Improved description of the πN-scattering phenomenology in covariant baryon chiral perturbation theory

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Part I

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- At high energies:
  - Allows to study the baryonic spectrum of QCD together with its properties.
- At low energies:
  - Test the chiral dynamics of QCD.
  - Study the role of isospin violation.
  - Provides important information about the internal structure of the nucleon.
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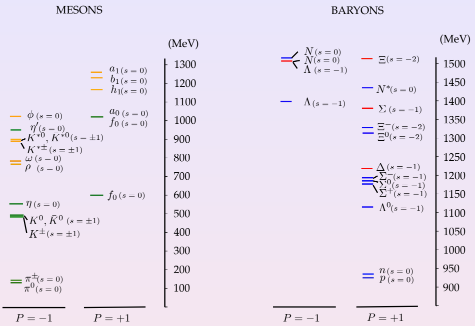

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- This (global) symmetry group also predicts that each hadron should have a chiral partner with opposite parity.
- Such parity doubling is not observed in the hadronic spectrum ⇒ The symmetry is *spontaneously* broken ⇒ Goldstone bosons.

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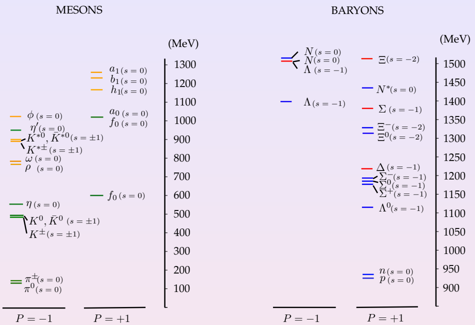

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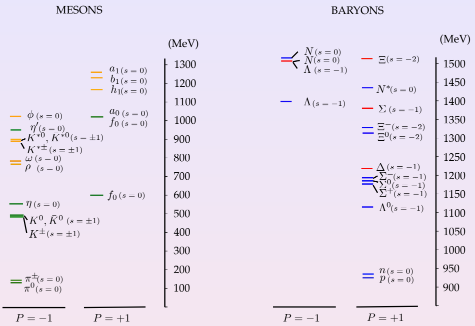

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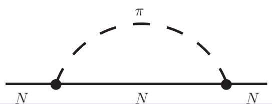

According to the power counting:

$$\nu = \sum_i V_i(d_i + 2m_i - 2 + \frac{n_i}{2}) + 2L - \frac{E_N}{2} + 2 = 3$$

However an explicit calculation ($\mu = m_N$) shows:

$$\delta m_N^{(3)} = \frac{3g_A^2 m_N M_\pi^2}{32\pi^2 f_\pi^2} + \mathcal{O}(M_\pi^3)$$

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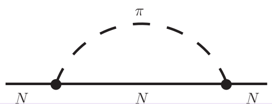

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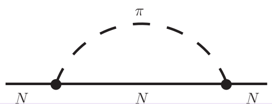

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The role of the $\Delta(1232)$ in πN scattering

- The $\Delta(1232)$ is a resonance with quantum numbers $J = 3/2$ and $I = 3/2$ that dominates the πN scattering at low energies.
- Most of the ChPT analyses of πN scattering do not include it as an explicit degree of freedom arguing that its contribution can be absorbed in the LECs of the πN Lagrangian (RS).
- However, the proximity of the Δ pole to the πN threshold makes that the behavior of this resonance cannot be well reproduced by a finite polynomial $\Rightarrow$ Worsening of the convergence of the chiral series.
- This resonance can be included *consistently* in our EFT using the consistent formulation of chiral Lagrangians of Pascalutsa **[Pascalutsa and Timmermans, PRC 60, (1999), Pascalutsa, PLB 503, (2001)]**.

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Part II

πN *scattering*

πN scattering

We calculate the πN scattering amplitude in covariant BChPT with EOMS up to $\mathcal{O}(p^3)$ exploring two possibilities:

- $\not\Delta$-ChPT: π and N are the only degrees of freedom $\Rightarrow$ Allows to compare with previous HBChPT and IR results.
- Δ-ChPT: We include the $\Delta(1232)$ as an explicit degree of freedom using consistent Lagrangians $\Rightarrow$ We expect an improvement of the convergence of the chiral series.
  - $\Rightarrow$ Can solve various open problems of BChPT when studying the πN scattering (convergence in the subthreshold region, $\sigma_{\pi N}$).

