Evolution of light-like Wilson loops and transverse momentum dependent correlators

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[Based on works with T. Mertens, F.F. Van der Veken]

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Gauge-Invariant Collinear Correlators

$$\Phi(k^+, \mu) \sim \int \frac{dz^-}{2\pi} \, \mathrm{e}^{-ik^+z^-} \langle h|\bar{\psi}(z^-, 0_\perp)\mathcal{W}_{n^-}[z^-, 0^-]\psi(0^-, 0_\perp)|h\rangle$$

Gauge invariance is saved by the **light-like Wilson line**

$$\mathcal{W}\,[y, x]^\Gamma_w = \mathcal{P}\exp\left[-ig\int_\Gamma d\tau\, w^\mu \mathcal{A}^a_\mu(w\tau)\right]$$

Saving gauge invariance, we get **path-dependence**: very important!

"Animal Farm" rule for the field correlators: [almost] all correlators are singular, but those on the light-cone are [expected to be] more singular than others.

Transverse Momentum-Depedent Correlators

"Trial" TMD with the light-like and transverse gauge links

[Belitsky, Ji, Yuan (2003); Boer, Mulders, Pijlman (2003)]

$$\Phi(k^+, k_\perp; \mathrm{scales}) \sim$$

$$\int dz^- d^2 z_\perp \ \ \mathrm{e}^{-ikz} \cdot \langle h|\bar{\psi}(z)\ \mathcal{W}_{n\cup l_\perp}[z^-, z_\perp; 0^-, 0_\perp]\psi(0)|h\rangle$$

Tree-level:

$$\Phi^{(0)}(k^+, k_\perp) = \delta(k^+ - p^+)\delta^{(2)}(k_\perp)$$

$$\int d^2 k_\perp \Phi(k^+, k_\perp) = \Phi(k^+) = \mathrm{collinear\ limit}$$

$$\Phi(k^+, \mu) = \int dz^- \ \mathrm{e}^{-ik^+ z^-} \langle h|\bar{\psi}(z)\ \mathcal{W}_n[z^-, 0^-]\psi(0)|h\rangle$$

One-loop corrections: $\rightarrow$ emergent (light-cone/rapidity) singularities!

Classification of Singularities

- Ultraviolet poles $\sim \frac{1}{\varepsilon}$: removed by the standard renormalization procedure;
- Overlapping divergences: contain the UV and rapidity poles simultaneously $\sim \frac{1}{\varepsilon} \ln\theta$: generalized renormalization procedure
- Pure rapidity divergences: $\sim \ln^{1,2}\theta$: can be safely summed up by means of the Collins-Soper equation.
- Specific self-energy divergences: stem from the gauge links, do not affect rapidity evolution; treated by modifications of the soft factors

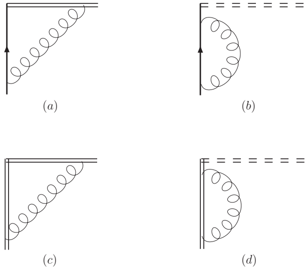
[ICh, Stefanis (2008, 2009, 2010); Collins (2003, 2008, 2011 etc.); Chiu, Jain, Neill, Rothstein (2011, 2012); Avsar (2012) Idilbi, Scimemi (2011, 2012)]

The similar classes of singularities arise in the collinear case as well. However, they cancel in the interplay of the virtual and real gluon contributions:

[Furmanski, Curci, Petronzio (1980); Fleming, Zhang (2012)]

Extra Divergences in TMD Correlators

One-loop corrections:



$$[\text{covariant}] = -\frac{\alpha_s}{\pi} C_{\rm F} \Gamma(\epsilon) \left[4\pi \frac{\mu^2}{-p^2}\right]^{\epsilon} \delta(1-x)\delta^{(2)}(k_\perp) \int_0^1 dx \, \frac{x^{1-\epsilon}}{(1-x)^{1+\epsilon}}$$

$$[\text{lightcone}] = -\frac{\alpha_s}{\pi} C_{\rm F} \, \Gamma(\epsilon) \left[4\pi \frac{\mu^2}{-p^2}\right]^{\epsilon} \delta(1-x)\delta^{(2)}(k_\perp) \int_0^1 dx \frac{(1-x)^{1-\epsilon}}{x^{\epsilon}[x]}$$

