Interface between Theoretical Framework of GPDs and Experimental Measurements of DVCS

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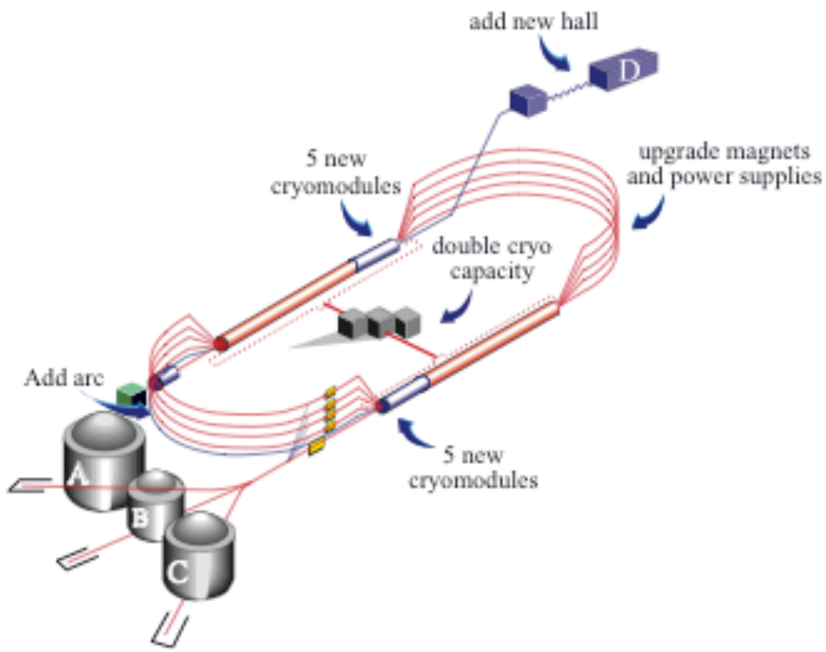
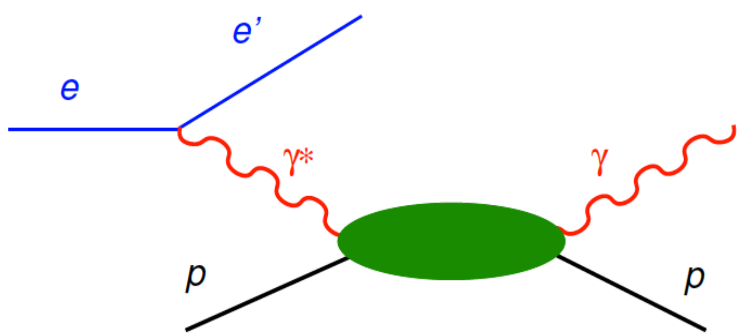
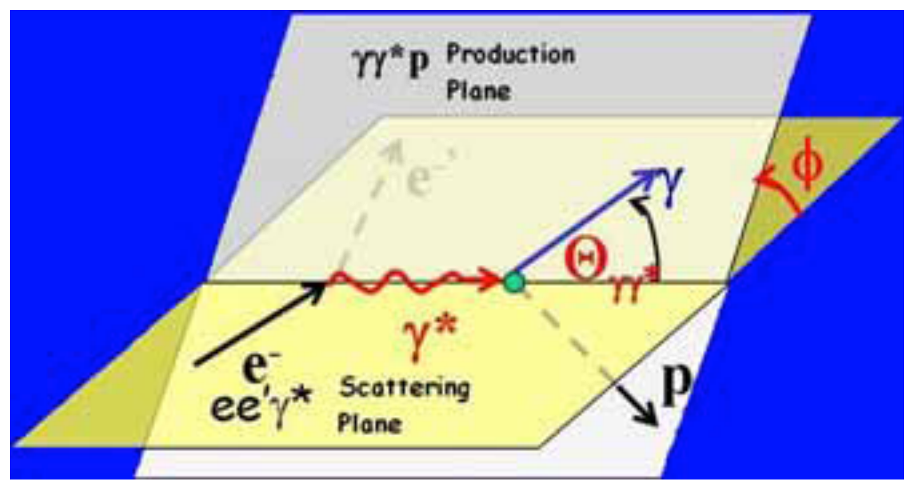
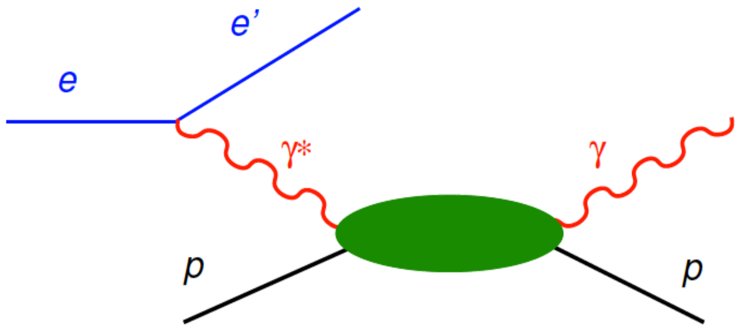
Bochum, March 7, 2013

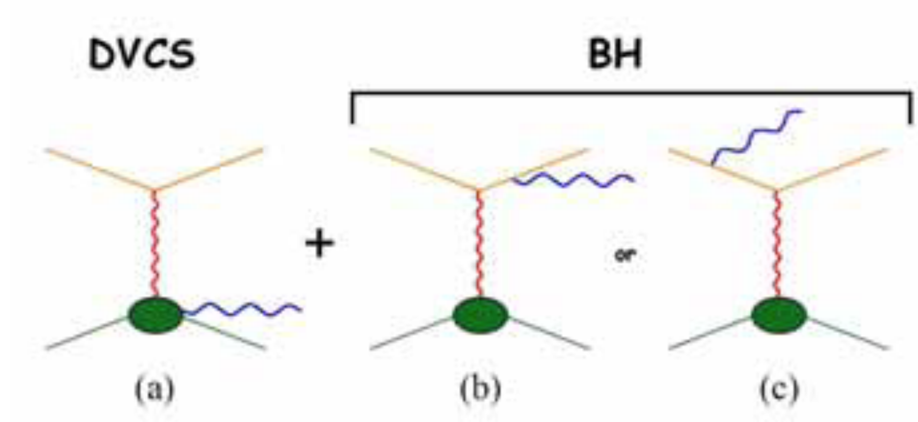
Hadron Physics at JLab

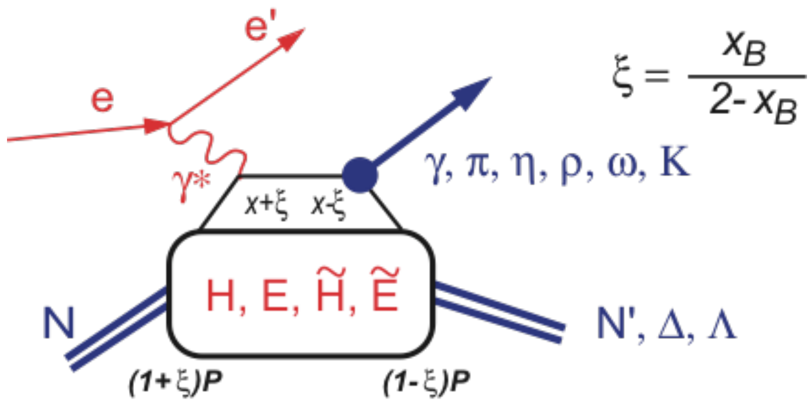


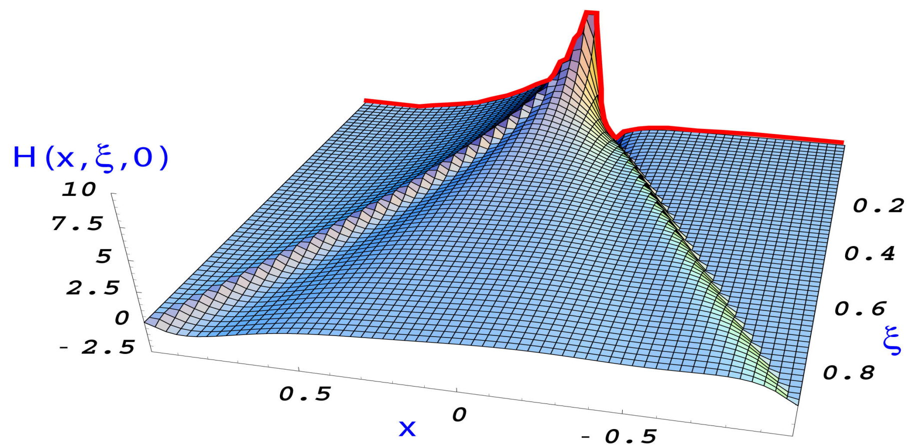












H, E - unpolarized, H̃, Ẽ - polarized GPD

The GPDs Define Nucleon Structure

Outline

- JLab Kinematics

($t < -|t_{min}| \neq 0$)

- Original Formulation of DVCS with GPDs

(Valid only at a limited t region)

- Comparison between

  Exact Tree-Level Result and the corresponding Results from Original Formulation

- Hadronic Tensors in DVCS

  Toward Generalization: Two Approaches

- Conclusion and Outlook

Review

International Journal of Modern Physics E
Vol. 22, No. 2 (2013) 1330002 (98 pages)