To fix the LECs of the chiral Lagrangians, we compare our theoretical amplitude to three different PWAs:

- PWA of the Karlsruhe group **[Koch, NPA 448 (1986) 707]** (KA85).
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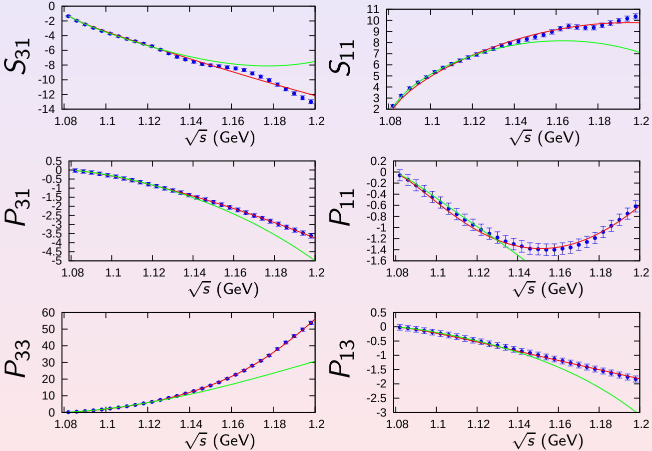
Fits

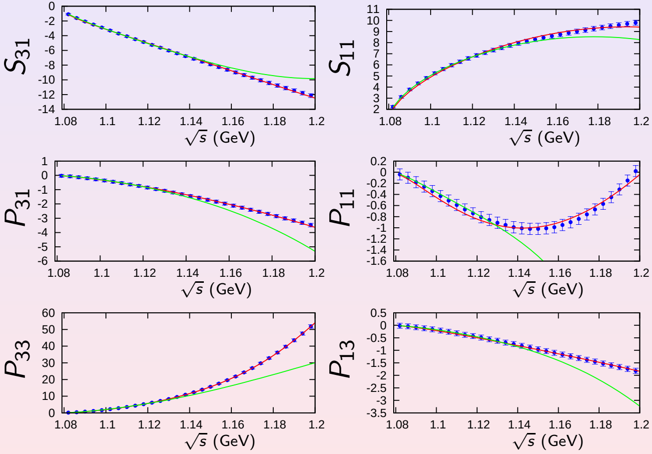
KA85



Red line: Δ-ChPT. Green line: $\not\Delta$-ChPT.

Fits

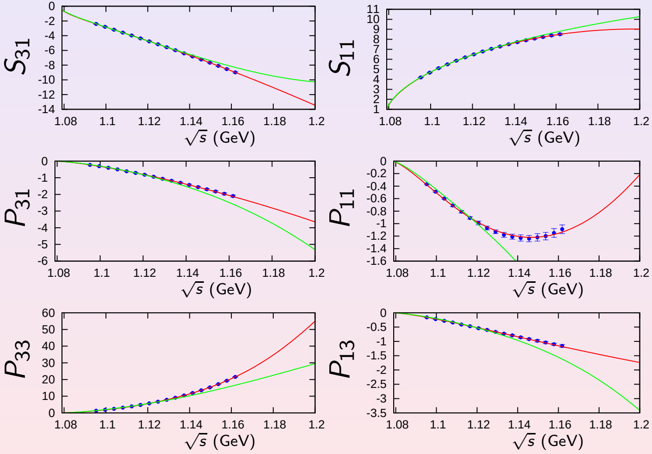
WI08



Red line: Δ-ChPT. Green line: $\not\Delta$-ChPT.

