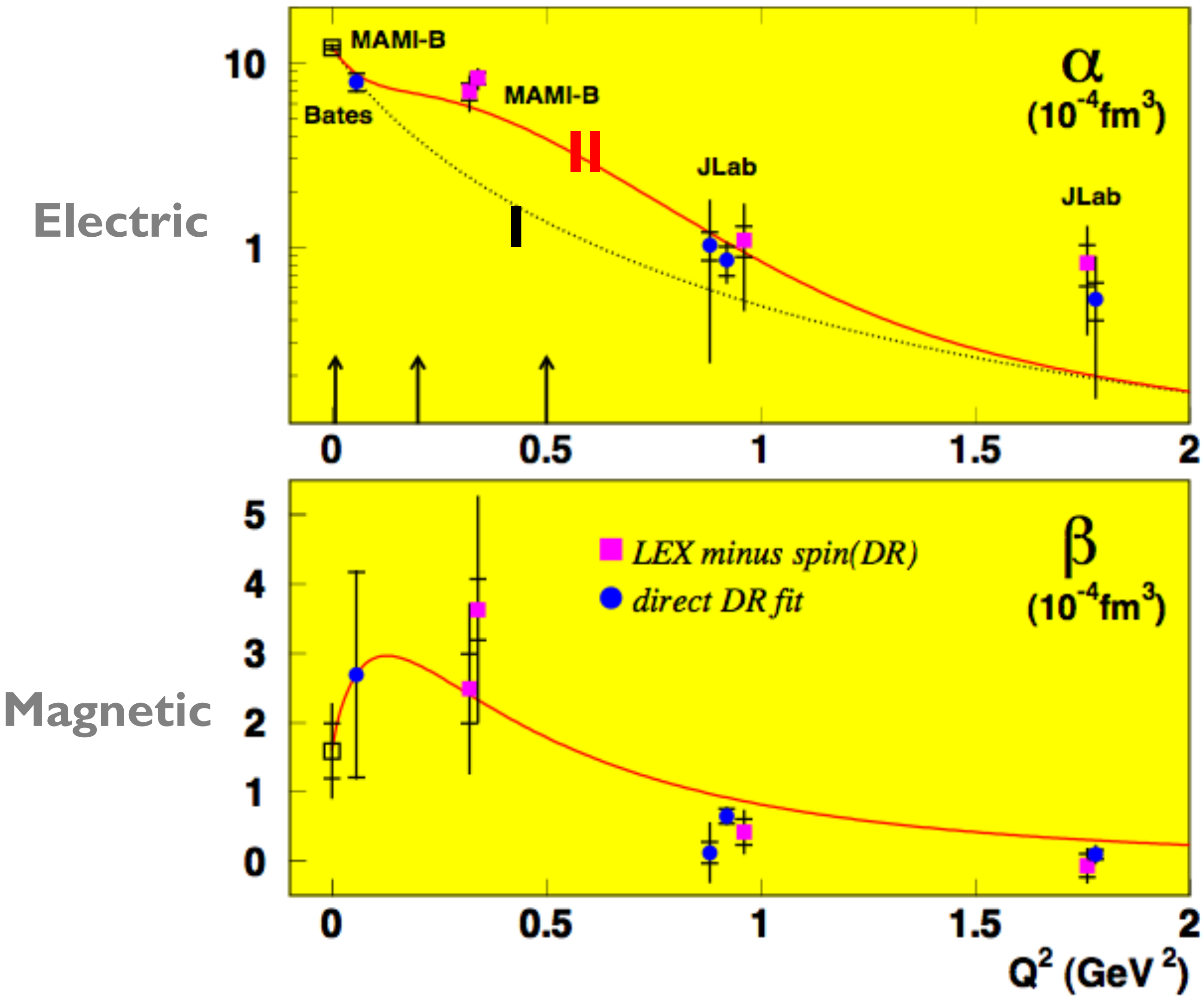
Jlab HallA (Fonvieille et al.),
Phys. Rev. Lett.93 (2004); Phys.Rev. C86 (2012)




LET
DRs
structure functions
subtraction of contribution
from spin-dependent GPs
(using DR model)
scalar electric and magnetic GPs

World data on proton

Generalized Polarizabilities (GPs)



parameterizations :
dispersive part +
asymptotic part (ASY)

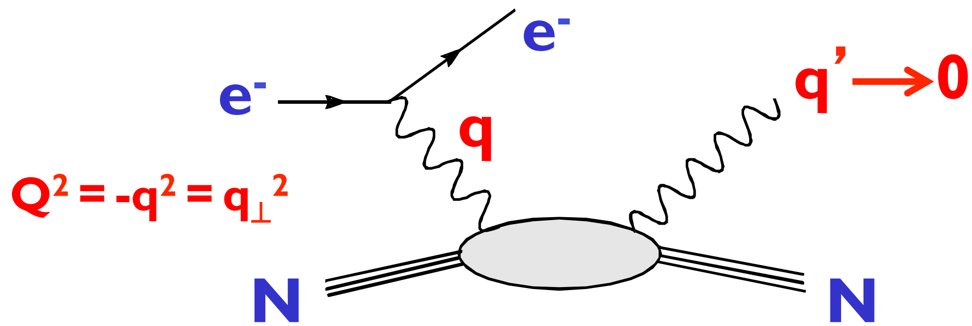
I : ASY ⟶ dipole

II : ASY ⟶ dipole
+ Gaussian

ASY: dipole

⟹ new experiments planned at MAMI in the intermediate Q^2 region

Induced polarization in proton



$$\nu = q \cdot P / M$$

$$\tau = Q^2/(4M^2)$$

$$H^{\mu\nu} = -i \int d^4x \, e^{-iq\cdot x} \langle p', \lambda'_N | T[J^\mu(x), J^\nu(0)] | p, \lambda_N \rangle$$

★ density interpretation : consider process in light-front frame

+ component of virtual photon ➡ VCS tensor $H^{+\nu}$

★ quasi-static electric field $\vec{E} \sim i q'^0 \vec{\varepsilon}'_\perp = i \frac{\nu}{(1+\tau)} \frac{P^+}{M} \vec{\varepsilon}'_\perp$

★ **induced dipole moment** minimizes energy $-\vec{E} \cdot \vec{P}_0$

light-front + component

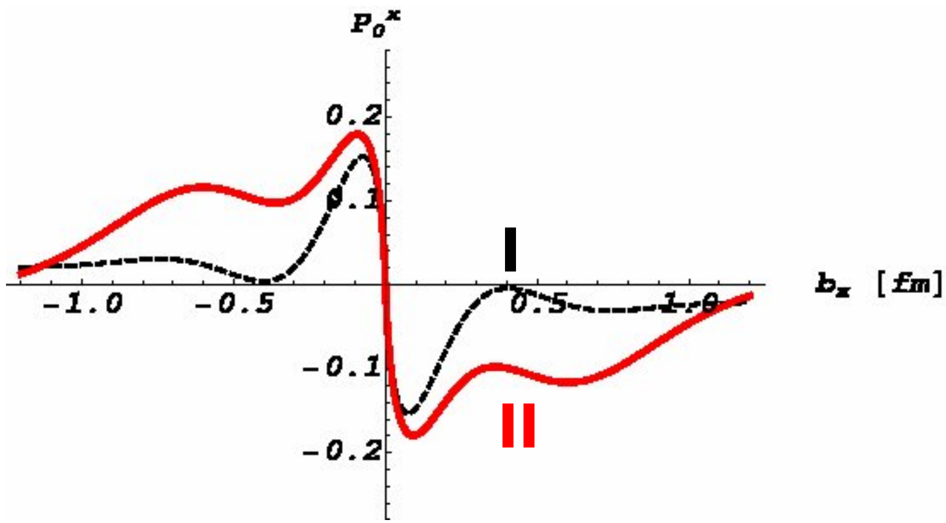
★ **linear response in outgoing photon energy ~** $i\vec{\varepsilon}'^*_\perp \cdot \vec{P}_0 \equiv \varepsilon'^*_\nu \frac{(1+\tau)}{(2P^+)} \frac{\partial H^{+\nu}}{\partial \nu}\Big|_{\nu=0}$