DOI: 10.1142/S0218301313300026

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CONCEPTUAL ISSUES CONCERNING GENERALIZED PARTON DISTRIBUTIONS

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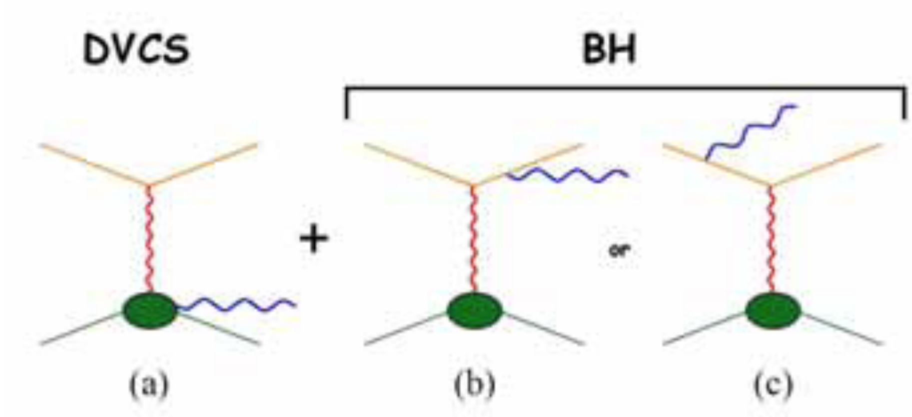
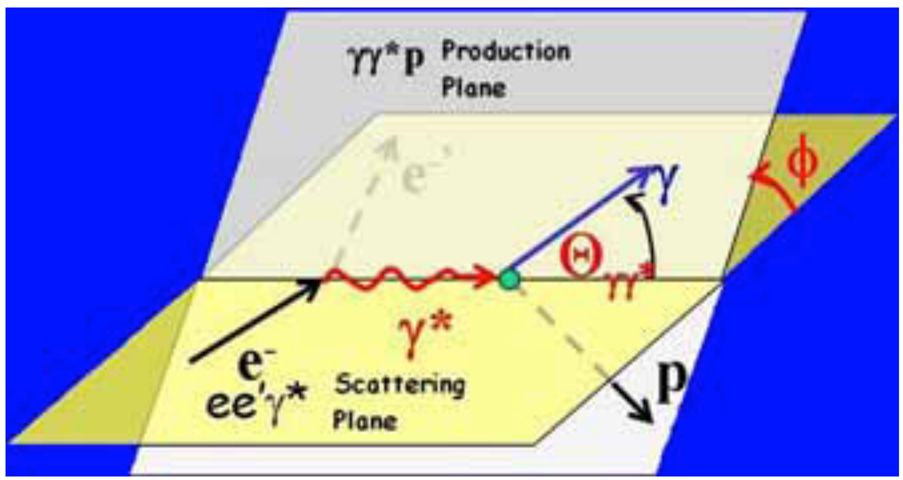
Received 15 July 2012
Revised 30 October 2012
Accepted 6 November 2012
Published 6 February 2013

JLab Kinematics $t < -|t_{min}| \neq 0$





$$t = \Delta^2 = -\frac{\varsigma^2 M^2 + \Delta_\perp^2}{1-\varsigma} \quad ; \quad \Delta^+ (\equiv \Delta^0 + \Delta^3) = \varsigma p^+ \quad ; \quad \Delta_\perp^2 > \Delta_{\perp\min}^2 \neq 0$$