Fits

EM06



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LECs

LEC	KA85 Δ-ChPT	WI08 Δ-ChPT	EM06 Δ-ChPT	KA85 $\not{\Delta}$-ChPT	WI08 $\not{\Delta}$-ChPT	EM06 $\not{\Delta}$-ChPT
c_1	-0.80(6)	-1.004(30)	-1.000(8)	−1.26(14)	−1.50(7)	−1.47(2)
c_2	1.12(13)	1.010(40)	0.575(25)	4.08(19)	3.74(26)	3.63(2)
c_3	-2.96(15)	-3.040(20)	-2.515(35)	−6.74(38)	−6.63(31)	−6.42(1)
c_4	2.00(7)	2.029(10)	1.776(20)	3.74(16)	3.68(14)	3.56(1)
$d_1 + d_2$	-0.15(21)	0.15(20)	-0.34(5)	3.3(7)	3.7(6)	3.64(8)
d_3	-0.21(26)	-0.23(27)	0.276(43)	−2.7(6)	−2.6(6)	−2.21(8)
d_5	0.82(14)	0.47(7)	0.2028(33)	0.50(35)	−0.07(16)	−0.56(4)
$d_{14} - d_{15}$	-0.11(44)	-0.5(5)	0.35(9)	−6.1(1.2)	−6.8(1.1)	−6.49(2)
d_{18}	-1.53(27)	-0.2(8)	-0.53(12)	−3.0(1.6)	−0.50(1.8)	−1.07(22)
h_A	3.02(4)	2.87(4)	2.99(2)	–	–	–
$\chi^2_{\mathrm{d.o.f.}}$	0.77	0.24	0.11	0.38	0.23	25.08

- Δ(1232) Breit-Wigner width $\Gamma_\Delta = 118(2)$ MeV (PDG) ⇒ $h_A = 2.90(2)$

Part III

The Goldberger-Treiman Relation

The Goldberger-Treiman Relation

- The Goldberger-Treiman relation is a pre-PCAC relation that relies on the conservation of the spontaneously broken chiral symmetry.
- The non-exact conservation of this symmetry due to the quark masses leads to a deviation from this relation (Δ_{GT}) that can be extracted from experimental information.

This deviation is usually defined as:

$$g_{\pi N} = \frac{g_A m_N}{f_\pi}(1 + \Delta_{GT})$$

Studies based on πN and NN PWA lead to $\Delta_{GT} = 1 - 3\%$
[Arndt, Workman and Pavan, PRC 49 (1994)], **[Schröder** *et al.*, **EPJ C 21 (2001)]**, **[de Swart, Rentmeester and Timmermans,** πN **Newsletter 13 (1997)]**.

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	KA85 [1] $\Delta\!\!\!/$-ChPT	WI08 [1] $\Delta\!\!\!/$-ChPT	EM06 [1] $\Delta\!\!\!/$-ChPT	KA85 [1] Δ-ChPT	WI08 [1] Δ-ChPT	EM06 [1] Δ-ChPT
Δ_{GT}	9(4)%	2(4)%	3.6(7)%	5.1(8)%	1.0(2.4)%	2.00(36)%
$g_{\pi N}$	14.03(52)	13.13(52)	13.34(10)	13.53(10)	13.00(31)	13.13(5)

	KA85 [2]	WI08 [3]	EM06 [4]	NN scattering [5]	Pionic atoms [6]
Δ_{GT}	4.5(7)%	2.1(1)%	0.2(1.0)%	1.9(6)%	1.9(7)%
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Part IV

Subthreshold Region

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- The subthreshold contains points that are connected to important low energies theorems.
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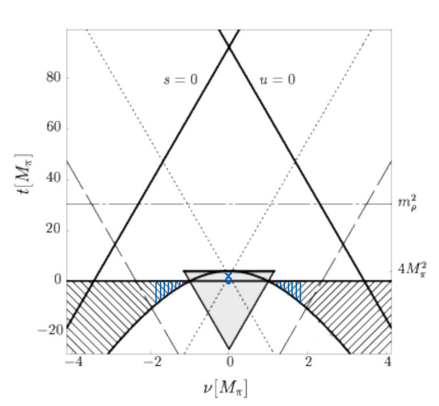

[T. Becher and H. Leutwyler, JHEP (2001)]

- The amplitude fitted in the physical region can be extrapolated into the subthreshold one and compare with PWAs.