TMD Definitions

- A_v-TMD: axial non-light-like gauge $(v \cdot A) = 0$; L_v-gauge links vanish; rapidity cutoff: $\zeta = (2P \cdot v)^2/|v^2| \rightarrow$ Collins, Soper
- C_v-TMD: covariant gauge; L_v-gauge links survive; rapidity cutoff: $\zeta = (2P \cdot v)^2/|v^2| \rightarrow$ Ji, Ma, Yuan
- A_n-TMD: light-like axial gauge $(n \cdot A) = 0 \ , \ n^2 = 0$; L_n-gauge links vanish; T-gauge links survive; regularization parameter: $\theta = (P \cdot n)/\eta$ from the gluon propagator; light-like gauge links in the soft factor $\rightarrow$ ICh, Stefanis
- C_n-TMD: covariant gauge; L_n-gauge links survive; T-gauge links vanish; regularization $\zeta = (2P \cdot v)^2/|v^2|$, $v^2 \neq 0$ in the soft factor $\rightarrow$ Collins, Hautmann
- L-TMD: lattice simulations; direct connector as gauge link, no regularization parameters, no light-like gauge links $\rightarrow$ Haegler, Musch
- $\sqrt{\ }$-TMD: combination of off- and on-light-like gauge links with their square roots $\rightarrow$ Collins; Idilbi, Scimemi

TMD Definitions
Recent development

Collins (2011, 2012, 2013); Collins, Rogers (2011, 2012); Aybat, Collins, Qui, Rogers (2011), García-Echevarría, Idilbi, Scimemi (2011, 2012), ICh, Stefanis (2011)...

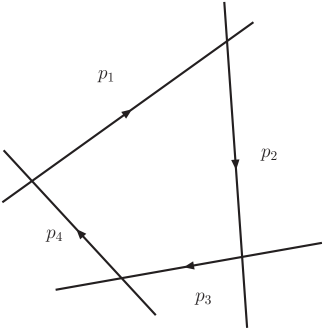
So it works, but:

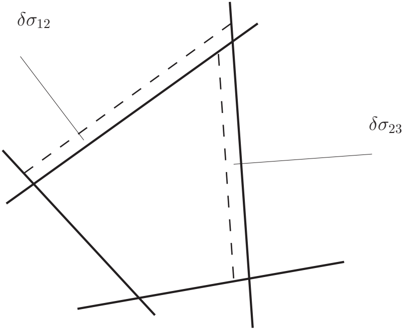
- too complicated and "arbitrary" soft factors;
- problems with reduction to the integrated PDF;
- too many "evolutions";
- connections between different approaches...

$\rightarrow$ looking for more "elegant"' ideas: singularity structure of the cusped light-like Wilson lines/loops.

Singularities of Light-like Cusped Wilson Loops

Generic light-like quadrilateral contour





Singularities of Light-like Cusped Wilson Loops

Generic light-like quadrilateral contour

Motivation: **duality** between 4-gluon planar scattering amplitude in $\mathcal{N} = 4$ SYM and the Wilson loop made up from four light-like segments:

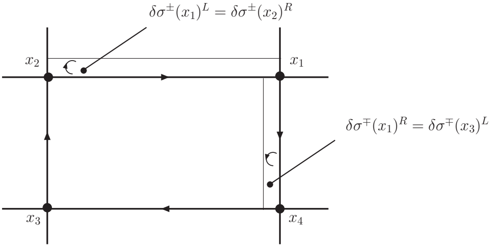
$$x_i - x_{i+1} = p_i$$

are equal to the external momenta of this 4-gluon amplitude. The IR evolution of the former is dual to the UV evolution of the latter: governed by the cusp anomalous dimension.

[Alday, Maldacena (2007); Makeenko (2003); Korchemsky, Drummond, Sokatchev (2008); Alday, Eden, Korchemsky, Maldacena, Sokatchev (2011); Beisert et al. (2012); Belitsky (2012)]

Singularities of Light-like Cusped Wilson Loops

Planar rectangular contour



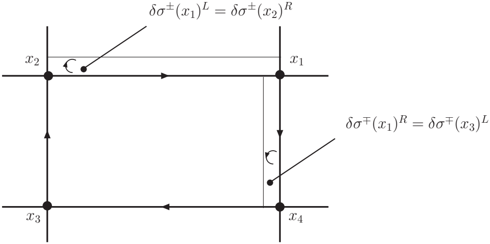
$$\sigma \equiv 2N^+N^- \ , \ \lim_{N_c \to \infty} \mathcal{W}[\Gamma] =$$

$$1 + \frac{\alpha_s N_c}{2\pi} \left\{ -\frac{1}{\epsilon^2} \left(\left[-\sigma\mu^2 + i0 \right]^{\epsilon} + \left[\sigma\mu^2 + i0 \right]^{\epsilon} \right) + \text{finite} \right\} + O(\alpha_s N_c)$$

[Korchemskaya, Korchemsky (1992); Bassetto, Korchemskaya, Korchemsky, Nardelli (1993)]

Singularities of Light-like Cusped Wilson Loops

Planar rectangular contour



$\mathcal{W}[\Gamma]$ is not multiplicatively renormalizable due to light-cone extra divergences—dual to the TMD case.