Ranges of Kinematic Variables

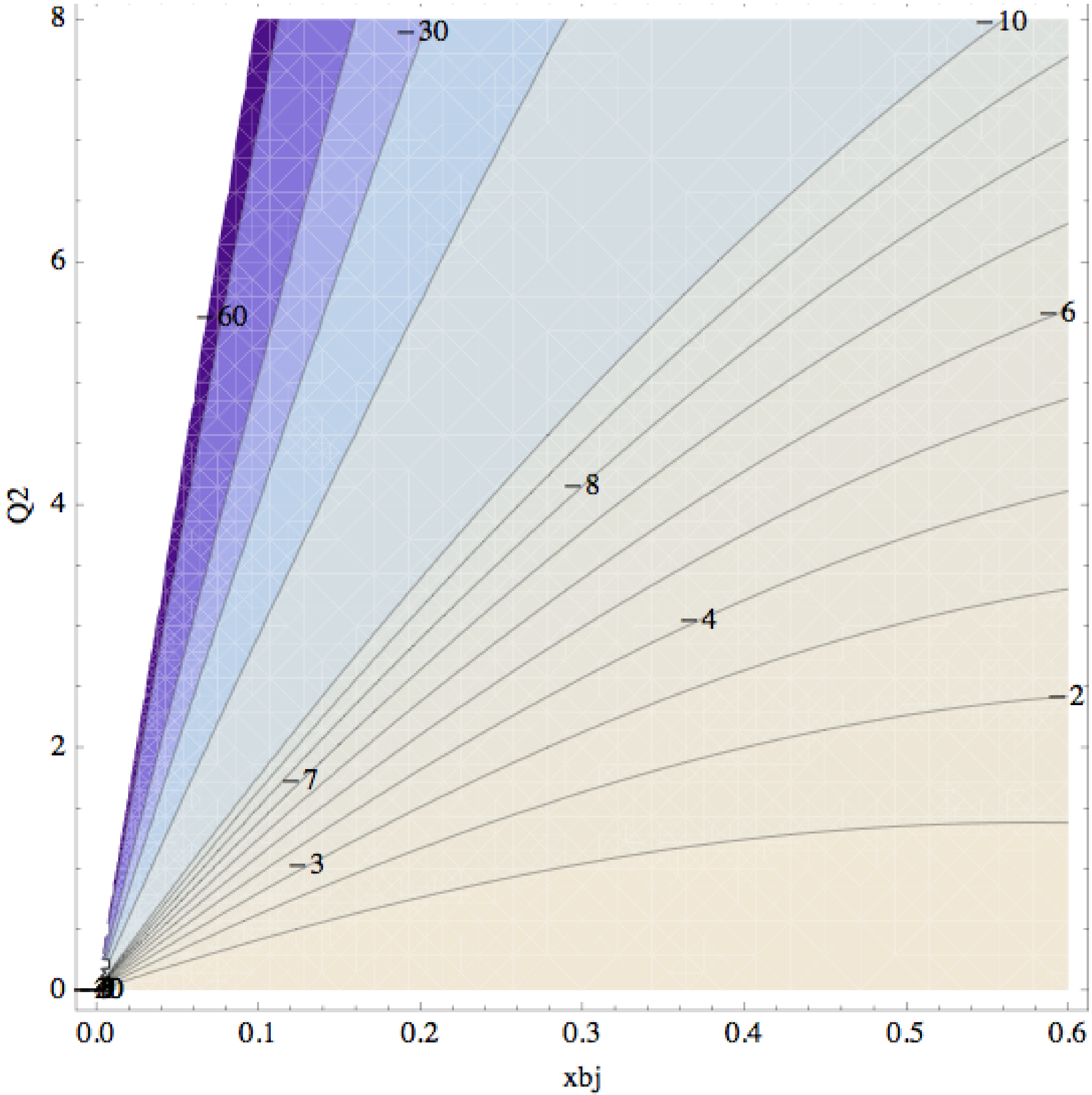
$$0 < Q^2 < \frac{4E^2 M}{2E + M}$$

$$\frac{Q^2}{2ME} < x_{Bj} = \frac{Q^2}{2p \cdot q} < 1$$

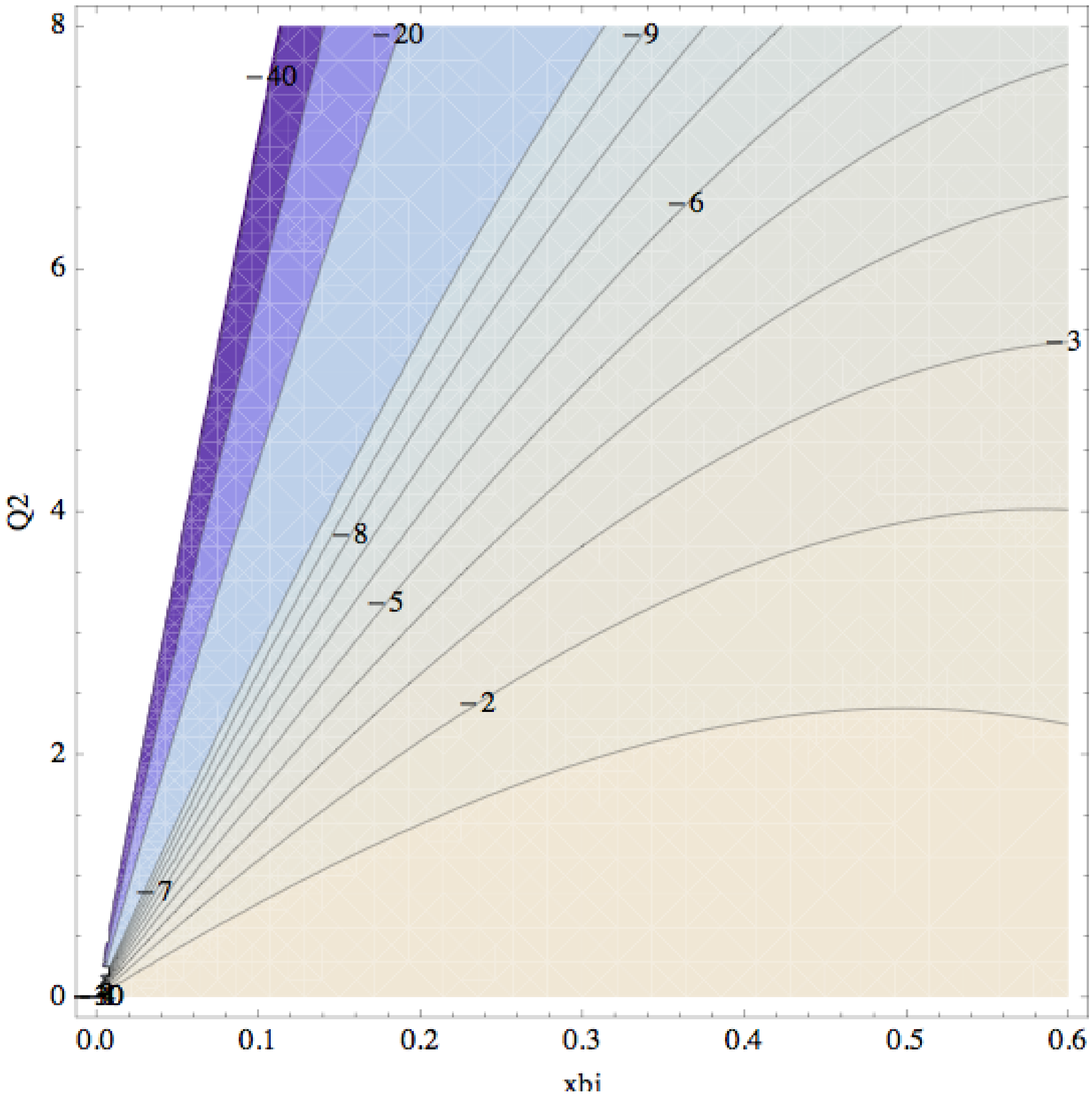
$$x_{Bj}\frac{-t}{Q^2} = \frac{Q^2 + 2x_{Bj}^2 M^2 - Q\sqrt{Q^2 + 4x_{Bj}^2 M^2}\cos\theta_\gamma}{Q^2 + 2x_{Bj} M^2 - Q\sqrt{Q^2 + 4x_{Bj}^2 M^2}\cos\theta_\gamma}$$

$$\cos\theta_\gamma = \frac{Q^4 + 2x_{Bj}^2 M^2 t + Q^2 x_{Bj}(t + 2x_{Bj} M^2)}{Q(Q^2 + x_{Bj} t)\sqrt{Q^2 + 4x_{Bj}^2 M^2}}$$

$\theta_\gamma = 28^\circ$



$\theta_{\gamma} = 14^{\circ}$



Nucleon GPDs in DVCS Amplitude

X.Ji,PRL78,610(1997): Eqs.(14) and (15)

$$p^\mu = \Lambda(\overset{ct}{1},\overset{x}{0},\overset{y}{0},\overset{z}{1}) \quad ,$$

$$n^\mu = (\overset{ct}{1},\overset{x}{0},\overset{y}{0},\overset{z}{-1})/(2\Lambda) \quad ,$$

$$\overline{P}^\mu = \frac{1}{2}(P+P')^\mu = p^\mu + \frac{M^2-\Delta^2/4}{2}n^\mu ,$$

$$q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi}n^\mu \quad , \quad \xi = \frac{Q^2}{2\overline{P}\cdot q} \quad ,$$

$$\Delta^\mu = -\xi\left[p^\mu - \frac{M^2-\Delta^2/4}{2}n^\mu\right] + \Delta_\perp^\mu \ .$$

$$T^{\mu\nu}(p,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int_{-1}^{+1}dx\left(\frac{1}{x-\frac{\xi}{2}+i\varepsilon}+\frac{1}{x+\frac{\xi}{2}-i\varepsilon}\right)$$
$$\times\left[H(x,\Delta^2,\xi)\overline{U}(P')\not{n}U(P)+E(x,\Delta^2,\xi)\overline{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$
$$-\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta\int_{-1}^{+1}dx\left(\frac{1}{x-\frac{\xi}{2}+i\varepsilon}-\frac{1}{x+\frac{\xi}{2}-i\varepsilon}\right)$$
$$\times\left[\tilde{H}(x,\Delta^2,\xi)\overline{U}(P')\not{n}\gamma_5U(P)+\tilde{E}(x,\Delta^2,\xi)\frac{\Delta\cdot n}{2M}\overline{U}(P')\gamma_5U(P)\right]$$

Just above Eq.(14),
``To calculate the scattering amplitude, it is convenient to define <u>a special system of coordinates</u>.”

Note here that $q'^2 = -\boldsymbol{\Delta}_\perp^2 = 0$.

Nucleon GPDs in DVCS Amplitude

A.V.Radyushkin, PRD56, 5524 (1997): Eq.(7.1)

$$q = q' - \zeta p \quad , \qquad \zeta = \frac{Q^2}{2p \cdot q'} \quad , \qquad r = p - p'$$

$$T^{\mu\nu}(p,q,q') = \frac{1}{2(p \cdot q')} \sum_a e_a^2 \Big[\left(-g^{\mu\nu} + \frac{1}{p \cdot q'} (p^\mu q'^\nu + p^\nu q'^\mu) \right) \times \left\{ \bar{u}(p') \not{q}' u(p) T_F^a(\zeta) + \frac{1}{2M} \bar{u}(p') (\not{q}' \not{r} - \not{r} \not{q}') u(p) T_K^a(\zeta) \right\} + i \varepsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q'_\beta}{p \cdot q'} \left\{ \bar{u}(p') \not{q}' \gamma_5 u(p) T_G^a(\zeta) + \frac{q' \cdot r}{2M} \bar{u}(p') \gamma_5 u(p) T_P^a(\zeta) \right\} \Big]$$

At the beginning of Section 2E (Nonforward distributions),
``Writing the momentum of the virtual photon as q=q’-ζp is equivalent to using the Sudakov decomposition in the light-cone `plus’ (p) and `minus’ (q’) components in a situation when there is no transverse momentum .”

Note here that $t = \Delta^2 = (\zeta P)^2 = \zeta^2 M^2 > 0$,i.e. only consistent at t=0, neglecting nucleon mass.

JLab Kinematics t < 0

- In JLab, the final hadron and final photon move off the z-axis.
- To see the effect of taking $t<0$, we mimic the kinematics at JLab and compute bare bone VCS amplitudes neglecting masses.

$$k'^{\mu} = \left((x-\zeta_{\text{eff}})P^{+}, \mathbf{\Delta}_{\perp}, \frac{\mathbf{\Delta}_{\perp}^{2}}{2(x-\zeta_{\text{eff}})P^{+})}\right)$$

$$q'^{\mu} = \left(\alpha\frac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}}P^{+}, -\mathbf{\Delta}_{\perp}, \frac{Q^{2}}{2\alpha P^{+}}\right)$$

The quantity ζ_{eff} is given by

$$\zeta_{\text{eff}} = \zeta + \alpha\frac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}} \to \zeta \text{ for } Q \to \infty$$

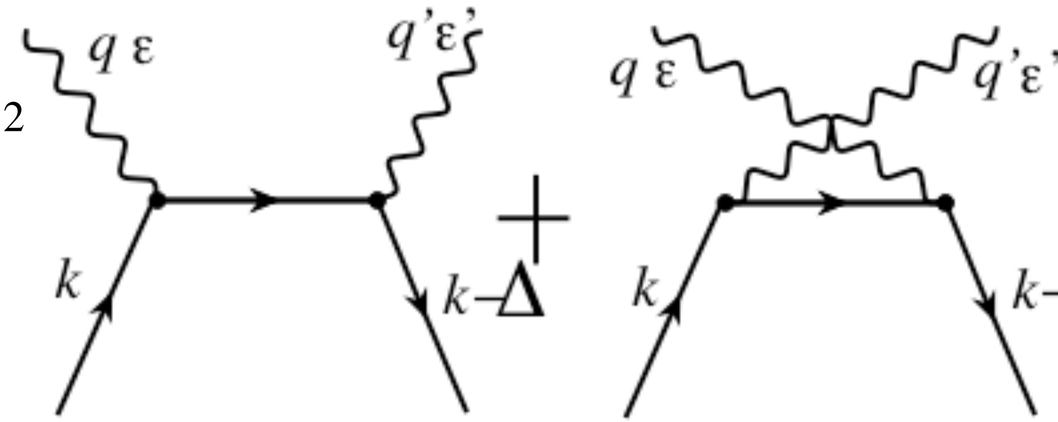
$$\alpha = \frac{x-\zeta}{2}\left(1-\sqrt{1-\frac{4\zeta}{x-\zeta}\frac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}}}\right) \to 0 \text{ for } Q \to \infty$$

B.L.G.Bakker and C.Ji, PRD83,091502(R) (2011); FBS 52, 285 (2012).