Subthreshold Region

	KA85 [1] $\not\Delta$-ChPT	WI08 [1] $\not\Delta$-ChPT	EM06 [1] $\not\Delta$-ChPT	KA85 [1] Δ-ChPT	WI08 [1] Δ-ChPT	EM06 [1] Δ-ChPT	KA85 [2]	WI08 [3]
d^+_{00}	$-2.02(41)$	$-1.65(28)$	$-1.56(5)$	$-1.48(15)$	$-1.20(13)$	$-0.98(4)$	-1.46	-1.30
d^+_{01}	1.73(19)	1.70(18)	1.64(4)	1.21(10)	1.20(9)	1.09(4)	1.14	1.19
d^+_{10}	1.81(16)	1.60(18)	1.532(45)	0.99(14)	0.82(9)	0.631(42)	1.14(2)	–
d^+_{02}	0.021(6)	0.021(6)	0.021(6)	0.004(6)	0.005(6)	0.004(6)	0.036	0.037
b^+_{00}	-6.5(2.4)	-7.4(2.3)	-7.01(1.1)	-5.1(1.7)	-5.1(1.7)	-4.5(9)	-3.54(6)	–
d^-_{00}	1.81(24)	1.68(16)	1.495(28)	1.63(9)	1.53(8)	1.379(8)	1.53(2)	–
d^-_{01}	-0.17(6)	-0.20(5)	-0.199(7)	-0.112(25)	-0.115(24)	-0.0923(11)	-0.134(5)	–
d^-_{10}	-0.35(10)	-0.33(10)	-0.267(14)	-0.18(5)	-0.16(5)	-0.0892(41)	-0.167(5)	–
b^-_{00}	17(7)	17(7)	16.8(7)	9.63(30)	9.755(42)	8.67(8)	10.36(10)	–
Σ	84(10)*	103(5)*	103(2)*	45(7)*	64(6)*	64(1)*	64(8)	79(7)

[d^+_{00} in units of M_π^{-1}. d^-_{00}, b^-_{00} in units of M_π^{-2}. d^+_{01}, d^+_{10}, b^+_{00} in units of M_π^{-3}. d^-_{01}, d^-_{10} in units of M_π^{-4}. d^+_{02} in units of M_π^{-5}. Σ in MeV.]

[1] J. M. Alarcón, J. Martín Camalich and J. A. Oller, arXiv: 1210.4450.

[2] R. Koch, Nucl. Phys. A **448** (1986) 707; R. Koch and E. Pietarinen, Nucl. Phys. A **336** (1980) 331.

[3] Computer code SAID, online program at http://gwdac.phys.gwu.edu/ , solution WI08.

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- The CD theorem states $\Sigma = \sigma(2M_\pi^2) + \Delta_R = \sigma_{\pi N} + \Delta_\sigma + \Delta_R$.
- If one writes Σ as $\Sigma = \Sigma_d + \Delta_D$, with $\Sigma_d \equiv f_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+)$ and Δ_D the reminder, the latter can be well approximated by $\Delta_D \approx 4f_\pi^2 M_\pi^4 d_{02}^+$.
- Neglecting Δ_R, the CD theorem takes the form

$$\sigma_{\pi N} = \Sigma_d + \Delta_D - \Delta_\sigma$$

- $\Delta_\sigma - \Delta_\sigma^{(3)} \approx 10$ MeV
- $\Delta_D - \Delta_D^{(3)} \approx 4M_\pi^4 f_\pi^2 (d_{02}^+ - d_{02}^{+(3)}) \approx 10$ MeV
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Our conclusions:

- Good agreement between EOMS-BChPT+$\Delta(1232)$ and PWAs!.
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Part V

The pion-nucleon σ-term

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- $\sigma_{\pi N}$ is an observable of fundamental importance that embodies the internal scalar structure of the nucleon, related to:
  - Origin of the mass of ordinary matter.
  - Used in estimations of DM-nucleon SI elastic scattering cross section.
  - Dependence of the abundance of fundamental elements on the quark masses. **[Berengut,** *et. al.*, **PRD 87, (2013)]**, **[Epelbaum,** *et. al.*, **arXiv:1303.4856]**
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- Good convergence of EOMS-BChPT+$\Delta(1232)$ $\Rightarrow$ Reliable LECs $\Rightarrow$ Reliable $\sigma_{\pi N}$.

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[1] J. M. Alarcón, J. Martín Camalich, J. A. Oller, PRD(R) **85**, (2012) and . arXiv: 1210.4450

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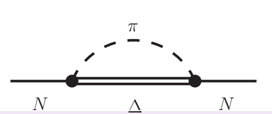
The pion-nucleon σ-term

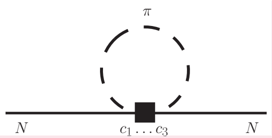
Higher order corrections:

- $\mathcal{O}(p^{7/2})$ (N^{2}LO):



$\Rightarrow -6$ MeV (to be compared with -19 MeV at $\mathcal{O}(p^3)$)

- $\mathcal{O}(p^4)$ (N^{3}LO):



$\Rightarrow -2 \cdots - 4$ MeV

(Extra contributions from $\mathcal{O}(p^4)$ LECs is estimated to be ~ 1 MeV)

The pion-nucleon σ-term

	LO	NLO	N^2LO	N^3LO
$\sigma_{\pi N}$ (MeV)	78–62	-19	-6	$-3(2)$

$\Rightarrow$ Chiral expansion shows a clear convergent pattern!
Comparison with independent phenomenology:

- h_A: Only WI08 Δ-ChPT is compatible with the $\Delta(1232)$ BW width.