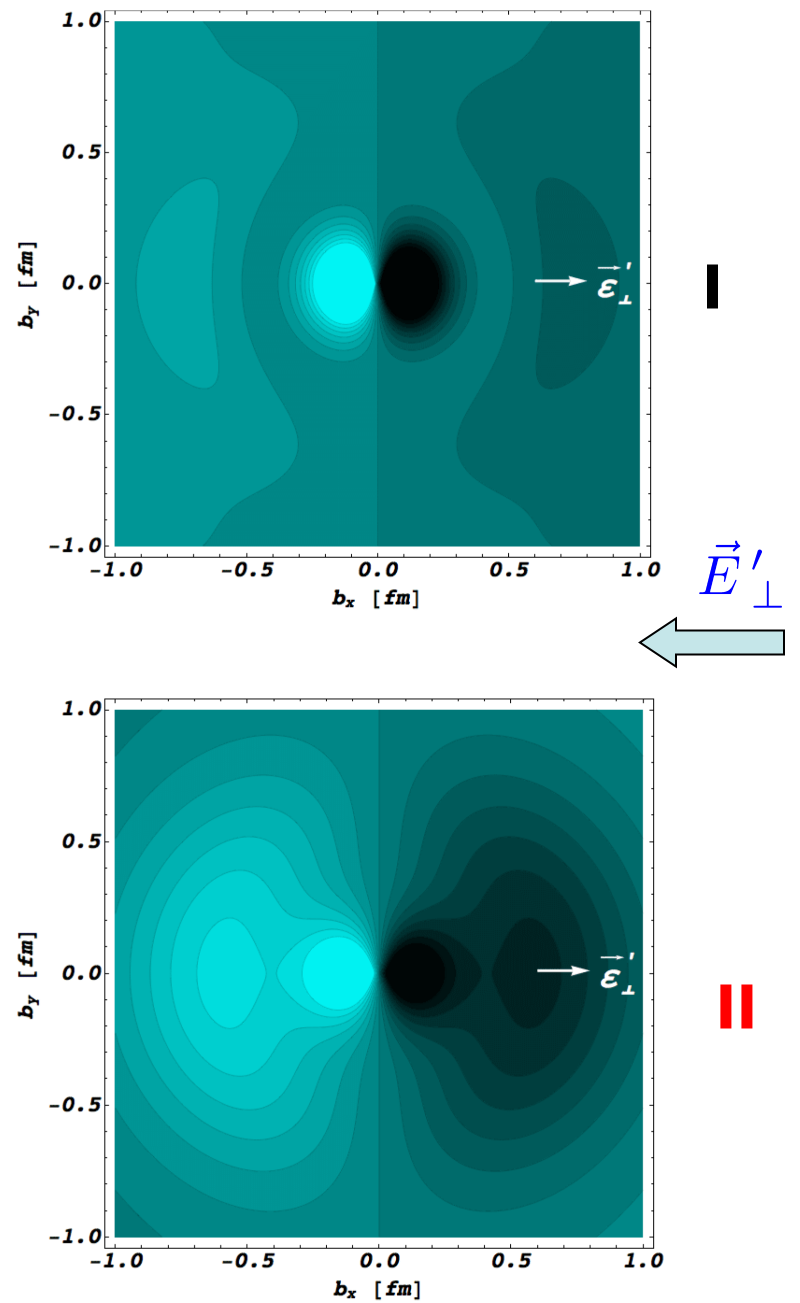
induced polarization in proton with definite helicity

$$\vec{P}_0(\vec{b}) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}\cdot\vec{b}_\perp}\, \vec{P}_0(\vec{q}_\perp)$$

$$= \hat{b}\int_0^\infty \frac{dQ}{(2\pi)}\, Q J_1(bQ) A(Q^2)$$

A expressed in terms of GPs : mainly α and β





[Gorchtein, Lorcé, BP, Vanderhaeghen, PRL(2009)]

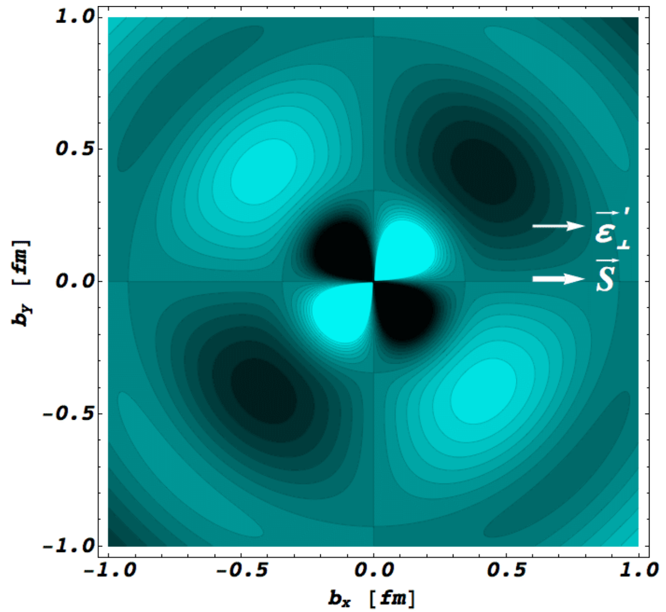
induced polarization in proton with **transverse spin**

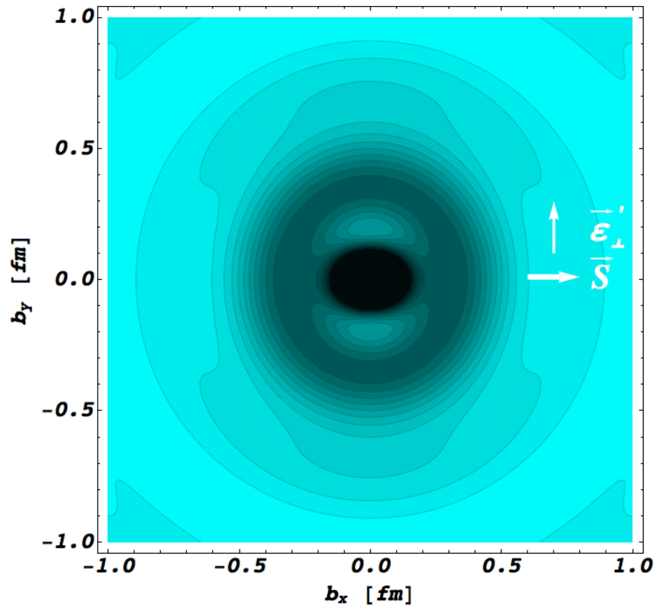
$$\vec{P}_T(\vec{b}) - \vec{P}_0(\vec{b}) =$$

$$-\vec{b}\left(\vec{S}_\perp \times \vec{\varepsilon}'_z\right) \cdot \vec{b} \int_0^\infty \frac{dQ}{(2\pi)} Q\, J_2(bQ) B(Q^2)$$

$$+\left(\vec{S}_\perp \times \vec{\varepsilon}'_z\right) \int_0^\infty \frac{dQ}{(2\pi)} Q \left[J_0(bQ)\, C(Q^2) + \frac{J_1(bq)}{bQ} B(Q^2)\right]$$