Bare Bone Structure

“Bare Bone” VCS Amplitude at Tree Level

$$S = (k+q)^2$$



$$U = (k-q')^2$$

$$= k'$$

Hadron Helicity Amplitude:

$$\mathrm{H}(h_q, h_{q'}, s_k, s_{k'}) = \varepsilon^*_\mu(q', h_{q'})\, \varepsilon_\nu(q, h_q) \left(T_S^{\mu\nu} + T_U^{\mu\nu}\right)$$

Neglecting masses,

$$T_S^{\mu\nu} = \frac{k_\alpha + q_\alpha}{S} \bar{u}(k', s_{k'}) \gamma^\mu \gamma^\alpha \gamma^\nu u(k, s_k)$$

$$T_U^{\mu\nu} = \frac{k_\alpha - q'_\alpha}{U} \bar{u}(k', s_{k'}) \gamma^\nu \gamma^\alpha \gamma^\mu u(k, s_k)$$

Identity: $\gamma^\mu \gamma^\alpha \gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\alpha\nu}\gamma^\mu - g^{\mu\nu}\gamma^\alpha + i\varepsilon^{\mu\alpha\nu\beta}\gamma_\beta \gamma_5$

Using Sudakov vectors

$$n(+)^\mu = (1,0,0,0), \; n(-)^\mu = (0,0,0,1)$$

we find

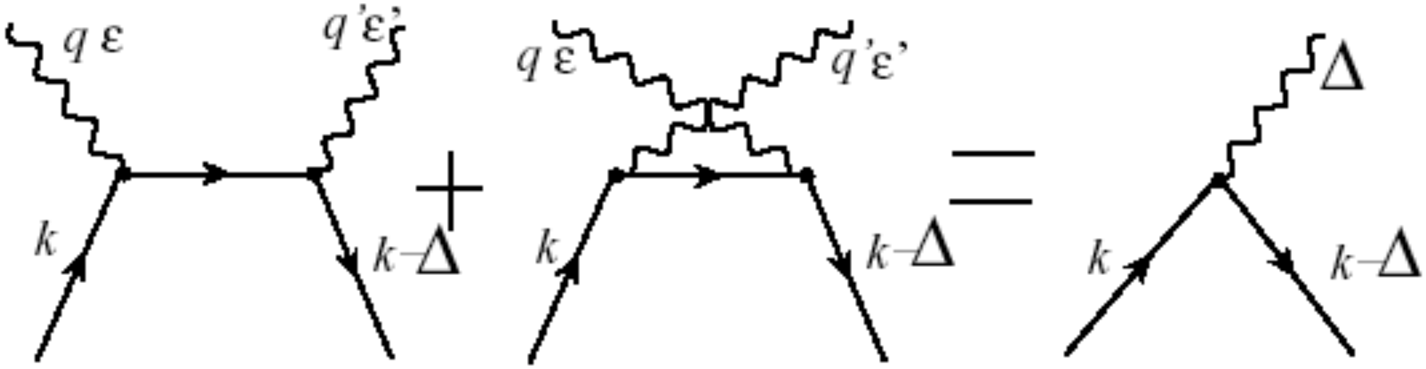
$$\begin{aligned} T_s{}^{\mu\nu} \;\; = \;\; & \frac{1}{s}\Big[\Big(\{(k^+ + q^+)n^\mu(+) + q^- n^\mu(-) + q_\perp{}^\mu\}n^\nu(+) \\ & +\{(k^+ + q^+)n^\nu(+) + q^- n^\nu(-) + q_\perp{}^\nu\}n^\mu(+) - g^{\mu\nu}q^-\Big) \\ & \times \bar{u}(k';s')\not{n}(-)u(k;s) \\ & -i\epsilon^{\mu\nu\alpha\beta}\{(k^+ + q^+)n_\alpha(+) + q^- n_\alpha(-) + q_{\perp\alpha}\}n_\beta(+) \\ & \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\Big] \,. \end{aligned}$$

Keeping no transverse momentum in DVCS, we agree on

$$\begin{aligned} T_s{}^{\mu\nu} \;\; = \;\; & \frac{q^-}{s}\Big[\{n^\mu(-)n^\nu(+) + n^\nu(-)n^\mu(+) - g^{\mu\nu}\} \\ & \times \bar{u}(k';s')\not{n}(-)u(k;s) \\ & -i\epsilon^{\mu\nu\alpha\beta}n_\alpha(-)n_\beta(+) \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\Big] \end{aligned}$$

equivalent to the expression given by X. Ji and A.V. Radyushkin.

Full Amp vs. Reduced Amp



S-channel:

$$\frac{\not\epsilon^*(\lambda')(\not k+\not q+m)\not\epsilon(\lambda)}{(k+q)^2-m^2} \longrightarrow \frac{\not\epsilon^*(\lambda')q^-\gamma^+\not\epsilon(\lambda)}{(x-\zeta)P^+q^-}$$

U-channel:

$$\frac{\not\epsilon(\lambda)(\not k'-\not q+m)\not\epsilon^*(\lambda')}{(k'-q)^2-m^2} \longrightarrow -\frac{\not\epsilon(\lambda)q^-\gamma^+\not\epsilon^*(\lambda')}{x\,P^+q^-}$$

Calculation for massless spinors

Complete amplitude

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\} h) \frac{1}{q^2} \mathcal{H}(\{s', s\}\{h', h\}),$$

Leptonic and hadronic parts

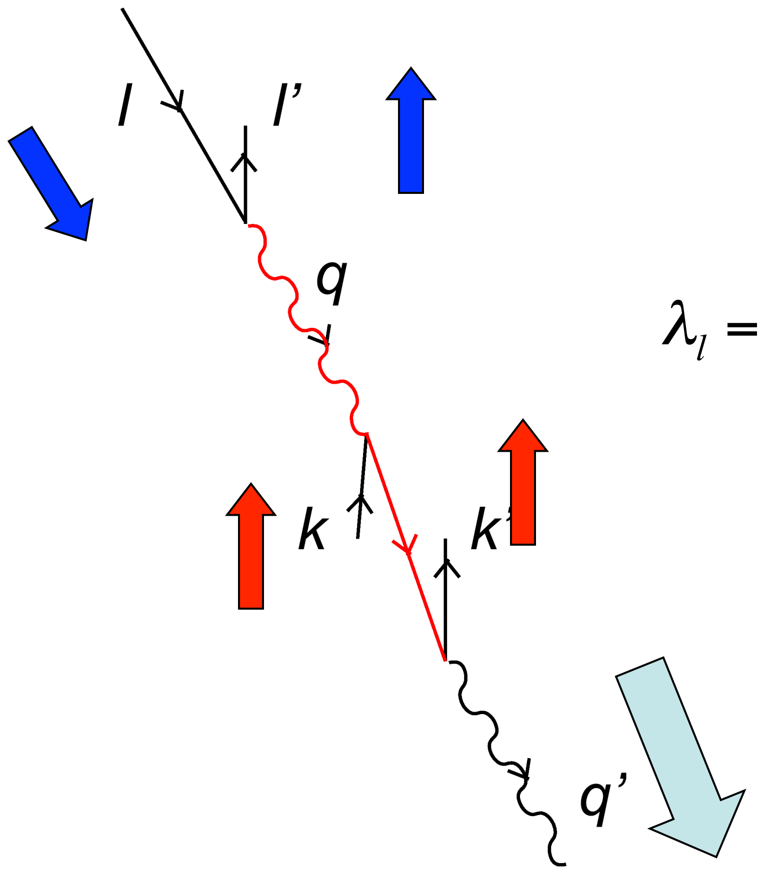
$$\begin{aligned} \mathcal{L}(\{\lambda', \lambda\} h) &= \bar{u}_{\mathrm{LF}}(\ell'; \lambda') \not\epsilon^{*}(q; h) u_{\mathrm{LF}}(\ell; \lambda), \\ \mathcal{H}(\{s', s\}\{h', h\}) &= \bar{u}_{\mathrm{LF}}(k'; s')(\mathcal{O}_s + \mathcal{O}_u) u_{\mathrm{LF}}(k; s), \end{aligned}$$

Operators

$$\begin{aligned} \mathcal{O}_s &= \frac{\not\epsilon^{*}_{\mathrm{LF}}(q'; h')(\not k + \not q)\not\epsilon_{\mathrm{LF}}(q; h)}{(k+q)^2}, \\ \mathcal{O}_u &= \frac{\not\epsilon_{\mathrm{LF}}(q; h)(\not k - \not q')\not\epsilon^{*}_{\mathrm{LF}}(q'; h')}{(k-q')^2} \end{aligned}$$

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.

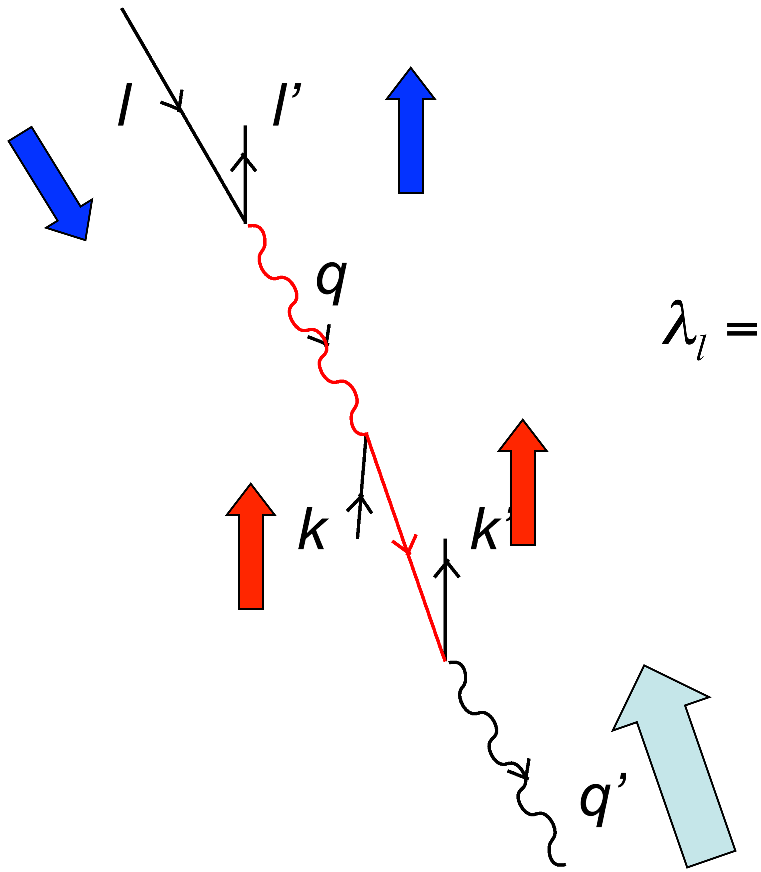


$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\; s_k = s_{k'} = +\frac{1}{2}\,,\; h_{q'} = +1\,;$$

Allowed !