	KA85 Δ-ChPT	WI08 Δ-ChPT	EM06 Δ-ChPT	PDG
Γ_Δ (MeV)	128(3)	115(3)	125(2)	118(2)

- Δ_{GT} : WI08 Δ-ChPT and EM06 Δ-ChPT give a Δ_{GT} compatible with independent determinations (NN scattering and π-atoms).

	KA85 Δ-ChPT	WI08 Δ-ChPT	EM06 Δ-ChPT	NN scattering [1]	π-atoms [2]
Δ_{GT}	5.1(8)%	1.0(2.4)%	2.00(36)%	1.9(7)%	1.9(7)%
$g_{\pi N}$	13.53(10)	13.00(31)%	13.13(5)%	13.12(8)%	13.12(9)%

[1] J. J. de Swart, M. C. M. Rentmeester and R. G. E. Timmermans, πN Newsletter **13** (1997) 96.
[2] Baru, Hanhart, Hoefrichter, Kubis, Nogga, Phillips, Phys. Lett. B 694, 437-477 (2011).

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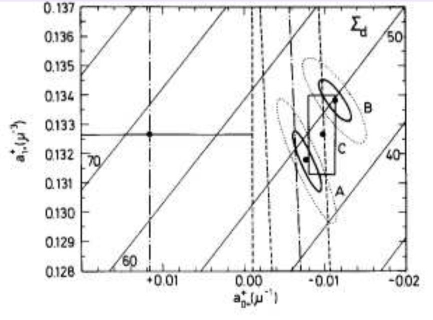
- a^+_{0+}: Strongly constrains the value of $\sigma_{\pi N}$:



Gasser, *et. al.*, PLB 253 (1991)

	a^+_{0+} $(10^{-3} M_\pi^{-1})$
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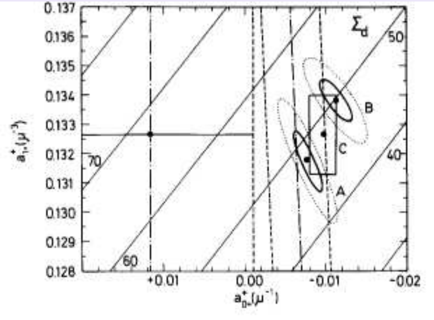

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Part VI

The strangeness puzzle

The strangeness puzzle

- Given a value of $\sigma_{\pi N}$, one can determine σ_s through σ_0.

$$\sigma_{\pi N} = \frac{\sigma_0}{1-y} \qquad \text{since} \qquad \sigma_s = \frac{m_s\,\sigma_{\pi N}}{2\hat{m}} y.$$

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle \qquad \sigma_s = \frac{m_s}{2m_N}\langle N|\bar{s}s|N\rangle$$

$$\sigma_0 \equiv \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle \qquad y \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

- σ_0 can be considered as the pion-nucleon sigma term with no s-quark contribution.
- Gasser calculated σ_0 from the hadron spectrum using a chiral model for hadronic interactions **[Gasser, Annals of Physics 136, (1981)]**.
  - The model is very close to covariant BChPT but with a cut-off (Λ).
  - $\sigma_0 = 35(5)$ MeV with $\Lambda = 700$ MeV.
- **[Borasoy and Meißner, Ann. of Phys. 254 (1997)]** obtained in $SU(3)$ HBChPT $\sigma_0 = 36(7)$ MeV.

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- PWAs extract $\sigma_{\pi N}$ from:

$$\Sigma = f_\pi^2 \bar{D}^+(0, 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R = \sigma_{\pi N} + \Delta_\sigma + \Delta_R$$

where $\Delta_R < 2$ MeV and it was believed that $\Delta_\sigma \approx 5$ MeV **[Gasser, Sainio and Svarc, NPB 307:779 (1988)]**.