However, the area logarithmic derivative does the job:

$$\frac{d \ln \mathcal{W}[\Gamma]}{d \ln \sigma} = -\frac{\alpha_s N_c}{2\pi} \frac{1}{\epsilon} \left([\sigma \mu^2 + i0]^\epsilon - [-\sigma \mu^2 + i0]^\epsilon \right)$$

Singularities of Light-like Cusped Wilson Loops

Planar rectangular contour

Anomalous dimension results from (large-N_c):

$$\mu \frac{d}{d\mu} \frac{d \ \mathcal{W}[\Gamma]}{d \ln \sigma} \sim -4 \ \Gamma_{\rm cusp} \cdot \mathcal{W}[\Gamma] \ , \ \Gamma_{\rm cusp} = \frac{\alpha_s N_c}{2\pi}$$

We get finite result by means of the area derivative: **dynamical properties** of the light-like Wilson loop are encoded in the cusp anomalous dimension

[Korchemsky, Radyushkin (1987)]

Local quantity: behavior in vicinity of an obstruction.
Path-dependence shows up in finite terms. We related the **geometry** of the loop space (area differentials) and the **dynamics** of the fundamental d.o.f., that is the light-like Wilson loops.

Loop Space

Makeenko-Migdal approach

Wilson loops as the (fundamental) gauge-invariant degrees of freedom:

$$\mathcal{W}_n[\Gamma_1, ...\Gamma_n] = \langle 0|\mathcal{T}\frac{1}{N_c}\Phi(\Gamma_1)\cdots\frac{1}{N_c}\Phi(\Gamma_n)|0\rangle$$

$$\Phi(\Gamma_i) = \mathcal{P}\ \exp\left[ig\int_{\Gamma_i} dz^\mu \mathcal{A}_\mu(z)\right]$$

The Wilson functionals obey the Makeenko-Migdal loop equations:

$$\partial^\nu\ \frac{\delta}{\delta\sigma_{\mu\nu}(x)}\ \mathcal{W}_1[\Gamma] = N_c g^2\ \oint_\Gamma\ dz^\mu\ \delta^{(4)}(x-z)\mathcal{W}_2[\Gamma_{xz}\Gamma_{zx}]$$

The equation is exact and non-perturbative, but not closed and difficult to solve in general

[Polyakov (1979); Makeenko, Migdal (1979, 1981); Kazakov, Kostov (1980); Brandt, Neri, Sato (1981); Brandt, Gocksch, Sato, Neri (1982); Stefanis et al. (1989, 2003)]

Loop Space

Makeenko-Migdal approach

Area derivative:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\Phi(\Gamma) = \lim_{|\delta\sigma_{\mu\nu}(x)|\to 0} \frac{\Phi(\Gamma\delta\Gamma_x) - \Phi(\Gamma)}{|\delta\sigma_{\mu\nu}(x)|}$$

Path derivative:

$$\partial_\mu\Phi(\Gamma) = \lim_{|\delta x_\mu|\to 0} \frac{\Phi(\delta x_\mu^{-1}\Gamma\delta x_\mu) - \Phi(\Gamma)}{|\delta x_\mu|}$$

Mandelstam formula:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\mathrm{Tr}\ \Phi(\Gamma) = ig\mathrm{Tr}\ [F_{\mu\nu}\Phi(\Gamma)]$$

No information about cusps, divergences, etc.

Loop Space

Makeenko-Migdal approach

$$\mathcal{W}[\Gamma] = \mathcal{W}^{(0)} + \mathcal{W}^{(1)} = 1 - \frac{g^2 C_F}{2} \oint_\Gamma \oint_\Gamma dz_\mu dz'_\nu \, D^{\mu\nu}(z - z') + O(g^4)$$

$$D^{\mu\nu}(z - z') = -g^{\mu\nu} \, \Delta(z - z')$$

$$\Delta(z - z') = \frac{\Gamma(1-\epsilon)}{4\pi^2} \frac{(\pi\mu^2)^\epsilon}{[-(z-z')^2 + i0]^{1-\epsilon}}$$

$$\frac{\delta \, \mathcal{W}[\Gamma]}{\delta \sigma_{\mu\nu}} == \frac{g^2 C_F}{2} \frac{\delta}{\delta \sigma_{\mu\nu}} \oint_\Gamma \oint_\Gamma dz_\lambda dz'^\lambda \, \Delta(z - z') + O(g^4)$$