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\ s_k = s_{k'} = +\frac{1}{2}\,,\ h_{q'} = -1\,;$$

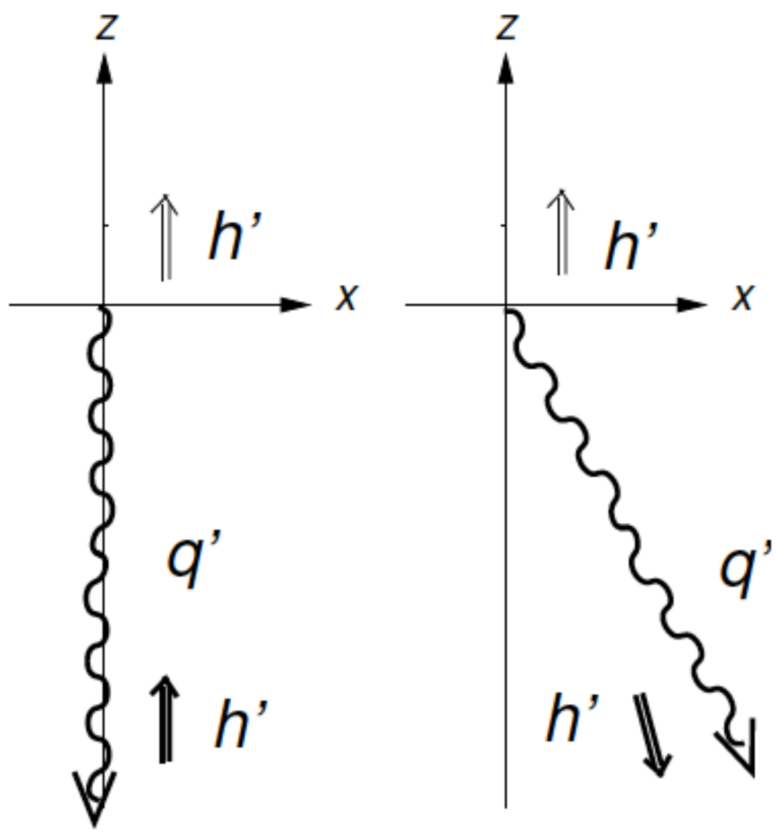
Prohibited !

Comparison

Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda',\lambda\},h)\frac{1}{q^2}\mathcal{H}(\{h',h\}\{s',s\})$ in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless, $\lambda'=\lambda$ and $s'=s$.

λ	h'	s	this work	AVR	XJ
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{4}{Q}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}\right)$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\boldsymbol{\Delta}_\perp^2}{Q^2}\right)$

Swap



Definition of Light-Front Helicity

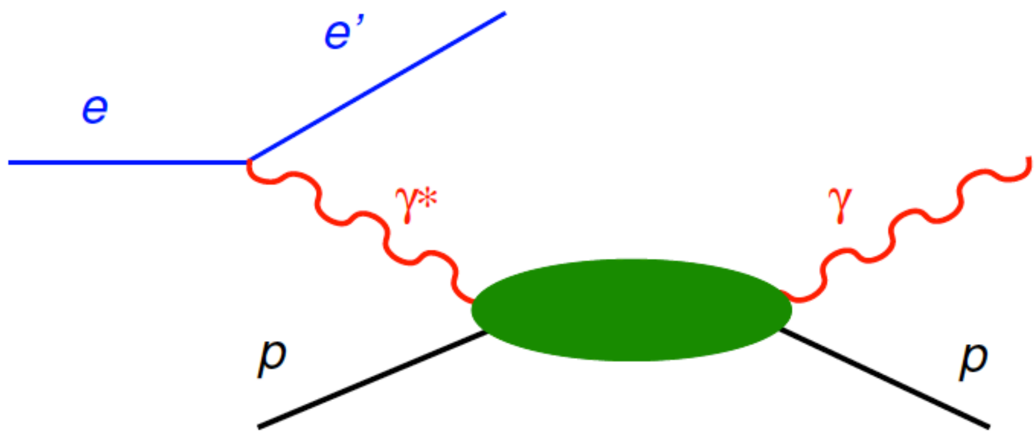
C.Carlson and C.Ji, Phys.Rev.D67,116002 (2003);
B.Bakker and C.Ji, Phys.Rev.D83,091502(R) (2011).

For any orders in Q

λ	h'	s	Exact $\mathcal{A}=\Sigma\mathcal{L}\frac{1}{q^2}\mathcal{H}$	Reduced $\mathcal{A}_{\rm red}=\Sigma\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\rm red}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$4\sqrt{\frac{x}{(x-\zeta)D_+}}\frac{Q^3}{Q^4-4(\zeta p^+)^4}$	$-4(\zeta p^+)^2\sqrt{\frac{x-\zeta}{xD_+}}\frac{4Q\Delta(\zeta p^+)^2-D_-Q^4}{\Delta(Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$2\frac{2Q\{Q^3(x-\zeta)-4\Delta\zeta((\zeta p^+)^2\}-D_-\{Q^4(x-\zeta)-4\zeta((\zeta p^+)^4\}}{\sqrt{x(x-\zeta)D_+}Q(Q^4-4(\zeta p^+)^4)}$	$-8\sqrt{\frac{xD_+}{x-\zeta}}\frac{(\zeta p^+)^4}{Q(Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	-1	$\frac{1}{2}$	$2\frac{4(\zeta p^+)^2\{2Q\Delta\zeta-(\zeta p^+)^2(x-\zeta)D_+\}-D_-Q^4\zeta}{\sqrt{x(x-\zeta)D_+}Q(Q^4-4(\zeta p^+)^4)}$	$2\sqrt{\frac{xD_+}{x-\zeta}}\frac{Q^3}{Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	$-16\sqrt{\frac{x}{(x-\zeta)D_+}}\frac{(\zeta p^+)^4}{Q(Q^4-4(\zeta p^+)^4)}$	$4\sqrt{\frac{x-\zeta}{xD_+}}\frac{Q^3\Delta-(\zeta p^+)^2D_-Q^2}{\Delta(Q^4-4(\zeta p^+)^4)}$

$$D=\frac{4\zeta\Delta^2}{(x-\zeta)Q^2},\quad D_\pm=1\pm\sqrt{1-D}.$$

Number of Independent Amplitudes in VCS



Nucleon Target

$$3 \times 2 \times 2 \times 2/2 = 12$$

12 independent tensor structures

M.Perrottet, Lett. Nuovo Cim. 7, 915 (1973);
R.Tarrach, Nuovo Cim. 28A, 409 (1975);
D.Drechsel et al.,PRC57,941(1998);
A.V.Belitsky, D.Mueller and A.Kirchner, NPB629, 323(2002);
A.V.Belitsky and D.Mueller, PRD82, 074010(2010)

A.V.Belitsky and D. Mueller,arXiv:1005.5209v1[hep-ph]

$$T_{\mu\nu} = -\mathcal{P}_{\mu\sigma} g_{\sigma\tau} \mathcal{P}_{\tau\nu} \frac{q \cdot V_1}{p \cdot q} + (\mathcal{P}_{\mu\sigma} p_\sigma \mathcal{P}_{\rho\nu} + \mathcal{P}_{\mu\rho} p_\sigma \mathcal{P}_{\sigma\nu}) \frac{V_{2\,\rho}}{p \cdot q} - \mathcal{P}_{\mu\sigma} i \varepsilon_{\sigma\tau q\rho} \mathcal{P}_{\tau\nu} \frac{A_{1\,\rho}}{p \cdot q},$$

$$\mathcal{P}_{\mu\nu} = g_{\mu\nu} - \frac{q_{1\mu} q_{2\nu}}{q_1 \cdot q_2},$$

$$V_{1\,\rho} = \frac{1}{p \cdot q} \bar{u}_2 \left(\not{q} \left[p_\rho \mathcal{H} + \Delta_{\perp\rho} \mathcal{H}_+^3 \right] + i\sigma_{\mu\nu} \frac{q_\mu \Delta_\nu}{2M} \left[p_\rho \mathcal{E} + \Delta_{\perp\rho} \mathcal{E}_+^3 \right] + \tilde{\Delta}_{\perp\rho} \left[\not{q} \widetilde{\mathcal{H}}_-^3 + \frac{q \cdot \Delta}{2M} \widetilde{\mathcal{E}}_-^3 \right] \gamma_5 \right) u_1,$$

$$A_{1\,\rho} = \frac{1}{p \cdot q} \bar{u}_2 \left(\not{q} \gamma_5 \left[p_\rho \widetilde{\mathcal{H}} + \Delta_{\perp\rho} \widetilde{\mathcal{H}}_+^3 \right] + \frac{q \cdot \Delta}{2M} \gamma_5 \left[p_\rho \widetilde{\mathcal{E}} + \Delta_{\perp\rho} \widetilde{\mathcal{E}}_+^3 \right] + \tilde{\Delta}_{\perp\rho} \left[\not{q} \mathcal{H}_-^3 + i\sigma_{\mu\nu} \frac{q_\mu \Delta_\nu}{2M} \mathcal{E}_-^3 \right] \right) u_1,$$

$$V_{2\,\rho} = \xi \left(V_{1\,\rho} - \frac{p_\rho}{2} \frac{q \cdot V_1}{p \cdot q} \right) + \frac{i}{2} \frac{\varepsilon_{\rho\sigma\Delta q}}{p \cdot q} A_{1\,\sigma},$$

$$\Delta_\rho^\perp \equiv \Delta_\rho - \frac{\Delta \cdot q}{p \cdot q} p_\rho \quad \text{and} \quad \tilde{\Delta}_\rho^\perp \equiv \frac{i\,\varepsilon_{\rho\Delta p q}}{p \cdot q}$$