B and **C** expressed in terms of GPs : include **spin GPs**





[Gorchtein, Lorcé, Pasquini, Vdh , PRL(2009)]

Summary

- **Nucleon polarizabilities** to learn about e.m. response of the internal degrees of freedom of the nucleon under the influence of a quasistatic e.m. field

- **Dispersion Relation formalism and EFTs** as powerful and complementary tools to extract polarizabilities from data

- **Planned extractions at MAMI and HIGS of scalar polarizabilities** using linearly polarized photons
  → allows **first ever independent extraction** of α_{E1} and β_{M1}

- **New double polarization experiments for RCS at MAMI and HIGS** will give first results for spin polarizabilities in RCS

- **Planned VCS experiments at MAMI** to learn about the GPs in the intermediate Q^2 region
  → mapping in the coordinate space of the deformation induced in the charge and magnetization densities of the proton in the presence of an external e.m. field

Compton scattering off Protons and Light Nuclei:
pinning down the nucleon polarizabilities

29 July - 2 August

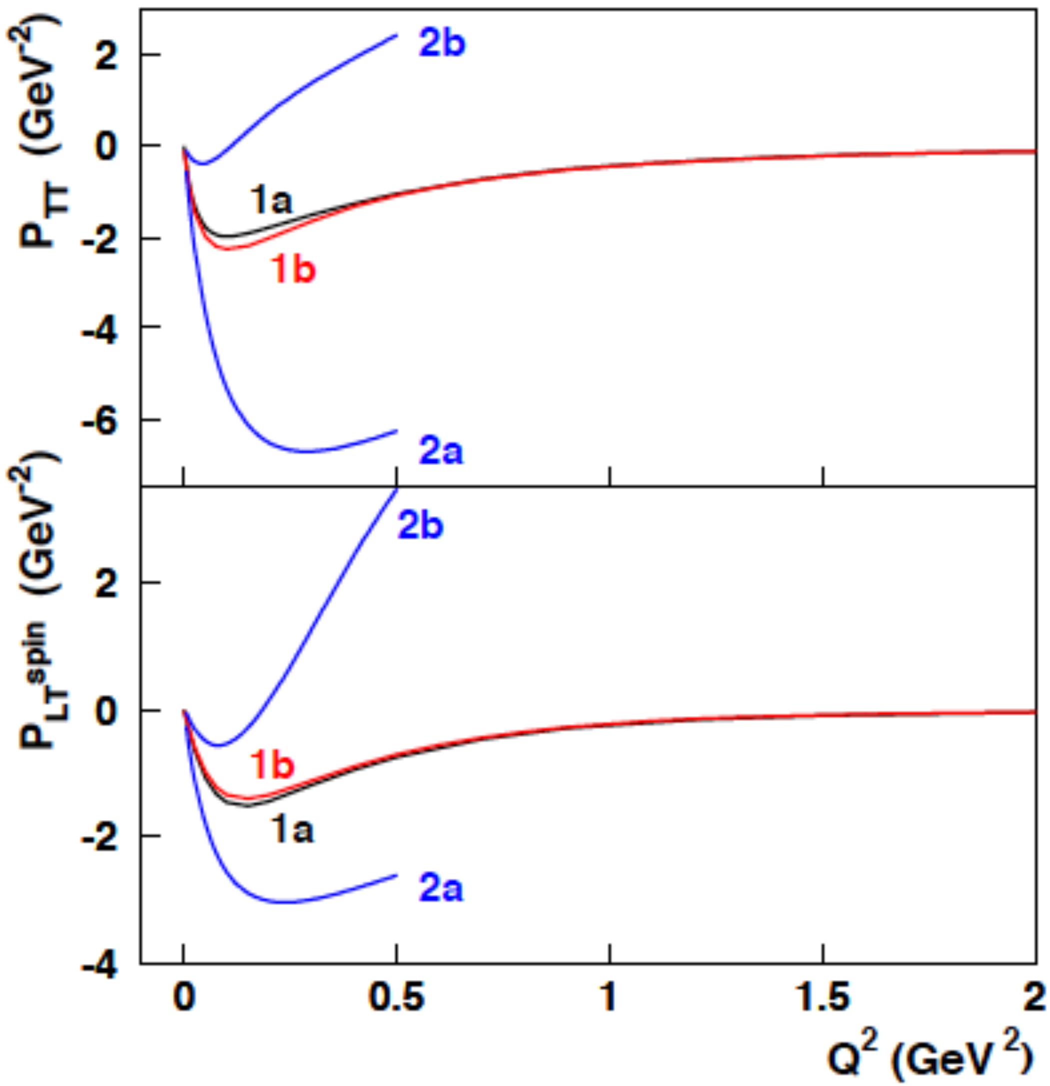
www.ectstar.eu

Organizers: E. Downie, H. Fonvieille, V. Pascalutsa, B. Pasquini

We hope to see you in Trento!