Loop Space

Makeenko-Migdal approach

Use the Stokes theorem

$$\oint_\Gamma dz_\lambda \ \mathcal{O}^\lambda = \frac{1}{2}\int_\Sigma \ d\sigma_{\lambda\rho}(\partial^\lambda \mathcal{O}^\rho - \partial^\rho \mathcal{O}^\lambda) \ , \ \mathcal{O}^\lambda = \oint_\Gamma dz^\lambda \ \Delta(z)$$

$$\partial_\mu \frac{\delta \ \mathcal{W}[\Gamma_{\rm smooth}]}{\delta\sigma_{\mu\nu}(x)} = \frac{g^2 N_c}{2} \oint_{\Gamma_{\rm smooth}} \ dy^\nu \ \delta^{(\omega)}(x-y) + O(g^4)$$

Example: 2D QCD

$$\mathcal{W}[\Gamma_{\rm smooth}]^{\rm 2D} = \exp\left[-\frac{g^2 N_c}{2}\Sigma\right] \ , \ \Sigma = \ \text{area inside } \Gamma_{\rm smooth}$$

Loop Space

Makeenko-Migdal approach

Problems:

- Most interesting loops are divergent and have obstructions: we are particularly interested in **cusped** loops. In that case, renormalized version of the MM equation is not available.
- The area functional derivative is not well-defined operation for arbitrary contour. In particular, the area differentiation for cusped loops is not (at least) straightforward.
- Problems with continuous deformation of the loops in the Minkowski space: consistent definition of the derivatives obscure.
- Connection of the loop functionals to observables.
- Solution of the MM equations in the four-dimensional Minkowskian space-time is not known.

Loop Space

Makeenko-Migdal approach

Simplifications:

- Large-N_c limit: factorization property
  $W_2(C_1, C_2) \approx W_1(C_1) \cdot W_2(C_2)$
- Null-plane light-cone rectangular contours are effectively two-dimensional
- Light-like polygons with conserved angles: no angle-dependent contributions which may break MM-equation
- Area differentiation: the power of divergency decreases

Therefore, the MM approach relates cusp dynamics, renormalization properties and geometry of the loop space. The problem now is how to extract reliable information.

Loop Space

Schwinger approach

[Schwinger (1951)]

Fundamental quantum dynamical principle

$$\delta\langle\ '\ |\ ''\ \rangle = \frac{i}{\hbar}\langle\ '\ |\delta S|\ ''\ \rangle$$

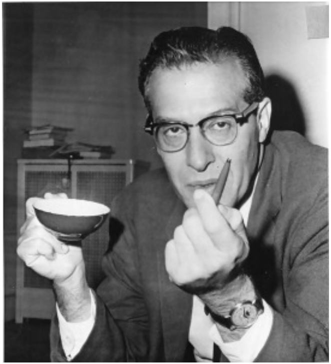
Application to the Wilson functionals $\Phi(\Gamma)$ + Mandelstam formula + Stokes theorem yields MM Eq.

$$\langle 0|\nabla_\mu F^{\mu\nu}\ \mathrm{Tr}\Phi(\Gamma)|0\rangle = i\hbar\langle 0|\frac{\delta}{\delta A_\nu}\ \mathrm{Tr}\Phi(\Gamma)|0\rangle$$

$$\partial^\nu\ \frac{\delta}{\delta\sigma_{\mu\nu}(x)}\ \mathcal{W}_1(\Gamma) = N_c g^2\ \oint_\Gamma\ dz^\mu\ \delta^{(4)}(x-z)\mathcal{W}_2(\Gamma_{xz}\Gamma_{zx})$$

Loop Space

Schwinger warning



Did you take care of singularities?