Biproducts of P,V,A,T, but not S

D.Drechsel,G.Knoechlein,A.Yu.Korchin,A.Metz and S.Scherer,
PRC 57, 941 (1998)

$$\mathcal{M}_B^{\gamma^*\gamma} = -ie^2\bar{u}(p_f,S_f)\sum_{i=1}^{12}\varepsilon_\mu\rho_i^{\mu\nu}\varepsilon_\nu'^* f_i(q^2,q\cdot q',q\cdot P)u(p_i,S_i)$$

$$\begin{aligned}
\varepsilon_\mu\rho_1^{\mu\nu}\varepsilon_\nu'^* &= \varepsilon_\mu\tilde{T}_1^{\mu\nu}\varepsilon_\nu'^* \\
&= \varepsilon\cdot q'\varepsilon'^*\cdot q - q\cdot q'\varepsilon\cdot\varepsilon'^*\,, \\
\varepsilon_\mu\rho_2^{\mu\nu}\varepsilon_\nu'^* &= \varepsilon_\mu\tilde{T}_2^{\mu\nu}\varepsilon_\nu'^* \\
&= q\cdot P(\varepsilon\cdot P\varepsilon'^*\cdot q + \varepsilon'^*\cdot P\varepsilon\cdot q') - q\cdot q'\varepsilon\cdot P\varepsilon'^*\cdot P - (q\cdot P)^2\varepsilon\cdot\varepsilon'^*\,, \\
\varepsilon_\mu\rho_3^{\mu\nu}\varepsilon_\nu'^* &= \varepsilon_\mu\tilde{T}_4^{\mu\nu}\varepsilon_\nu'^* \\
&= q\cdot Pq^2\varepsilon\cdot\varepsilon'^* - q\cdot P\varepsilon\cdot q\varepsilon'^*\cdot q - q^2\varepsilon'^*\cdot P\varepsilon\cdot q' + q\cdot q'\varepsilon'^*\cdot P\varepsilon\cdot q\,, \\
\varepsilon_\mu\rho_4^{\mu\nu}\varepsilon_\nu'^* &= \varepsilon_\mu\tilde{T}_7^{\mu\nu}\varepsilon_\nu'^* \\
&= \varepsilon\cdot P\varepsilon'^*\cdot PQ\cdot\gamma - q\cdot P(\varepsilon\cdot P\varepsilon'^*\cdot\gamma + \varepsilon'^*\cdot P\varepsilon\cdot\gamma) \\
&\quad + iq\cdot P\gamma_5\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\mu\varepsilon_\nu'^* Q_\alpha\gamma_\beta\,,
\end{aligned}$$

...

Biproducts of S,V,A,T, but not P

Possible Reconciliation

Gordon Decomposition and Extension

$$(p+p')_\mu \quad \leftrightarrow \quad 2M\gamma_\mu - i\sigma_{\mu\nu}q^\nu$$

$$i\varepsilon_{\mu\nu\alpha\beta}\gamma^5\gamma^\nu p^\alpha p'^\beta \quad \leftrightarrow \quad \frac{q^2}{2}\gamma_\mu - iM\sigma_{\mu\nu}q^\nu$$

$$q_\mu\gamma^5 \quad \leftrightarrow \quad 2M\gamma_\mu\gamma^5 + \varepsilon_{\mu\nu\alpha\beta}(p+p')_\nu\sigma_{\alpha\beta}$$

All are equivalent!

$$J^{\mu} = \gamma^{\mu} F_1 + i\frac{\sigma^{\mu\nu} q_{\nu}}{2M} F_2$$ V T

$$= \gamma^{\mu}(F_1 + F_2) + \frac{(p+p')^{\mu}}{2M} F_2$$ V S

$$= \frac{(p+p')^{\mu}}{2M}\frac{4M^2 F_1 + q^2 F_2}{4M^2 - q^2} - i\varepsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_{\nu} p_{\alpha} p'_{\beta}\frac{2(F_1+F_2)}{4M^2 - q^2}$$ S A

$$= \frac{(p+p')^{\mu}}{2M} F_1 + i\frac{\sigma^{\mu\nu} q_{\nu}}{2M}(F_1 + F_2)$$ S T

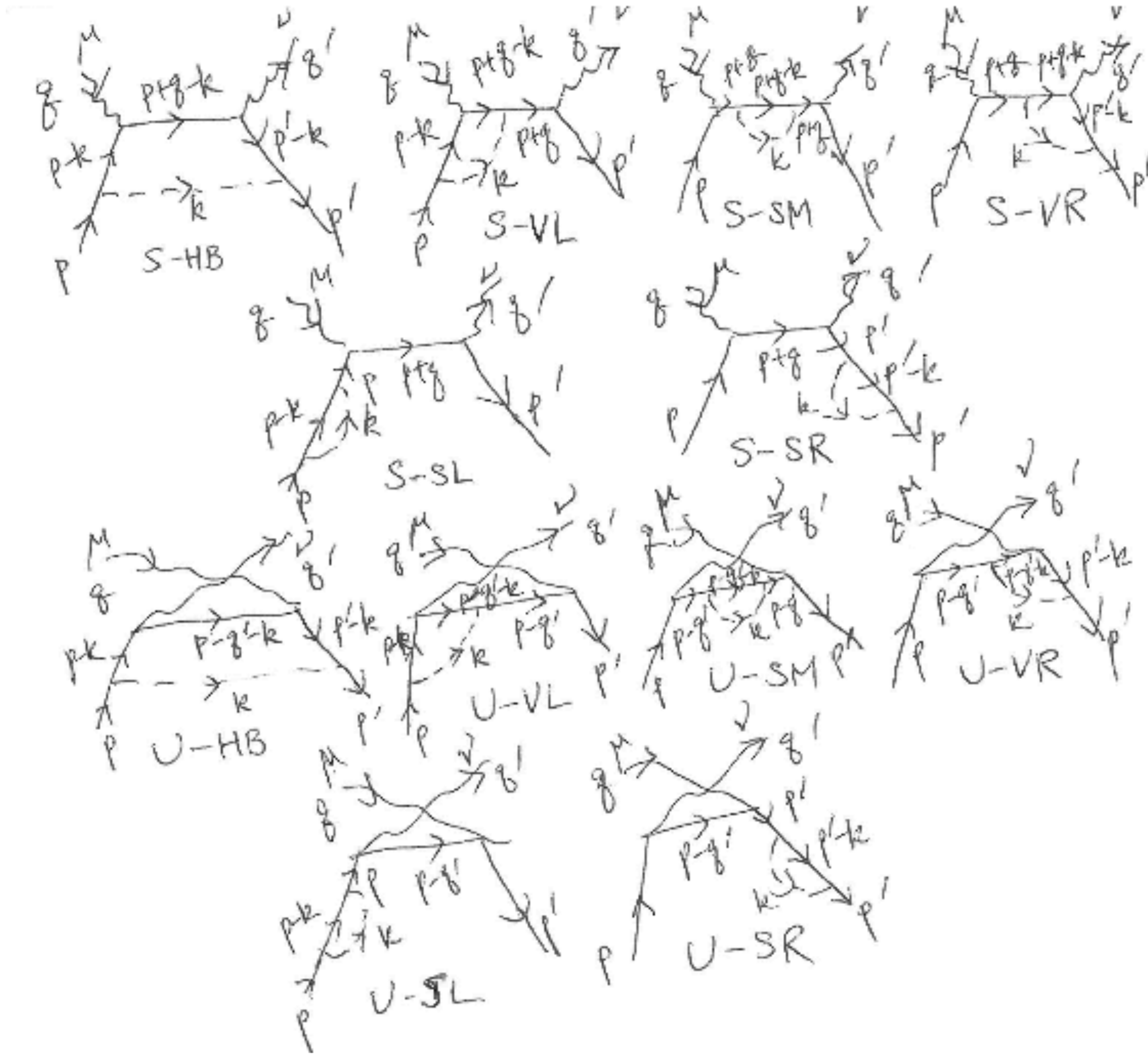
$$= \gamma^{\mu}(F_1 + \frac{q^2}{4M^2} F_2) - i\varepsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_{\nu} p_{\alpha} p'_{\beta}\frac{F_2}{2M^2}$$ V A

$$= i\frac{\sigma^{\mu\nu} q_{\nu}}{2M}(\frac{4M^2}{q^2} F_1 + F_2) + i\varepsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_{\nu} p_{\alpha} p'_{\beta}\frac{2F_1}{q^2}$$ T A