Backup slides

Contribution to structure functions from spin-dependent GPs



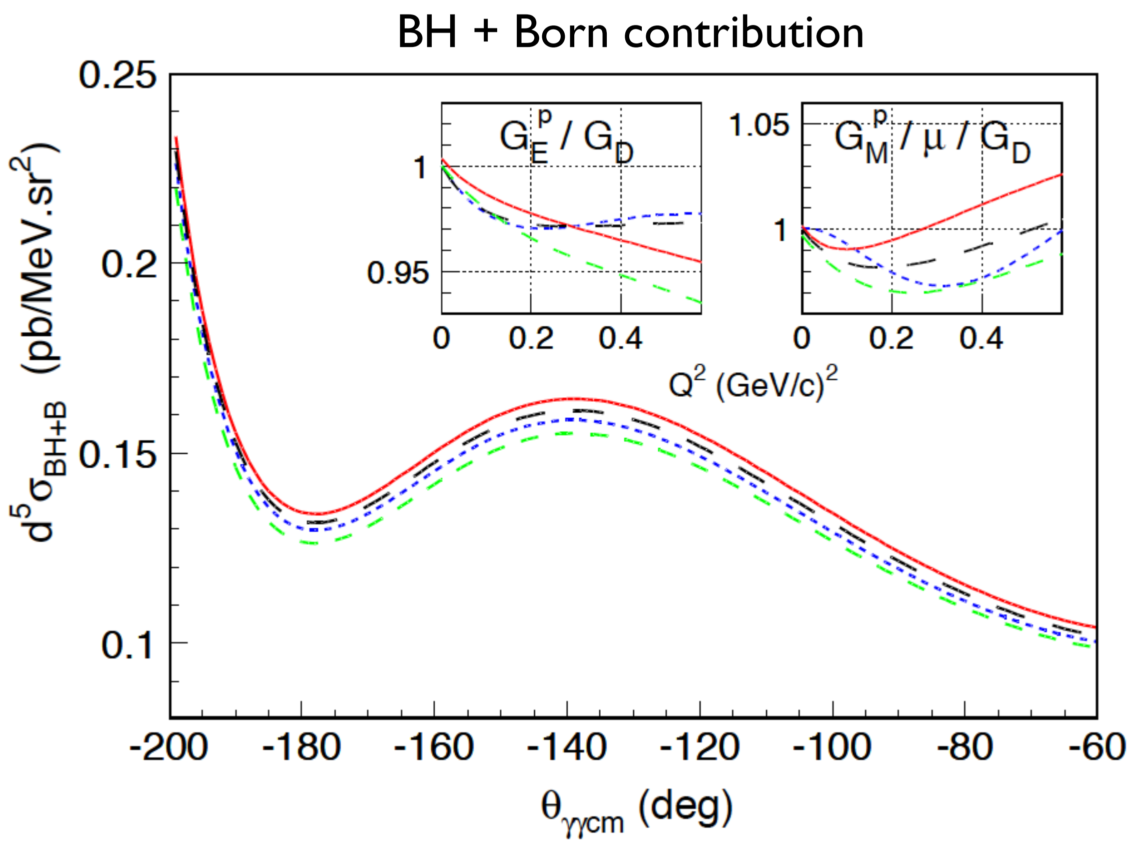
1a: DRs-MAID2000

1b: DRs-MAID2000

2a: HBChPT O(p^3)

2b: HBChPT O(p^4)

Dependence on the proton em form factors



1: Friedrich, Walcher (2003)

2: Mergel, Meissner, Drechsel (1996)

3: Belushkin, Hammer, Meissner (2007)

4: Hoehler (1976)

Effects on the structure functions

P_{LT} $+23\%$ -48%

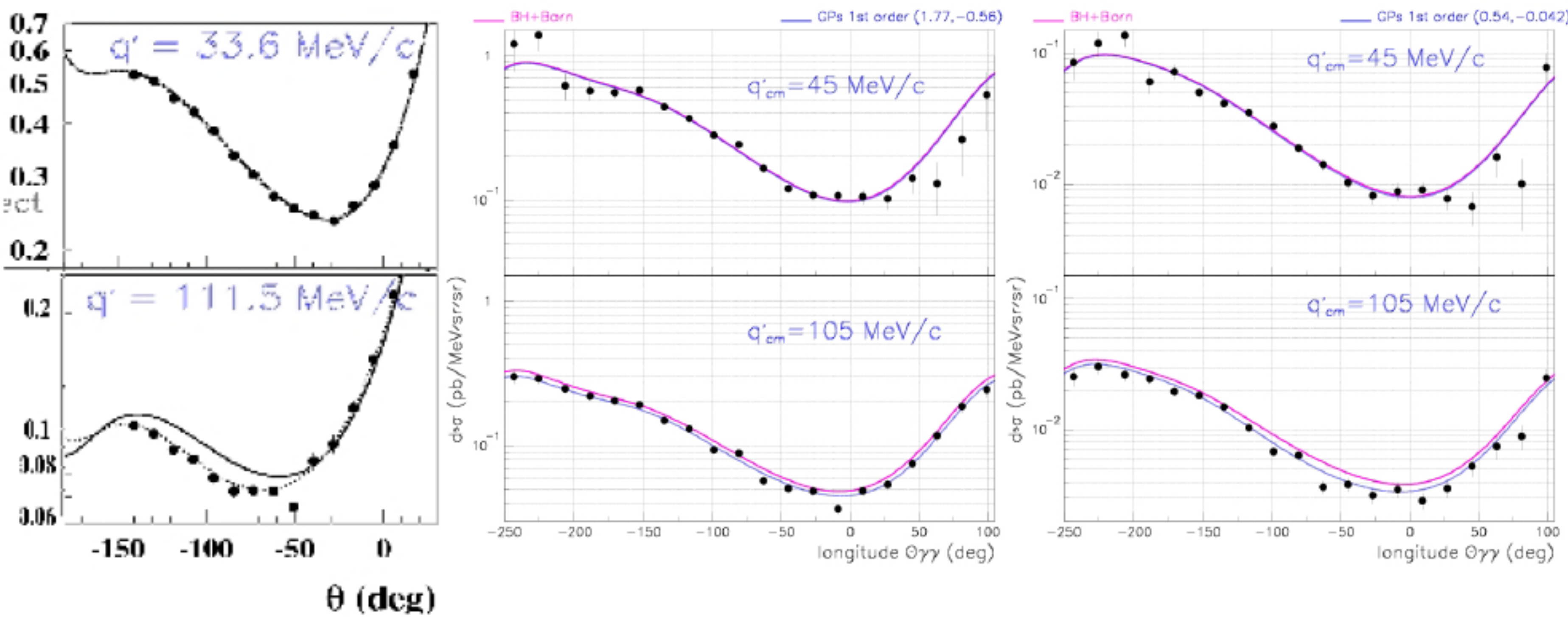
$P_{LL} - P_{TT}/\epsilon$ $+1\%$ $+4\%$

Measured ep $\rightarrow$e'pγ cross sections below threshold

MAMI $Q^2 = 0.33\ \mathrm{GeV}^2$

JLab $Q^2 = 0.92\ \mathrm{GeV}^2$

JLab $Q^2 = 1.76\ \mathrm{GeV}^2$



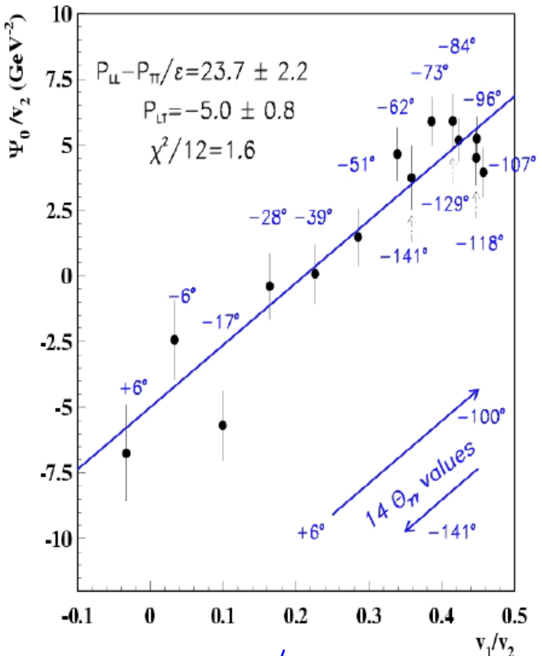
Fit of the VCS structure functions

LEX analysis

$$(\mathrm{d}\sigma_{\mathrm{exp}} - \mathrm{d}\sigma_{\mathrm{BH+Born}})/\Phi/v_2 = P_{LT} + v_1/v_2(P_{LL} - P_{TT}/\epsilon)$$