Loop Space

Shape variations without Stokes theorem

$$\mathcal{W}^{(1)}[\Gamma_{\Box}] = \frac{g^2 C_F}{2} \frac{\Gamma(1-\epsilon)(\pi\mu^2)^\epsilon}{4\pi^2} \cdot$$

$$\sum_{i,j} (v_j^\lambda v_j^\lambda) \cdot \int_0^1 \int_0^1 \frac{d\tau d\tau'}{[-(x_i - x_j - \tau_i v_i + \tau_j v_j)^2 + i0]^{1-\epsilon}}$$

$$2(v_1 v_2) = 2N^+ N^- \ , \ \frac{\delta}{\delta \ln \sigma} \equiv \sigma_{+-} \frac{\delta}{\delta \sigma_{+-}} + \sigma_{-+} \frac{\delta}{\delta \sigma_{-+}}$$

$$\frac{\delta \ \mathcal{W}[\Gamma_{\Box}]}{\delta \sigma_{\mu\nu}} = -\frac{\alpha_s N_c}{2\pi} \Gamma(1-\epsilon)(\pi\mu^2)^\epsilon \frac{\delta}{\delta \sigma_{\mu\nu}} (-2N^+ N^-)^\epsilon \ \frac{1}{2} \int_0^1 \int_0^1 \frac{d\tau d\tau'}{[(1-\tau)\tau']^{1-\epsilon}}$$

$$\mu \frac{d}{d\mu} \left[\frac{\delta}{\delta \ln \sigma} \ \ln \ \mathcal{W}[\Gamma_{\Box}] \right] = -\sum \Gamma_{\rm cusp}$$

Loop Space

Makeenko-Migdal approach / preliminary!

Non-Abelian exponentiation

[Gatheral (1983); Frenkel, Taylor (1984); Korchemsky, Radyushkin (1987)]

$$W(\Gamma; \epsilon; g; s, t) = \exp\left[\sum_{k=1} \alpha_s^k \; C_k(W) F_k(W)\right] \; , \; C_k \sim C_F \; N_c^{k-1} \to \frac{N_c^2}{2}$$

Perturbative expansion of the MM equation:

$$\mu \frac{d}{d\mu} \; \frac{dW}{d \ln s} = Z \; \alpha_s \; W$$

$$W(\epsilon; g; s, t) = 1 + \alpha_s C_1 F_1 + \alpha_s^2 \left(C_2 F_2 + \frac{1}{2!} C_1^2 F_1^2 \right) + O(\alpha_s)$$

—a closed chain of perturbative equations:

$$C_1 \frac{dF_1}{d \ln s} = Z(\epsilon; s, t) \alpha_s \; , \; C_2 \frac{dF_2}{d \ln s} = Z \; C_1 F_1 - \frac{1}{2!} C_1^2 \frac{dF_1^2}{d \ln s} \; \ldots$$

$Z(\epsilon; s, t)$ is universal factor related to the cusp anomalous dimension

[Cherednikov, Mertens, Van der Veken (2013) [in preparation]]

Loop Space

Schwinger approach for light-like planar contours

Return to the definition of the area derivative and consider special area differentials (do not make use of the Stokes theorem or Mandelstam formula)
Take into account renormalization group invariance

$$\mu \frac{d}{d\mu} \left[\sigma_{\mu\nu} \frac{\delta}{\delta \sigma_{\mu\nu}} \mathcal{W}[\Gamma] \right] = - \sum \Gamma_{\text{cusp}} \cdot \mathcal{W}[\Gamma]$$

Works for the rectangular light-like Wilson loop in the null-plane;
Works for the TMD "on the light-cone":

$$\mu \frac{d}{d\mu} \left[\frac{d}{d \ln \theta} \ \ln \ \Phi(k^+, k_\perp) \right] = 2\Gamma_{\text{cusp}}$$

→ complete evolution of the TMDs + further development...

Outlook I:

- Makeenko-Migdal approach provides a full and consistent description of the geometrical properties of the loop space. Fundamental degrees of freedom are closed Wilson loops and the MM Eqs. resemble the Schwinger-Dyson Eqs. in the loop space. In general, the system of the MM Eqs. is not closed and cannot be straightforwardly applied to calculate any useful quantity.

- However, in the large-N_c limit, in the null-plane $z_\perp = 0$, for the rectangular planar light-like Wilson loops, the area functional derivative is reduced to the normal derivative for the dimensionally regularized (not renormalized!) loops and the MM Eqs. appear to be equivalent to the energy/rapidity evolution equations.

Outlook II:

- Geometrical properties of the Wilson loop space provide a hint for understanding singularities and evolution of gauge invariant quantum field correlators with light-like and off-light-cone Wilson lines and loops with cusps and self-intersectons (collinear PDFs, TMDs, high-energy amplitudes, heavy quarks, etc.)
- To relate geometrical properties of the loop space and the dynamics encoded in cusps is a challenge. The cusps are introduced by externally-driven obstructions of (initially) smooth Wilson loops.
- Conjecture: since the quantum dynamical Schwinger approach is universal, it can be applied to construction of the energy/rapidity evolution equations in many interesting situations. Specific properties of the Wilson loops are determined by the contours with cusps (and/or self-intersections).

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