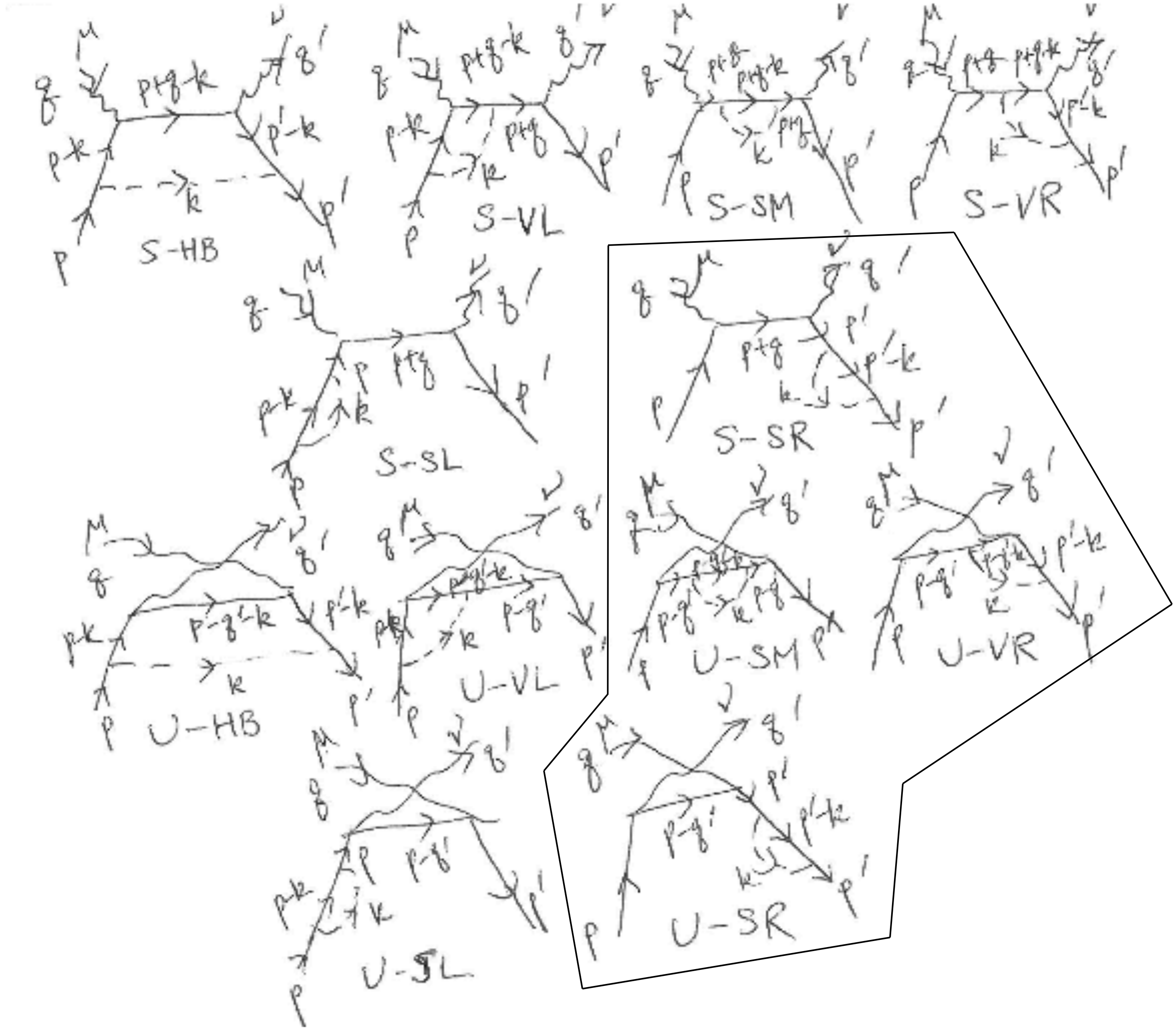
Conclusion and Outlook

- We find that the XJ and AVR amplitudes for DVCS in terms of GPDs for $t < 0$ are not satisfactory.
- The determination of all independent structures is important for the discussion of GPDs.
- Maintaining EM gauge invariance is an important constraint.
- If all invariant structures are identified, the question whether one can measure GPDs in experiments where Q^2 does not go to infinity may become more focused.

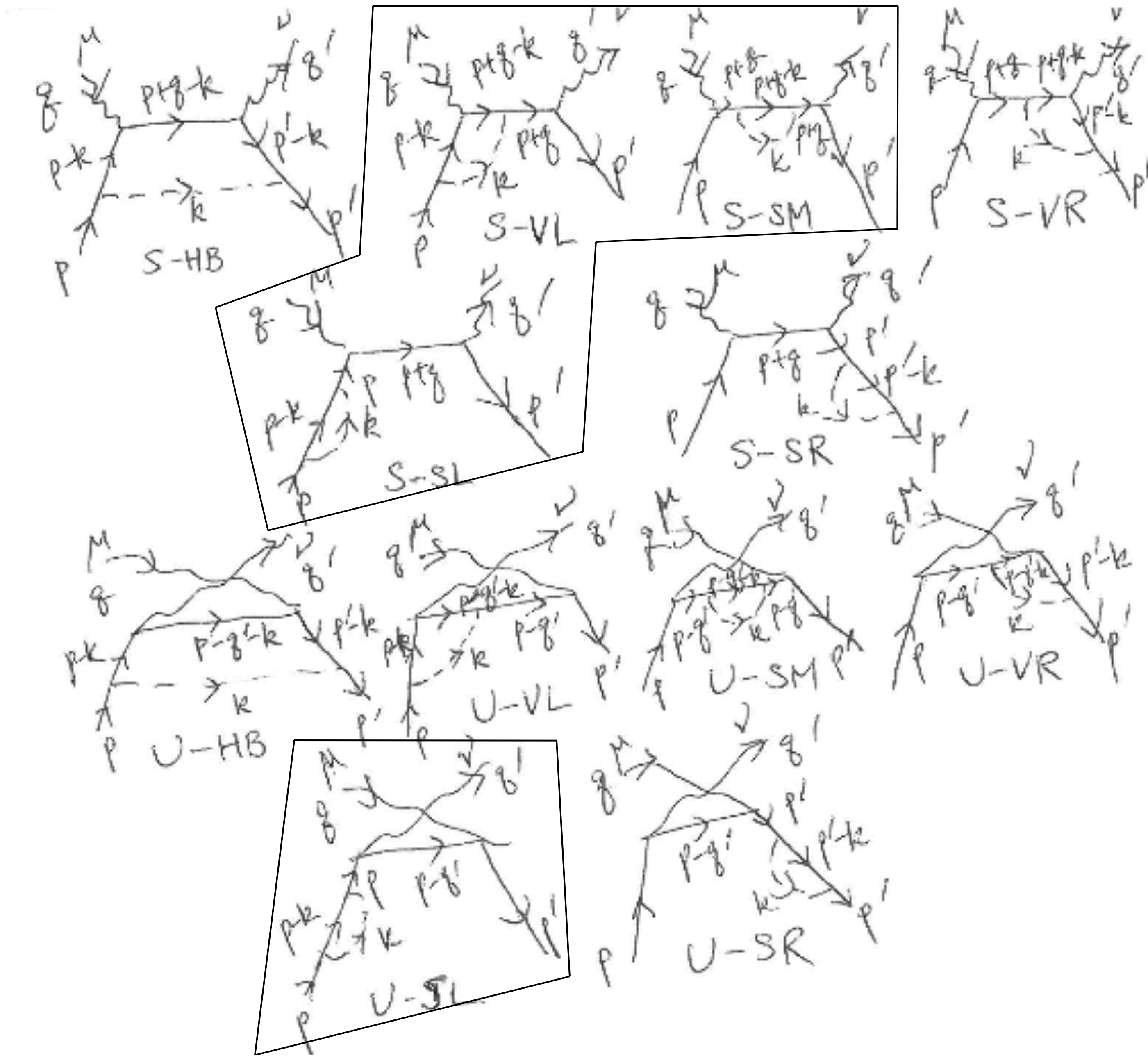
Gauge Invariance Check in One-Loop Level