MAMI

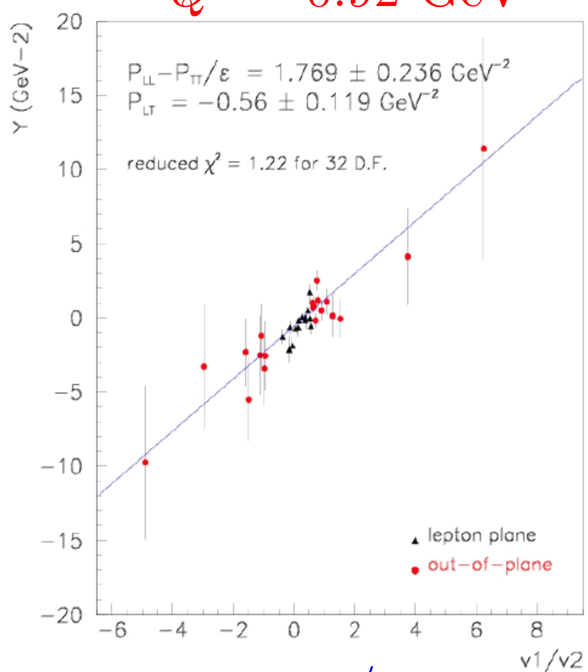
$Q^2 = 0.33\ \mathrm{GeV}^2$



v_1/v_2

JLab

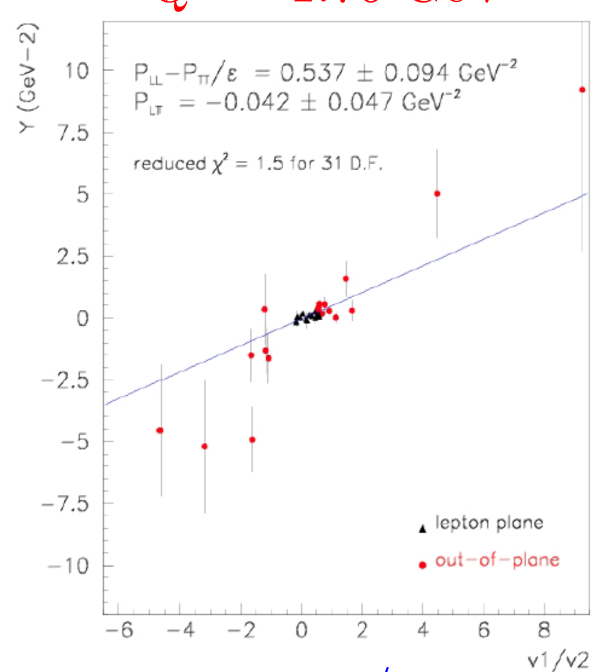
$Q^2 = 0.92\ \mathrm{GeV}^2$



v_1/v_2

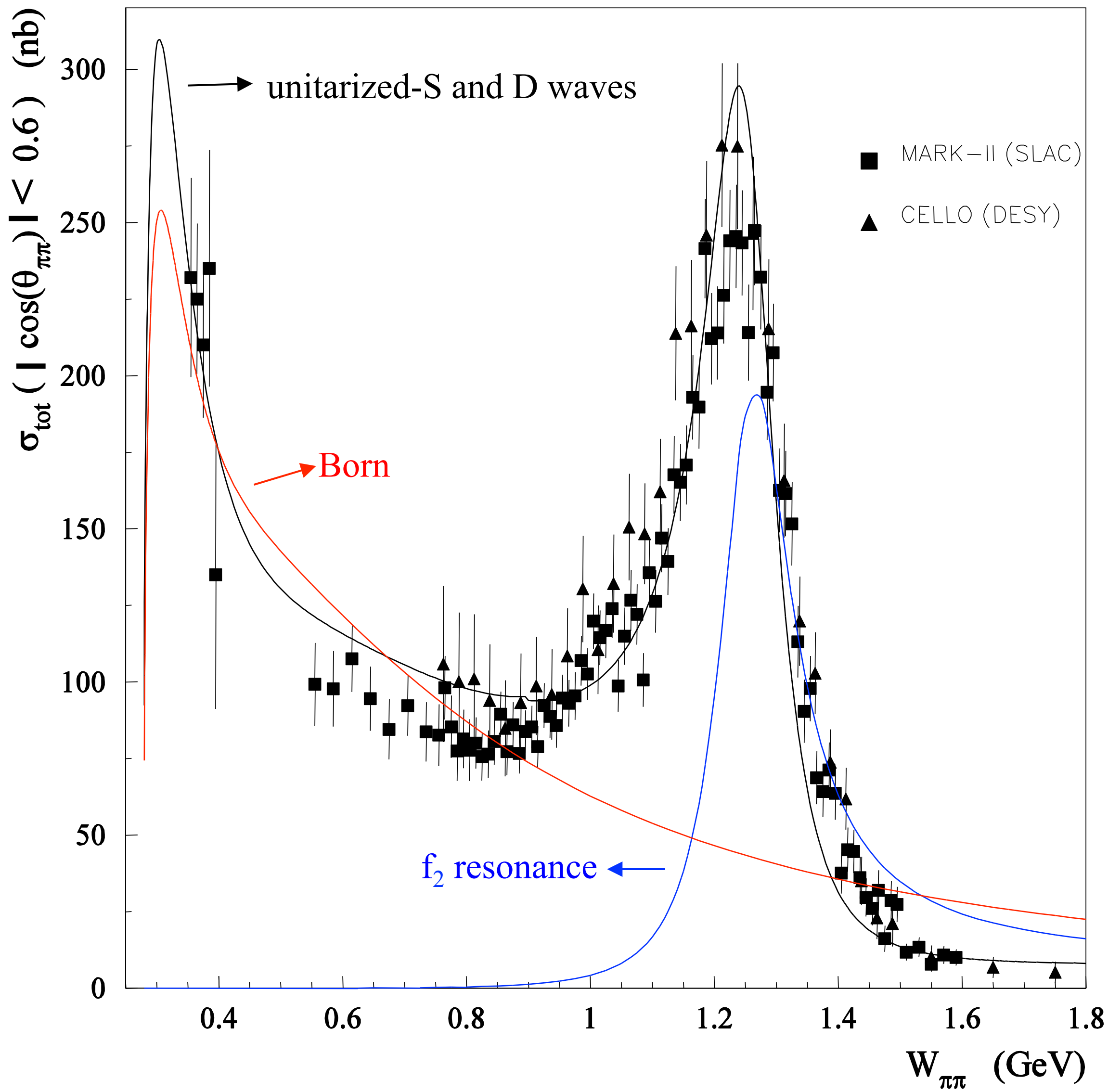
JLab

$Q^2 = 1.76\ \mathrm{GeV}^2$



v_1/v_2

$\gamma\gamma \to \pi^+\pi^-$ total cross section



Asymptotic behavior

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu' \mathrm{d}\nu'}{\nu'^{\,2}-\nu^2}$$