- Using $\Sigma = 64$ MeV **[Koch, NPA 448 (1986) 707]**, $\sigma_{\pi N} \approx 60$ MeV $\Rightarrow$ $y \approx 0.40 \Rightarrow \sigma_s \sim 300$ MeV.

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Possible solutions:

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- This shifts the value of $\sigma_{\pi N}$ to $\sigma_{\pi N} \approx 45$ MeV $\Rightarrow y \approx 0.23 \Rightarrow \sigma_s \approx 130$ MeV.

This seems to solve the strangeness puzzle, although

- Is odds with recent experimental determinations related to electromagnetic structure **[Ahmed et al., PRL 108 (2012)]** and spin content **[Alekseev et al. PLB 693 (2010)]** of the nucleon.
- $y = 0.23$ is far from lattice results $y = 0.030(16)$ **[Ohki et al., PRD 78 (2008)]**.

- Our result, $\sigma_{\pi N} = 59(7)$ MeV, seems to resurrect the strangeness puzzle.
- However, the value of σ_0 needs to be reexamined with the modern formalism of covariant BChPT, which eliminates Λ dependence and allow to calculate the uncertainties systematically.

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- In ChPT:

$$\sigma_{\pi N} = -4(2b_0 + b_D + b_F)\frac{M_\pi^2}{2} + \sigma_{\pi N}^{loops}(octet) + \sigma_{\pi N}^{loops}(decuplet)$$

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	Octet ($\mathcal{O}(p^3)$)	Octet+Decuplet ($\mathcal{O}(p^3)$)
σ_0 (MeV)	46(8)	58(8)

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- The old value $\sigma_{\pi N} = 45$ MeV gives a large strangeness content $\Rightarrow$ The problem of a too large strangeness content still remains.
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Part VII

Summary and Conclusions

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- We use different PWAs as an input to fix the LECs of the chiral Lagrangians.
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  - Accurate and reliable value of $\sigma_{\pi N} \Rightarrow \sigma_{\pi N} = 59(7)$ MeV.
    - Compatible with updated phenomenology!
    - Favors a small strangeness content in the nucleon, $\sigma_s = 16(80)$ MeV $\Rightarrow$ compatible with experimental determinations and LQCD!
    - New scenario where the σ-terms are compatible with updated phenomenology, recent experimental determinations and LQCD.
- $\Rightarrow$The inclusion of the $\Delta(1232)$ gives a boost to BChPT!

Summary and Conclusions

- We performed a novel analysis of πN scattering in covariant BChPT within the EOMS scheme up to $\mathcal{O}(p^3)$ including the $\Delta(1232)$.
- We use different PWAs as an input to fix the LECs of the chiral Lagrangians.
- We show how covariant BChPT+$\Delta(1232)$ achieves the best convergence both in the physical and subthreshold regions.
  - Excellent description of the data up to 1.20 GeV.
  - Good description of the subthreshold region with amplitude fitted in the physical region. $\Rightarrow$ BChPT is reliable above the threshold.
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FIN

- T. Becher and H. Leutwyler, JHEP 0106 (2001) 017.
- N. Fettes and U.-G. Meißner, Nucl. Phys. A 693 (2001) 693.
- F. James, Minuit Reference Manual D 506 (1994).
- N. Fettes, U. G. Meißner and S. Steininger, Nucl. Phys. A 640 (1998) 199.
- N. Fettes, U. G. Meißner and S. Steininger, Nucl. Phys. A 640 (1998) 199.
- P. Buettiker and U. G. Meißner, Nucl. Phys. A 668 (2000) 97.
- V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. A 615 (1997) 483.
- V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. A 615 (1997) 483.
- K. Torikoshi and P. J. Ellis, Phys. Rev. C 67 (2003) 015208.
- Computer code SAID, online program at http://gwdac.phys.gwu.edu/ , solution WI08. R. L. Workman,