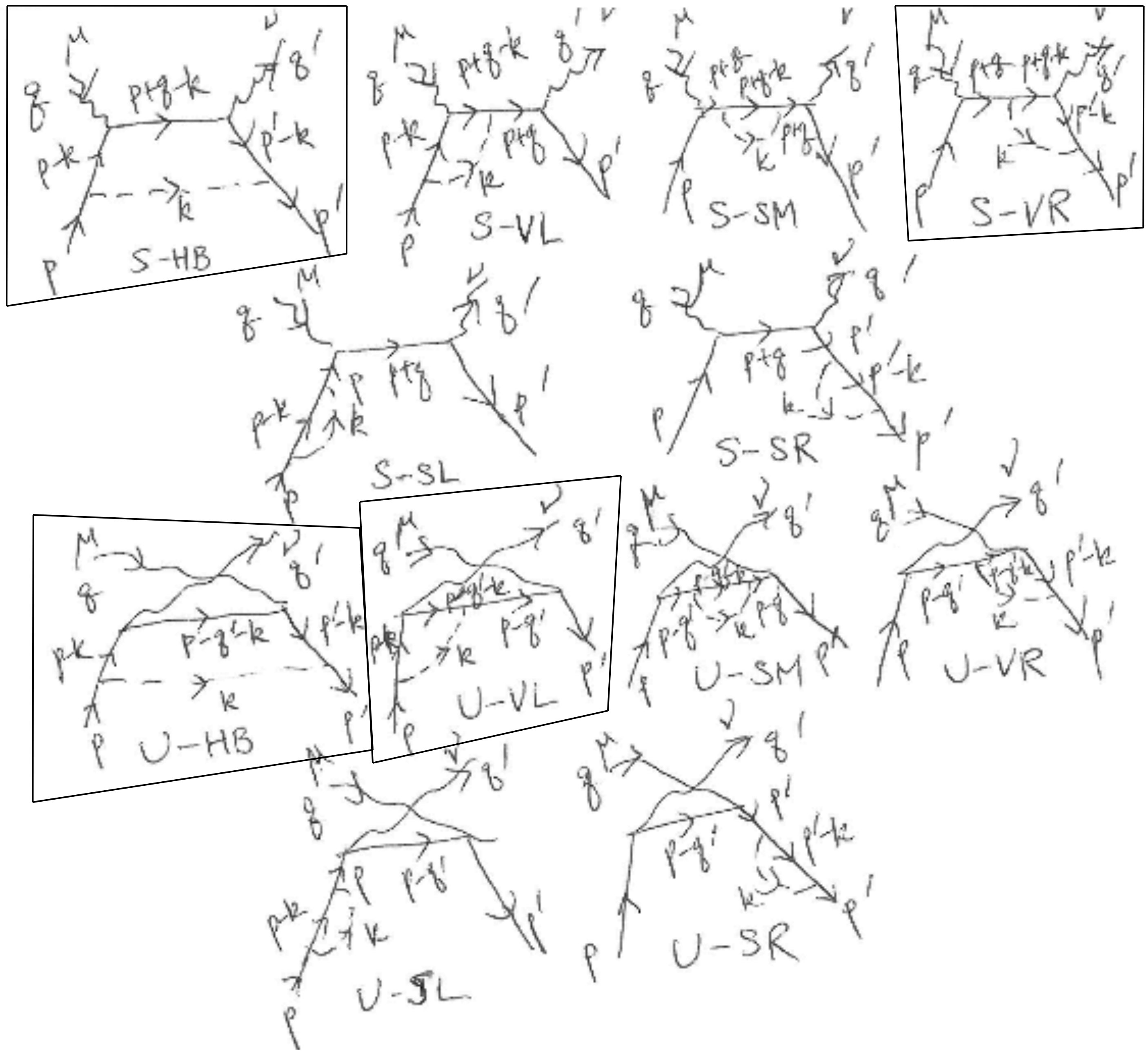
Gauge Invariance Check in One-Loop Level



Gauge Invariance Check in One-Loop Level



Gauge Invariance Check in One-Loop Level

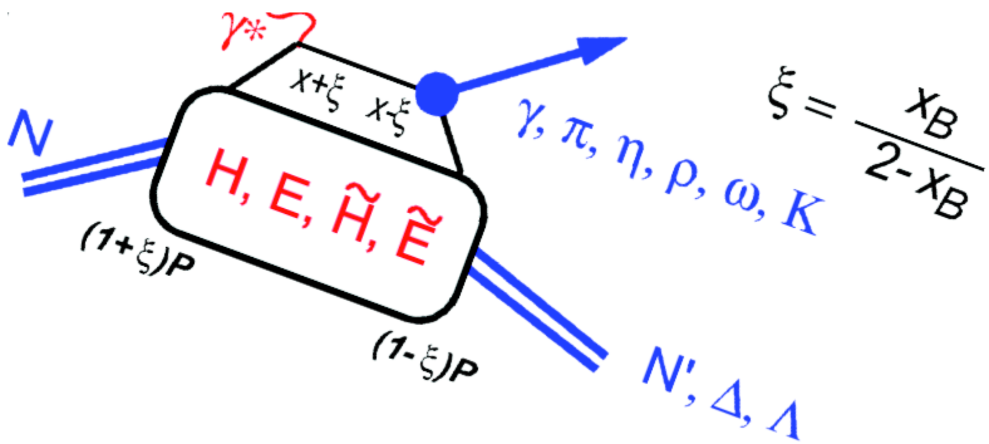


Investigation of Complete Ampltude

Attach the lepton current and check the spin filter for the DVCS amplitude.

$$\epsilon_{LF}(q;\pm 1) = \frac{1}{\sqrt{2}}\left(0, \mp 1, -i, \mp \frac{q_x \pm i q_y}{q^+}\right)$$

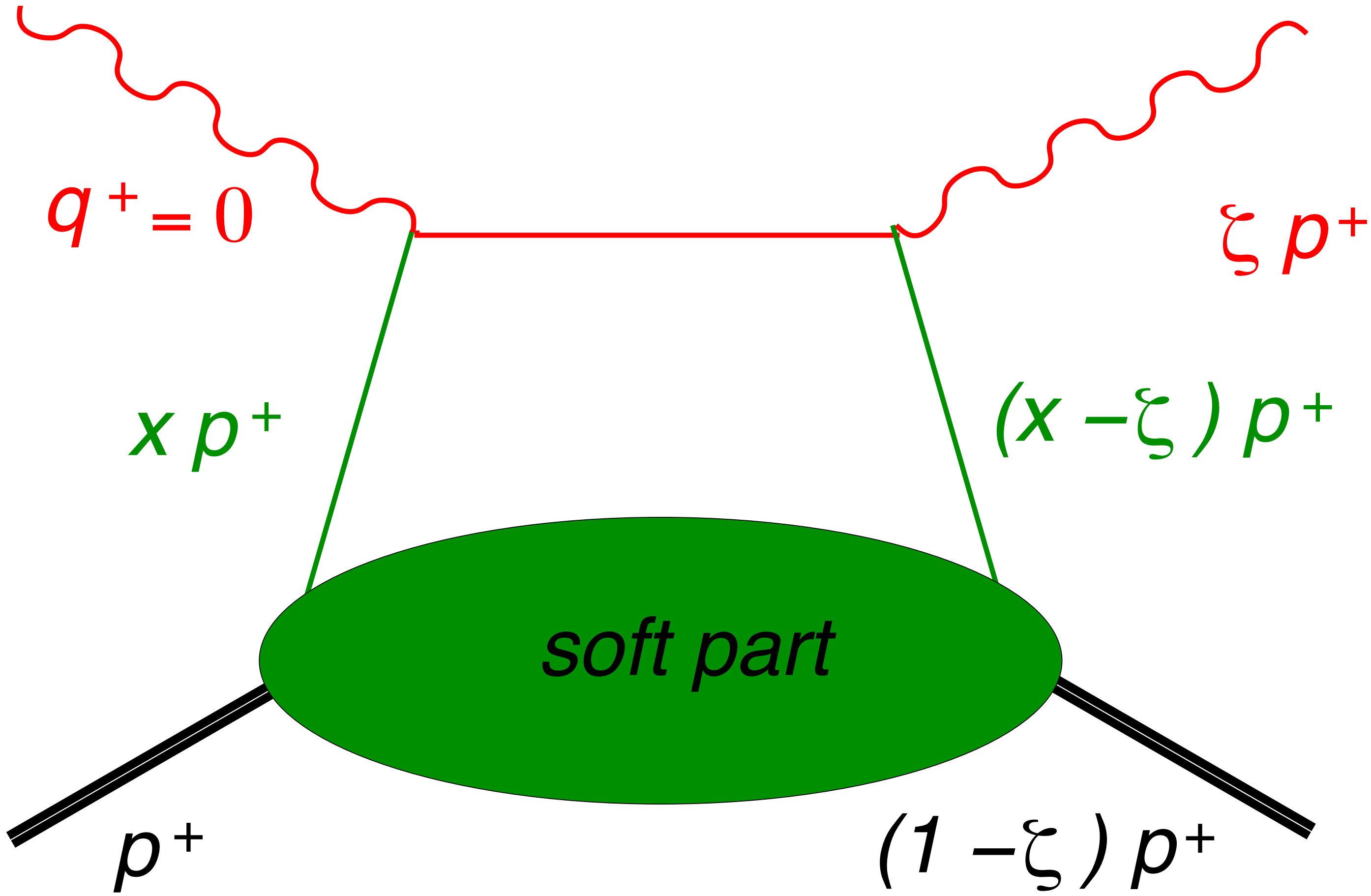
$$\epsilon_{LF}(q;0) = \frac{1}{\sqrt{q^2}}\left(q^+, q_x, q_y, \frac{q_\perp^2 - q^2}{2q^+}\right)$$



Singularities develop in the polarization vector as $q^+ \to 0$.

The amplitudes being obtained by contraction with the polarization vectors may be sensitive to the neglected parts.

GPDs rely on the handbag dominance in DVCS; i.e.
Q^2 >> any soft mass scale



$q^2 = q^+q^- - q_\perp^2 = -q_\perp^2 = -Q^2 < 0$, e.g.
S.J.Brodsky,M.Diehl,D.S.Hwang, NPB596,99(01)