- Behaviour at ν→ ∞ (at fixed t and Q²)

$$F_1,\ F_5 \quad \sim \quad \nu^{\alpha_P(t)-2}\,, \qquad \nu^{\alpha_M(t)}$$

$$F_7 \quad \sim \quad \nu^{\alpha_P(t)-3}\,, \qquad \nu^{\alpha_M(t)-1}$$

$$F_5+4F_{11} \quad \sim \quad \nu^{\alpha_P(t)-2}\,, \qquad \nu^{\alpha_M(t)-1}$$

$$F_3,\ F_8 \quad \sim \quad \nu^{\alpha_P(t)-3}\,, \qquad \nu^{\alpha_M(t)-2}$$

$$F_2,\ F_6,\ F_{10} \quad \sim \quad \nu^{\alpha_P(t)-2}\,, \qquad \nu^{\alpha_M(t)-2}$$

$$F_9,\ F_{12} \quad \sim \quad \nu^{\alpha_P(t)-4}\,, \qquad \nu^{\alpha_M(t)-2}$$

$$F_4 \quad \sim \quad \nu^{\alpha_P(t)-4}\,, \qquad \nu^{\alpha_M(t)-3}$$

$\alpha_P(0) \equiv 1.08$ **(Pomeron);** $\alpha_M(0) \leq 0.5$ **(Meson)**

- for i ≠ 1, 5: unsubtracted dispersion relations

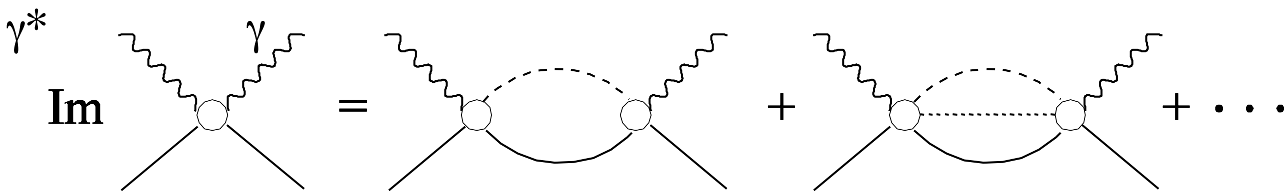
$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu' \mathrm{d}\nu'}{\nu'^{\,2}-\nu^2}$$

- for i = 1, 5: finite energy sum rule

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\nu_{max}} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu' \mathrm{d}\nu'}{\nu'^{\,2}-\nu^2} + F_i^{\mathrm{as}}(Q^2,\nu,t)$$

➢ Dispersion Integrals

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}F_i^{\mathrm{NB}}(Q^2,\nu',t)\frac{\nu'\,\mathrm{d}\nu'}{\nu'^2-\nu^2}$$



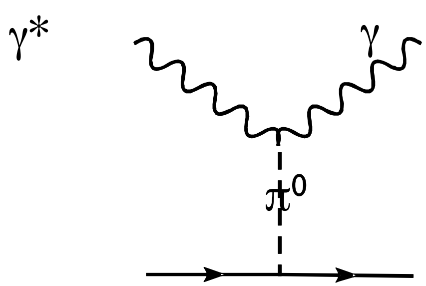
$s \geq (M + m_\pi)^2$ $\quad$ $s \geq (M + 2\, m_\pi)^2$

- for $s \leq (M_N+2m_\pi)^2$ the dominant contribution is from one-pion intermediate states

$$\mathrm{Im}F_i \Longrightarrow \gamma^{(*)}N \rightarrow \pi N \quad \text{from MAID2003}$$

MAID2003: Drechsel, Hanstein, Kamalov,Tiator, NPA 645 (1999)

➢ Asymptotic contribution to $F_5 = -\,4\,F_{11}$



π^0 exchange in the t-channel

➢ Asymptotic parts and contributions beyond π N

F_1^{as} : energy independent t-channel pole

$$F_1^{\mathrm{NB}}(Q^2,\nu,t) - F_1^{\pi N}(Q^2,\nu,t) \simeq \frac{f(Q^2)}{1-t/m_\sigma^2}$$

$$f(Q^2 f(Q^2) = [F_1(t=-Q^2,\nu=0) - F_1^{\pi N}(t=-Q^2,\nu=0)](1+Q^2/m_\sigma^2)'m_\sigma^2)$$

$$\beta(Q^2) - \beta^{\pi N}(Q^2) = \frac{\left(\beta^{\exp}(0) - \beta^{\pi N}(0)\right)}{\left(1+Q^2/\Lambda_\beta^2\right)^2}$$

contribution beyond π N to F_2 :energy independent constant

$$F_2^{\mathrm{NB}}(Q^2,\nu,t) - F_2^{\pi N}(Q^2,\nu,t) \simeq F_2^{\mathrm{NB}}(t=-Q^2,\nu=0,Q^2) - F_2^{\pi N}(t=-Q^2,\nu=0,Q^2)$$

$$\alpha(Q^2) - \alpha^{\pi N}(Q^2) = \frac{\left(\alpha^{\exp}(0) - \alpha^{\pi N}(0)\right)}{\left(1+Q^2/\Lambda_\alpha^2\right)^2}$$

Scalar polarizabilities $\alpha(Q^2)$ ad $\beta(Q^2)$
(two parameters Λ_α and Λ_β)
fitted from experiments

Subtracted Dispersion Relations

$$\mathrm{Re}A_i(\nu,t) = A_i^{\mathrm{B}}(\nu,t) + \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}_s\, A_i(\nu',t)\frac{\nu'\mathrm{d}\nu'}{\nu'^2-\nu^2}$$

❖ t = const. , subtraction point at $(\nu = 0, t)$

$$\mathrm{Re}\, A_i(\nu,t) = A_i^B(\nu,t) + \left[A_i(0,t) - A_i^B(0,t)\right]$$

$$+ \frac{2}{\pi}\nu^2\, \mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im_s}\, A_i(\nu',t)\,\frac{d\nu'}{\nu'(\nu'^2-\nu^2)}$$

→ convergence for all 6 amplitudes

❖ subtraction functions $A_i(0,t) - A_i^B(0,t)$ determined from subtracted DRs in t at fixed $\nu = 0$

$$A_i(0,t) - A_i^B(0,t) = a_i + \frac{t}{\pi}\left(\int_{\mathrm{pos.-t\ cut}} dt' - \int_{\mathrm{neg.-tcut}} dt'\right)\frac{\mathrm{Im}_t\, A_i(0,t')}{t'(t'-t)}$$

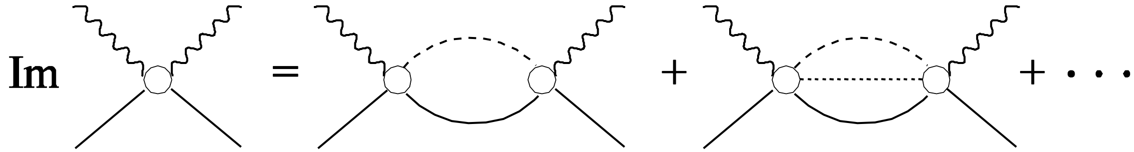
❖ subtraction constants $a_i = A_i(0,0) - A_i^B(0,0)$ directly related to linear combinations of static polarizabilities

Subtracted Dispersion Relations

$$A_i(\nu,t) = a_i + A_i^s(\nu,t) + A_i^t(0,t)$$

Drechsel, Gorchtein, BP, Vanderhaeghen, PRC61 (1999)

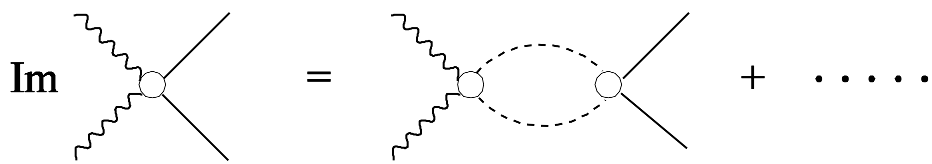
s-channel dispersion integral



- one-pion intermediate states: 1π photoproduction multipoles from MAID analysis
- resonance contribution from multipion intermediate states

t-channel dispersion integral

- positive – t cut: $t \geq 4\, m_\pi^2$



$\gamma\gamma \rightarrow \pi\pi$: unitarized S and D waves amplitudes

$\pi\pi \rightarrow N\bar{N}$: extrapolation of the crossed π N → π N helicity amplitudes [Hoehler, 1983]