R. A. Arndt, W. J. Briscoe, M. W. Paris and I. I. Strakovsky, Phys. Rev. C **86**, 035202 (2012).

J. M. Alarcón, J. Martín Camalich, J. A. Oller, L. Alvarez-Ruso, Phys. Rev. C 83 (2011) 055205.

E. Matsinos, William S. Woolcock, G.C. Oades, G. Rasche, A. Gashi. Nucl.Phys. A 778 (2006) 95-123.

J. M. Alarcón, J. Martín Camalich and J. A. Oller, PRD(R) **85** (2012) and arXiv:1210.4450.

J. M. Alarcón, J. Martín Camalich and J. A. Oller, PRD(R) **85** (2012) and arXiv:1210.4450.

J. M. Alarcón, J. Martín Camalich and J. A. Oller, PRD(R) **85** (2012) and arXiv:1210.4450.

J. M. Alarcón, J. Martín Camalich, J. A. Oller, L. Alvarez-Ruso, Phys. Rev. C 83 (2011) 055205.

J. Bsaisou, C. Hanhart, S. Liebig, U. -G. Mei [] ner, A. Nogga and A. Wirzba, arXiv:1209.6306 [hep-ph].

- U. G. Meißner and J. A. Oller, Nucl. Phys. A **673**, 311 (2000).
- J. A. Oller and U. G. Meißner,
- J. Gasser, M. E. Sainio and A. Svarc, NPB 307:779 (1988)
- J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B **253**, 252 (1991).
- J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B **253**, 260 (1991). .
- E. E. Jenkins and A. V. Manohar, Phys. Lett. B 255 (1991) 558.
- T. Becher and H. Leutwyler, Eur. Phys. J. C 9 (1999) 643
- R. Koch, Nucl. Phys. A 448 (1986) 707; R. Koch and E. Pietarinen, Nucl. Phys. A 336 (1980) 331.
- R. A. Arndt, R. L. Workman and M. M. Pavan, Phys. Rev. C 49 (1994) 2729.
- H.-Ch. Schröder et al., Eur. Phys. J. C 21 (2001) 473.

- J. J. de Swart, M. C. M. Rentmeester and R. G. E. Timmermans, πN Newsletter 13 (1997) 96.
- L. Castillejo, R. H. Dalitz and F. J. Dyson, Phys. Rev. 101 (1956) 453.
- J. A. Oller and E. Oset, Phys. Rev. D 60, 074023 (1999).
- J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).
- T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D 68, 056005 (2003).
- V. Bernard, N. Kaiser, Ulf-G. Meissner, Int.J.Mod.Phys.E4:193-346,1995.
- T. P. Cheng and R. Dashen, Phys.Rev.Lett. 26 (1971) 594
- J.Gegelia, G.Japaridze, K.Turashvili, Theoretical and Mathematical Physics, Vol. 101, No. 2, 1994 (Translated from russian)
- L. S. Geng, J. Martín Camalich, L. Alvarez-Ruso and M. J. Vicente Vacas, Phys. Rev. Lett. 101, 222002 (2008)

- Baru, Hanhart, Hoefrichter, Kubis, Nogga, Phillips, Phys. Lett. B 694, 437-477 (2011)
- G. Höler, "Pion-nucleon scattering", edited by H. Schopper, Landolt-Börnstein, New Series, Group I, Vol. 9, Pt. B2 (Springer-Verlag, Berlin, 1983).
- S. Weinberg, Phys. Lett. B **251** (1990) 288; Nucl. Phys. B **363** (1991) 3.
- V. Pascalutsa, arXiv:1110.5792 [nucl-th].
- R. A. Arndt, R. L. Workman and M. M. Pavan, Phys. Rev. C **49** (1994) 2729.
- H.-Ch. Schröder *et al.*, Eur. Phys. J. C **21** (2001) 473.
- J. J. de Swart, M. C. M. Rentmeester and R. G. E. Timmermans, πN Newsletter **13** (1997) 96.
- J. A. Oller and U. G. Meissner, Phys. Lett. B **500**, 263 (2001).

U. G. Meissner and J. A. Oller, Nucl. Phys. A **673**, 311 (2000).

J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999) [hep-ph/9908377].

V. Pascalutsa and D. R. Phillips, Phys. Rev. C **67**, 055202 (2003) [nucl-th/0212024].

V. Pascalutsa, M. Vanderhaeghen and S. N. Yang, Phys. Rept. **437**, 125 (2007) [hep-ph/0609004].

V. Pascalutsa and R. Timmermans, Phys. Rev. C **60**, 042201 (1999) [nucl-th/9905065].

V. Pascalutsa and R. Timmermans, Phys. Rev. C **60**, 042201 (1999) [nucl-th/9905065].

V. Pascalutsa, Phys. Lett. B **503**, 85 (2001) [hep-ph/0008026].

E. Matsinos, W. S. Woolcock, G. C. Oades, G. Rasche, A. Gashi, Nucl. Phys. A **778** (2006) 95.

- G. C. Oades, G. Rasche, W. S. Woolcock, E. Matsinos and A. Gashi, Nucl. Phys. A **794**, (2007) 73 .
- J. M. Alarcon, J. Martin Camalich, J. A. Oller, Phys. Rev. D **85**, 051503 (2012).

- J. M. Alarcon, J. Martin Camalich, J. A. Oller, Phys. Rev. D **85**, 051503 (2012).

- N. Fettes and U. -G. Meissner, Nucl. Phys. A **693**, 693 (2001) [hep-ph/0101030].
- J. M. Alarcón, J. Martín Camalich, J. A. Oller, L. Alvarez-Ruso, Phys. Rev. C 83 (2011) 055205.
- J. A. Oller, E. Oset, J. R. Pelaez, Phys. Rev. D **59** (1999) 074001.
- Baru, *et. al.*, PLB 694 (2011).
- B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **74**, 091103 (2006); Phys. Rev. D **76**, 012008 (2007).

C. P. Shen *et al.* [Belle Collaboration], Phys. Rev. D **80**, 031101 (2009).

Z. G. Wang, Nucl. Phys. A **791**, 106 (2007).

H.-X. Chen, X. Liu, A. Hosaka and S.-L. Zhu, Phys. Rev. D **78**, 034012 (2008).

G.-J. Ding and M.-L. Yan, Phys. Lett. B **650**, 390 (2007).

N. Isgur and J. E. Paton, Phys. Rev. D **31**, 2910 (1985); N. Isgur, R. Kokoski and J. E. Paton, Phys. Rev. Lett. **54**, 869 (1985).

T. Barnes, N. Black and P. R. Page, Phys. Rev. D **68**, 054014 (2003).

A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale and E. Oset, Phys. Rev. D **78**, 074031 (2008).

J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997); (E)-ibid. A **652**, 407 (1999).

- C. A. Vaquera-Araujo and M. Napsuciale, Phys. Lett. B **681**, 434 (2009).
- J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997); (E)-ibid. A **652**, 407 (1999).
- L. Alvarez-Ruso, J. A. Oller and J. M. Alarcón, Phys. Rev. D **80**, 054011 (2009).
- L. Alvarez-Ruso, J. A. Oller and J. M. Alarcón, Phys. Rev. D **82**, 094028 (2010).
- N. Fettes and U. G. Meissner, Nucl. Phys. A **679**, 629 (2001).
- J. Gasser, Annals Phys. **136**, 62 (1981).
- H. Ohki, H. Fukaya, S. Hashimoto, T. Kaneko, H. Matsufuru, J. Noaki, T. Onogi and E. Shintani *et al.*, Phys. Rev. D **78**, 054502 (2008).
- M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino and A. Ukawa, Phys. Rev. D **52**, 3003 (1995).

S. R. Beane, W. Detmold, T. C. Luu, K. Orginos, A. Parreno, M. J. Savage, A. Torok and A. Walker-Loud, Phys. Rev. D **79**, 114502 (2009).

Z. Ahmed *et al.* [HAPPEX Collaboration], Phys. Rev. Lett. **108**, 102001 (2012) [arXiv:1107.0913 [nucl-ex]].

M. G. Alekseev *et al.* [COMPASS Collaboration], Phys. Lett. B **693**, 227 (2010) [arXiv:1007.4061 [hep-ex]].

J. Giedt, A. W. Thomas and R. D. Young, Phys. Rev. Lett. **103**, 201802 (2009) [arXiv:0907.4177 [hep-ph]].

L. M. Sehgal. in ,.Proceedings of the International Conference on High Energy Physics. Geneva. 27 June–l July 1979Ó (European Physical Society. Ed.), p. 98.
Phys. Lett. B **253**, 260 (1991).

J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B 253, 260 (1991)

- J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B 253, 252 (1991)
- W. Heisenberg. Z. Phys. 77, 1 (1932).
- J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U. -G. Meissner, J. Nebreda and J. R. Pelaez, Phys. Rev. D 87, 085018 (2013).
- E. Epelbaum, H. Krebs, T. A. Lähde, D. Lee and U. -G. Mei§ner, arXiv:1303.4856 [nucl-th].
- B. Borasoy and U. -G. Meissner, Annals Phys. **254**, 192 (1997) [hep-ph/9607432].