- negative – t cut: $t \leq -2m_\pi\,(M+m_\pi)$

extrapolation of the s-channel amplitudes (Δ(1232) and non-resonant π N exchange) in the unphysical region at $\nu = 0$ and $t \leq 0$

$$A_i(\nu, t) = a_i + A_i^s(\nu, t) + A_i^t(0, t)$$

- $A_i^s(\nu, t)$ and $A_i^t(0, t)$: input from available experimental information of different processes ($\gamma\pi \to \pi N$, $\gamma\gamma \to \pi\pi$ and $\pi\pi \to NN$)

- a_i : subtraction constants given by the polarizabilities

free parameters to be fitted to RCS data

$$\alpha_{E1} + \beta_{M1} = -\frac{1}{2\pi}(a_3 + a_6)$$

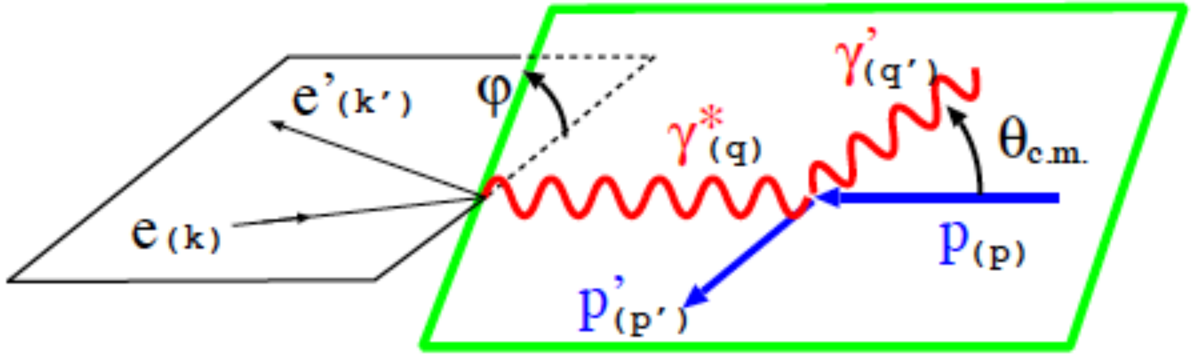
$$\alpha_{E1} - \beta_{M1} = -\frac{1}{2\pi}a_1$$

$$\gamma_{E1E1} = \frac{1}{8\pi M}(a_2 - a_4 + 2a_5 + a_6)$$

$$\gamma_{M1M1} = -\frac{1}{8\pi M}(a_2 + a_4 + 2a_5 - a_6)$$

$$\gamma_{E1M2} = \frac{1}{8\pi M}(a_2 - a_4 - a_6)$$

$$\gamma_{M1E2} = -\frac{1}{8\pi M}(a_2 + a_4 + a_6)$$



e'(k')
φ
γ*(q)
γ'(q')
θc.m.
e(k)
p(p)
p'(p')

forward spin-dependent amplitude from experimental data

$$T(\nu, \theta = 0) \quad = \quad \vec{\varepsilon}\,'^{*} \cdot \vec{\varepsilon}\, f(\nu) \; + \; i\vec{\sigma} \cdot (\vec{\varepsilon}\,'^{*} \times \vec{\varepsilon})\, g(\nu)$$

$$g(\nu) = \frac{-e^2 \kappa_N^2}{8\pi M^2}\, \nu + \nu^3 \gamma_0^{\rm dyn}(\nu) \quad \xrightarrow{\text{LEX}} \quad \gamma_0^{\rm dyn}(\nu) = \gamma_0 + \bar{\gamma}_0\, \nu^2 + \mathcal{O}(\nu^4)$$

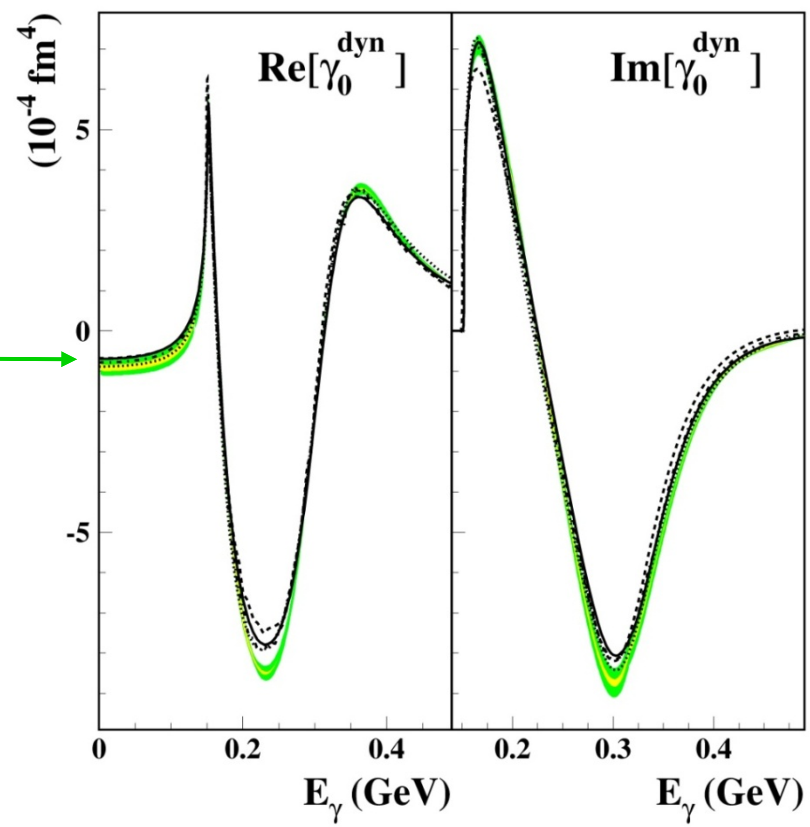
$$\mathrm{Re}[\gamma_0^{\rm dyn}(\nu)] = \frac{1}{4\pi^2} \mathcal{P} \int_{\nu_0}^{\infty} \frac{\sigma_{1/2}(\nu') - \sigma_{3/2}(\nu')}{\nu'(\nu'^2 - \nu^2)}\, d\nu' \qquad \mathrm{Im}[\gamma_0^{\rm dyn}(\nu)] = \frac{\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)}{8\pi\nu^2}$$

± sd ± syst.

GDH Coll. (MAMI & ELSA)

Ahrens et al., PRL87 (2001); Dutz et al. PRL91 (2003)

$$\gamma_0 = -0.9 \pm 0.08 \pm 0.11$$



HDT MAID

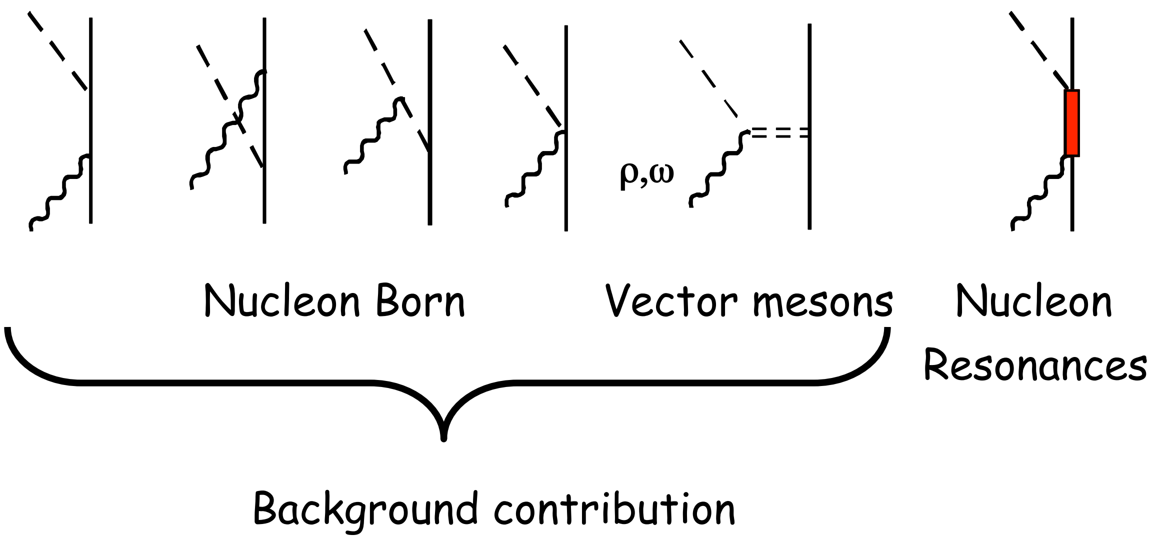
SAID DMT

input of DRs: phenomenological analysis of pion-photoproduction multipoles

[B.P., Pedroni , Drechsel , PLB(2010)]

MAID05: input of dispersion integrals

Mainz-Dubna Unitary Isobar Model



- ✓ Background and resonance separately unitarized
- ✓ Pion cloud is absorbed in the dressed resonances
- ✓ Vector meson and Resonance parameters fitted to $\gamma + N \rightarrow \pi N$ and $\gamma^* + N \rightarrow \pi N$ experimental data

Nucleon resonance included
(all 13 **** below 2 GeV)

Resonance	Partial Wave	Multipoles
$\Delta(1232)$	P_{33}	M_{1+}, E_{1+}, L_{1+}
$N^*(1440)$	P_{11}	M_{1-}, L_{1-}
$N^*(1520)$	D_{13}	M_{2-}, E_{2-}, L_{2-}
$N^*(1535)$	S_{11}	E_{0+}, L_{0+}
$\Delta(1620)$	S_{31}	E_{0+}, L_{0+}
$N^*(1650)$	S_{11}	E_{0+}, L_{0+}
$N^*(1680)$	F_{15}	M_{3-}, E_{3-}, L_{3-}
$\Delta(1700)$	D_{33}	M_{2-}, E_{2-}, L_{2-}
$N^*(1675)$	D_{15}	M_{2+}, E_{2+}, L_{2+}
$N^*(1720)$	P_{13}	M_{1+}, E_{1+}, L_{1+}
$\Delta(1910)$	P_{31}	M_{1-}, L_{1-}
$\Delta(1905)$	F_{35}	M_{3-}, E_{3-}, L_{3-}
$\Delta(1950)$	F_{37}	M_{3+}, E_{3+}, L_{3+}

Drechsel, Hanstein, Kamalov, Tiator, NPA645 (1999)

Asymptotic behaviour

Behavior at ν !1 at fixed Q^2 and t

$$
\begin{array}{rcll}
F_1,\ F_5 & \sim & \nu^{\alpha_P(t)-2}\,, & \nu^{\alpha_M(t)} \\
F_5 + 4F_{11} & \sim & \nu^{\alpha_P(t)-2}\,, & \nu^{\alpha_M(t)-1} \\
F_2,\ F_6,\ F_{10} & \sim & \nu^{\alpha_P(t)-2}\,, & \nu^{\alpha_M(t)-2} \\
F_7 & \sim & \nu^{\alpha_P(t)-3}\,, & \nu^{\alpha_M(t)-1} \\
F_3,\ F_8 & \sim & \nu^{\alpha_P(t)-3}\,, & \nu^{\alpha_M(t)-2} \\
F_9,\ F_{12} & \sim & \nu^{\alpha_P(t)-4}\,, & \nu^{\alpha_M(t)-2} \\
F_4 & \sim & \nu^{\alpha_P(t)-4}\,, & \nu^{\alpha_M(t)-3}
\end{array}
$$

$$\alpha_P(0) \equiv 1.08 \text{ (Pomeron)},\ \alpha_M(0) \leq 0.5 \text{ (Meson)}$$

for i ≠ 1, i ≠ 5 UNsubtracted Dispersion Relations

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu' \mathrm{d}\nu'}{\nu'^{\,2}-\nu^2}$$

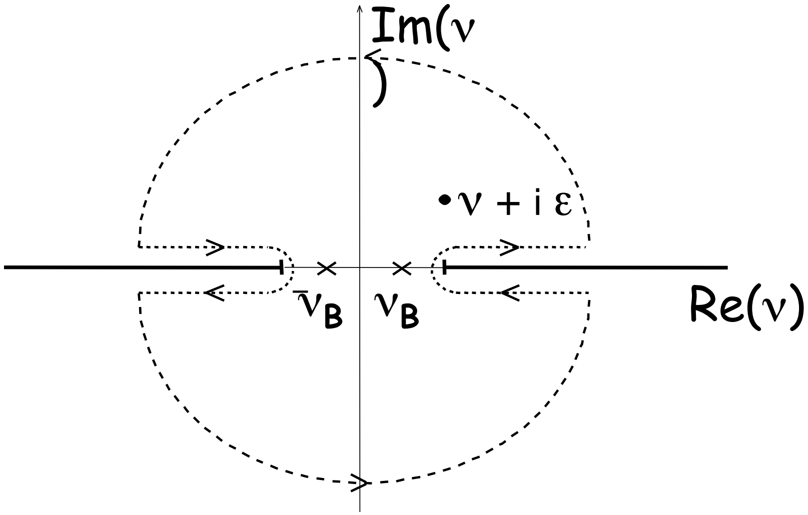
for i = 1, i = 5 :

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\nu_{max}} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu' \mathrm{d}\nu'}{\nu'^{\,2}-\nu^2} + F_i^{\mathrm{as}}(Q^2,\nu,t)$$

Dispersion Relations at fixed t and fixed Q^2

$$\mathrm{T}^{\mathrm{VCS}} = \varepsilon_\mu \varepsilon'^{*}_\nu \sum_{i=1}^{12} F_i(Q^2,\nu,t)\rho_i^{\mu\nu}$$

$F_i(Q^2,\nu,t)$: 12 analytical functions in the complex ν plane with cuts and poles on the real axis



❖ Analyticity in ν, Causality ➡ Cauchy integral formula

$$F_i(Q^2,\nu,t) = F_i^B(Q^2,\nu,t) + \oint_C F_i(Q^2,\nu',t)\frac{\nu' d\nu'}{\nu'^2-\nu^2}$$

❖ Crossing symmetry and analyticity

$$F_i(Q^2,\nu,t) = F_i(Q^2,-\nu,t) \qquad F_i(Q^2,\nu^*,t) = F_i^*(Q^2,\nu,t)$$

⇩

$$\mathrm{Re}F_i^{\mathrm{NB}}(Q^2,\nu,t) = \frac{2}{\pi}\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im}_s\, F_i(Q^2,\nu',t)\frac{\nu' d\nu'}{\nu'^{\,2}-\nu^2}$$